Determination of Natural Frequencies of an Engine Crankshaft Using Finite Elements

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Abstract

To get accurate natural frequencies of an engine crankshafts, finite element equations of motion are developed, taking real geometries of the shaft into account. For the crankshaft with wide crank webs, a specialized rotating web element is developed. This includes the effects of rotary inertia, gyroscopic moment, and shear. After the finite element equations are constructed, eigenvalues are extracted from the system equations to get natural frequencies, based on the Sturm sequence method which exploits the banded forms of the system matrices to reduce computations. The scheme developed can be used for the free vibration analysis of any type of spinning structures which include skew symmetric gyroscopic moment matrix in the system matrices. The results are compared with experimental data in order to confirm the study.

I. Introduction

Many papers have reported that the main source of unpleasant cabin noise in passenger cars usually originates from crankshaft vibrations[1]. Crankshafts are subject to many different forces during operation[2], and a accurate dynamic model of the shafts is indispensable to study noise and vibration problems. There are various types of crankshafts. These depend on specific requirements of engines for different applications. Since each crankshaft has its peculiar characteristics, it is impossible to apply the acquired characteristics in the design of other engines of different size, power, speed, and operating conditions. One of most important characteristics in crankshaft design is critical speeds, which are natural frequencies of the crankshaft vibration. In general, natural frequencies of a crankshaft were estimated by reducing the crankshaft to a pure torsional system according to rules of thumb, and were approximated as shaft portions and disks using Holzer's method[3, 4]. Due to the complex geometry, investigations of crankshafts considering real geometries are limited. Most of research used simplified models applying Myklestad method. Bagci[5] determined natural frequencies of crankshafts using spatial finite line element, but his element is not suitable for a crankshaft with wide crank webs and the gyroscopic moment effect is not considered, which may be considerable in high speed engines. Bargis et al[6] investigated crankshaft design and evaluation methods based on critical analysis, experiments, modal analysis, and direct integration as a series. Recently, the works of Okamura and other Japanese researchers[1] were reported, which are on three dimensional vibration of automobile engine crankshafts. They used a dynamic stiffness approach and simplified crank web geometry as a rectangular beam. They neglect the effects of gyroscopic moment and shear deformation in constructing the equations of motion.

In this study, for the crankshaft with wide crank webs, a specialized finite element, rotating web element, is developed to get the natural frequencies, considering the real geometry and the effects of rotary inertia, gyroscopic moment, and shear. The results are compared with experimental data in order to confirm the scheme developed.

II. Modelling

An engine crankshaft body can be modelled by rotating shaft and rotating web elements. Main and crank pin journals are modelled as three dimensional rotating shaft elements and crank webs are modelled as rotating web elements developed in this study.

The shaft element is considered to be initially straight and the cross section is circular and modelled as a 2-node element of length L[7]. Each node has 5 degrees of freedom, 2 translations and 3 rotations. The finite element equations are represented as

$$[M]_{eq}\{\ddot{q}\}-\Omega[G]\{\dot{q}\}+[K]\{q\}=\{R\}_{eq} \tag{1}$$

where

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$$[M]_{eq} = \rho A \int_0^L N_t^T N_t dx + \rho I \int_0^L N_r^T N_r dx + \rho I_p \int_0^L N_\psi^T N_\psi dx$$

$$[G] = \int_0^L (N_\theta^T N_\phi - N_\phi^T N_\theta) dx$$

$$[K]_{eq} = EI \int_0^L B_r^T B_r dx + kAG \int_0^L B_s^T B_s dx + GI_p \int_0^L B_\psi^T B_\psi dx$$

ρ, A, I, I_p, and Ω represent density, cross sectional area, moment of inertia, polar moment of inertia, and rotating speed, respectively. E and G are Young's modulus and shear modulus. k is shear constant. Matrices N_i and B_i represent shape functions and their derivatives, $i = t, r, \psi, \theta, \phi, s$.

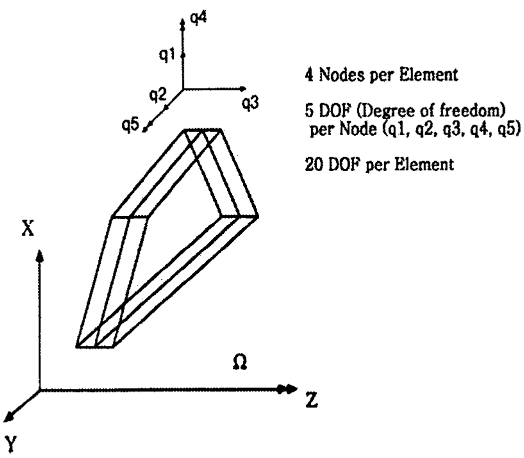

Figure 1. Rotating web element.

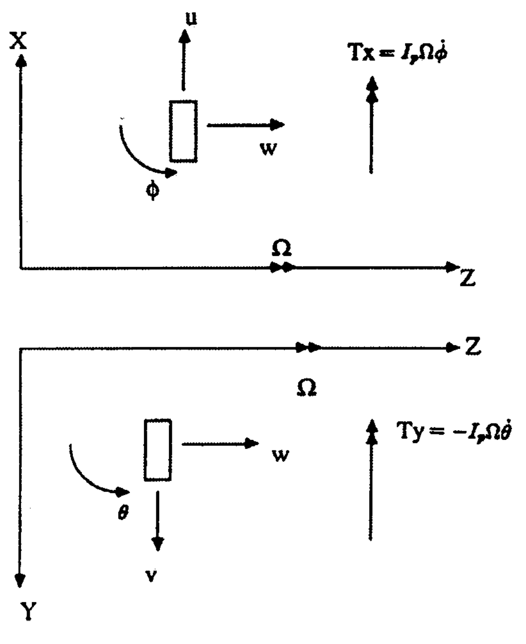

Figure 2. Free body diagram for the rotating web element.

Fig. 1 shows a typical rotating web element, of which free body diagrams in the x-z plane and the y-z plane are shown in Fig. 2, the element rotates with respect to z-axis. When the element rotates at a high speed, gyroscopic moment is not negligible. From the free body diagrams, we can get gyroscopic moment, T_x and T_y, from the right hand rule of mechanics.

$$T_x = \rho I_p \Omega \dot{\phi} \qquad T_y = -\rho I_p \Omega \dot{\theta} \tag{2}$$

Then the external virtual work due to the gyroscopic moments, δw_g, becomes

$$\begin{aligned} \delta w_g = & \rho I_p \Omega \left(\int_0^V \dot{\phi} \delta\theta dV - \int_0^V \dot{\theta} \delta\phi dV \right) \\ & = \rho I_p \Omega \left(\int_0^V \dot{w}_{,y} \delta w_{,x} dV - \int_0^V \dot{w}_{,x} \delta w_{,y} dV \right) \end{aligned} \tag{3}$$

From Figure 1, the coordinates of any points in the element are

$$x = \sum_{i=1}^4 N_i x_i \quad y = \sum_{i=1}^4 N_i y_i \quad z = \sum_{i=1}^4 N_i z_i + \sum_{i=1}^4 N_i \zeta \frac{t_i}{2} \tag{4}$$

x_i, y_i, z_i, and t_i are coordinates and thickness at the nodes of the element, ξ, η, and ζ are the natural coordinates in the x, y, and z direction, N_i's are shape or interpolation functions. In this study, the shape functions of the plane linear isoparametric element are adapted[7]. Then, this model assures convergence, because trial displacement functions satisfy the compatibility criterion and the completeness criterion for convergence.

Generic displacements in terms of nodal displacements are, for i = 1, 2, 3, 4,

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} N_i & 0 & 0 & 0 & N_i \zeta t_i/2 \\ 0 & N_i & 0 & -N_i \zeta t_i/2 & 0 \\ 0 & 0 & N_i & 0 & 0 \end{bmatrix} \begin{Bmatrix} q_{1i} \\ q_{2i} \\ q_{3i} \\ q_{4i} \\ q_{5i} \end{Bmatrix} \tag{5}$$

Jacobian matrix and its inverse are

$$[J] = \begin{bmatrix} x_{,\xi} & y_{,\xi} & z_{,\xi} \\ x_{,\eta} & y_{,\eta} & z_{,\eta} \\ x_{,\zeta} & y_{,\zeta} & z_{,\zeta} \end{bmatrix}, \quad [J]^{-1} = \begin{bmatrix} \eta_{,x} & \xi_{,x} & 0 \\ \eta_{,y} & \xi_{,y} & 0 \\ 0 & 0 & \zeta_{,z} \end{bmatrix} = [J]^* \tag{6}$$

$$\begin{Bmatrix} u,_x \\ u,_y \\ u,_z \\ v,_x \\ v,_y \\ v,_z \\ w,_x \\ w,_y \\ w,_z \end{Bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} a_i & 0 & 0 & 0 & 0 \\ b_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_i \\ 0 & a_i & 0 & 0 & 0 \\ 0 & b_i & 0 & 0 & 0 \\ 0 & 0 & 0 & -g_i & 0 \\ 0 & 0 & a_i & 0 & 0 \\ 0 & 0 & b_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} q_{1i} \\ q_{2i} \\ q_{3i} \\ q_{4i} \\ q_{5i} \end{Bmatrix} \tag{7}$$

where

$$a_i = J_{11}^* N_{i,\xi} + J_{12}^* N_{i,\eta}\ ,\quad b_i = J_{21}^* N_{i,\xi} + J_{22}^* N_{i,\eta}\ ,\quad g_i = \frac{t_i}{2} J_{33}^* N_i$$

Strains are expressed as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} u,_x \\ v,_y \\ w,_z \\ u,_y + v,_x \\ v,_z + w,_y \\ w,_x + u,_z \end{Bmatrix} = [B]\{q\} \tag{8}$$

where

$$[B] = [B_1 \quad B_2 \quad B_3 \quad B_4]\ ,\quad B_i = B_{Ai} + \zeta B_{Bi}\ ,\quad i = 1, 2, 3, 4$$

$$B_{Ai} = \begin{bmatrix} a_i & 0 & 0 & 0 & 0 \\ 0 & b_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & a_i & b_i & 0 & 0 \\ 0 & 0 & b_i & 0 & 0 \\ 0 & 0 & a_i & 0 & 0 \end{bmatrix},\quad B_{Bi} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{t_i}{2} a_i \\ 0 & 0 & 0 & -\frac{t_i}{2} b_i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{t_i}{2} a_i & -\frac{t_i}{2} b_i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In the finite element method , generic displacements are expressed as

$$\{u(t)\} = [N]\{q(t)\} \tag{9}$$

and strain vector is

$$\{\varepsilon(t)\} = [B]\{q(t)\} \tag{10}$$

Then time varying stress vector becomes

$$\{\sigma(t)\} = [E][B]\{q(t)\} \tag{11}$$

From the virtual work principal

$$\delta u_e = \delta w_e \tag{12}$$

where δu_e is the strain energy of internal stresses, and δw_e is the virtual work of external action on the element.

$$\delta u_e = \int_0^V \delta\varepsilon^T \sigma(t)\, dV \tag{13}$$

$$\delta w_e = \delta q^T p(t) + \int_0^V \delta u^T b(t)\, dV - \int_0^V \delta u^T \rho \ddot{u}\ dV + \delta w_g$$

where δw_g is the external virtual work due to gyroscopic moment. Then from equation(12)

$$\int_0^V \delta\varepsilon^T \sigma(t)\, dV = \delta q^T p(t) + \int_0^V \delta u^T b(t)\, dV - \int_0^V \delta u^T \rho \ddot{u}\ dV + \delta w_g \tag{14}$$

Then from equations(9) to (12)

$$\begin{aligned} \{\delta q\}^T & \int_0^V [B]^T [E][B] dV \{q(t)\} \\ &= \{\delta q\}^T p(t) + \{\delta q\}^T \int_0^V [N]^T \{b(t)\} dV \\ &\quad - \{\delta q\}^T \int_0^V \rho [N]^T [N] dV \{\ddot{q}(t)\} \\ &\quad + \{\delta q\}^T \rho I_p \Omega \int_0^V [H_1]^T [H_2] - [H_2]^T [H_1] dV \{\dot{q}(t)\} \end{aligned} \tag{15}$$

Then,

$$\begin{aligned} &\int_0^V \rho [N]^T [N] dV \{\ddot{q}(t)\} - \rho I_p \Omega \int_0^V [H_1]^T [H_2] - [H_2]^T [H_1] dV \{\dot{q}(t)\} \\ &+ \int_0^V [B]^T [E][B] dV \{q(t)\} = p(t) + \int_0^V [N]^T \{b(t)\} dV \end{aligned} \tag{16}$$

and

$$\begin{aligned} [M] &= \int_0^V \rho [N]^T [N] dV \\ [G] &= \rho I_p \Omega \int_0^V [H_1]^T [H_2] - [H_2]^T [H_1] dV \\ [K] &= \int_0^V [B]^T [E][B] dV \end{aligned} \tag{17}$$

Then, the finite element equation of the motion for the rotating web element becomes

$$[M]\{\ddot{q}\} - \Omega [G]\{\dot{q}\} + [K]\{q\} = \{p(t)\} + \{p_{b(t)}\} \tag{18}$$

where [M], [G], [K], {p(t)} and {pb(t)} are mass matrix, gyroscopic moment matrix, stiffness matrix, applied force and body force vectors, respectively.

Each journal bearing can be modelled by a set of springs and dash pots at the journal center, as a point element with 3 degrees of freedom, 2 translations and 1 rotation. In expanded form, fluid film journal bearings can also be idealized by two or three sets of springs and dash pots at the end points of the journal, or the three points located equidistantly along the journal axis, respectively.

The equation of motion for the complete crankshaft structure can be obtained by assembling the appropriate element equations. This assembling is accomplished by relating the element coordinates to the chosen set of system coordinates through statements of displacement compatibility which insure the connectivity of the system. The configuration of the system equations is designed by generalized coordinates of order equal to the number of total node points of the system times the number of degrees of freedom per node. The finite element equation of motion of the crankshaft body becomes

$$[M]\{\ddot{q}\} - \Omega[G]\{\dot{q}\} + [K]\{q\} = \{R(t)\} \tag{19}$$

where [M],[G], and [K] are mass, gyroscopic moment, and stiffness matrices. They are highly banded, and the banded forms can be utilized to reduce the computational efforts. The eigenvalue problem for the system equations has to be solved to get critical speeds, or natural frequencies of the system. Algorithms for the eigenvalue problems of dynamic systems are abundant[8, 9].

III. Eigenvalue Problems

In practical dynamic analysis, we usually want the few lowest eigenvalues. We have sparse matrices with a great many degrees of freedom in finite element analysis, but the matrices are usually highly banded. In finding roots of the characteristic polynomial equation, there exist no closed form expressions for the solution if there are more than four roots, so all algorithms are iterative. In large systems such as finite element equations, the determinant search method can effectively be used to extract an eigenvalues at a time[7,8]. In the determinant search method, we can find a root of the characteristic equation, with which the determinant vanishes. The determinant search method requires no special form of the coefficient matrices, but the determinant must be scaled to prevent overflow, and it is easy to miss a root. The subspace iteration method can be extended to get all eigenvalues in a specified interval, where load patterns should be chosen, so that a linear combination of the deformation patterns will probably represent important modes. In most of iterative methods for eigenvalues, prior knowledge of root distribution is required. By the way, the Sturm sequence property is to find the number of eigenvalues in an arbitrary range, and it can be utilized to bracket a root of the characteristic equation as closely as desired, combined with the determinant search method and other techniques such as the bisection technique.

In this study, a generalized eigenvalue algorithm based on the Sturm sequence property and the bisection technique is developed for the efficient critical analysis of spinning structures. The routine is numerically stable, and fully automatic in nature, so that no prier knowledge of root distribution or mode shapes is required for the eigenproblem solution under consideration, different from most of iterative methods. The scheme can be used in free vibration analysis of any type of spinning structures which include skew symmetric gyroscopic moment matrix in their system equations. The few lowest roots in a certain range are isolated using the Sturm sequence property of the characteristic equations, and the bisection technique, which assures convergence, is used to get the associated roots. The bisection technique involves the simultaneous determination of upper and lower limits of all relevant roots at any particular step. During such procedure successively, smaller bounds for the associated roots are achieved.

We have governing equations for free motions

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{0\} \tag{20}$$

Then

$$\begin{bmatrix} 0 & -M \\ M & C \end{bmatrix}\begin{Bmatrix} \ddot{q} \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix}\begin{Bmatrix} \dot{q} \\ q \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{21}$$

which is written as

$$A\dot{y} + By = 0 \tag{22}$$

When M and K are symmetric and C is skew symmetric, A is skew symmetric and B is symmetric, and both are usually highly banded.

Let

$$y = e^{pt} \tag{23}$$

Then

$$(B + pA)y = 0 \tag{24}$$

Let

$$p = \frac{1}{\omega i} \tag{25}$$

where ω is real. Then

$$(A^* - \omega B)y = 0 \tag{26}$$

where $A^* = Ai$ is a Hermitian matrix. To get a non-trivial solution,

$$det(A^* - \omega B) = 0 \quad (27)$$

In the eigenvalues of equation(27), ω will be in pairs, $\omega_1, -\omega_1, \omega_2, -\omega_2, \cdots, \omega_m, -\omega_n$ where n is the order of M, G, and K. Then p will be a pure imaginary number $1/\omega i$ as in equation(25), and appears in complex conjugate pairs. In finding roots equation(27), the Sturm sequence property is used to isolate the roots.

$$p(\omega) = det(A^* - \omega B) = 0 \quad (28)$$

Let $p_r(\omega)$ denote the determinant of the leading principal minors of $(A^* - \omega B)$. If we define $p_0 = 1$, then

$$p_1(\omega) = d_1 - \omega \quad (29)$$

where d_1 is the first diagonal element. Let $s(\lambda)$ denote the number of the sequence in signs between successive members of the sequence $\{p_r\}$, for example, $+ + - +$ gives $s = 1$. If p_r is zero then its sign is taken to be that of p_{r-1}. Then from the Sturm sequence property, the number of agreements in sign, $s(\lambda)$ of successive minors of the sequence $\{p_r(\lambda)\}$ is equal to the number of eigenvalues or roots of the characteristic polynomial equation which are strictly greater than λ. Thus if we find that

$$s(\lambda_1) = k_1, \quad s(\lambda_2) = k_1 + 1, \quad \lambda_1 > \lambda_2 \quad (30)$$

it follows that there is only one eigenvalue in the range $[\lambda_2, \lambda_1]$. After the roots in a certain range are isolated, they are approximated using the bisection technique. The algorithm developed here is efficient and stable for the determination of eigenvalues of the high speed spinning structures, which include skew symmetric gyroscopic moment matrices in their system equations.

IV. Results

To confirm the results, the analysis procedure is applied to the crankshaft of a single cylinder engine, which is the same as in reference[1]. Fig. 3 (a) and (b) show a brief sketch of the single cylinder engine crankshaft. The simplified discretization of the crank web is also shown in Fig. 3 (c). The natural frequencies of an engine crankshaft can be classified as in-plane and out-of-plane modes. Fig. 4 and 5 show the measured and predicted natural frequencies of in-plane and out-of-plane modes, respectively. Circle points represent the predicted natural frequencies and asterisk points represent the measured natural frequencies. Since a significant coincidence between the predicted and the measured results is shown, the basic procedure is confirmed.

In Fig. 6, the natural frequencies of the single cylinder engine crankshaft in both modes are shown. The difference between the modes is dependent upon the crankshaft geometry. In-plane and out-of-plane modes can be independently induced in the case of planar shape crankshafts. When the crank webs phase three dimensionally, any deflection in crank throw plane may induce deflection in other orthogonal planes. Thus, in-plane and out-of-plane modes would not be induced independently.

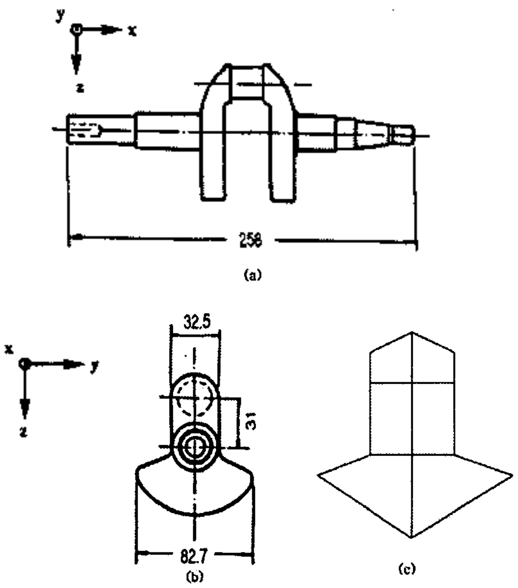

Figure 3. A single cylinder engine crankshaft (a), (b) unit; mm and discretization of the crank web (c).

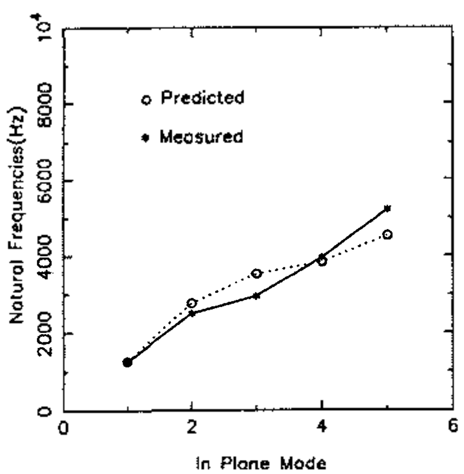

Figure 4. In-plane mode natural frequencies.

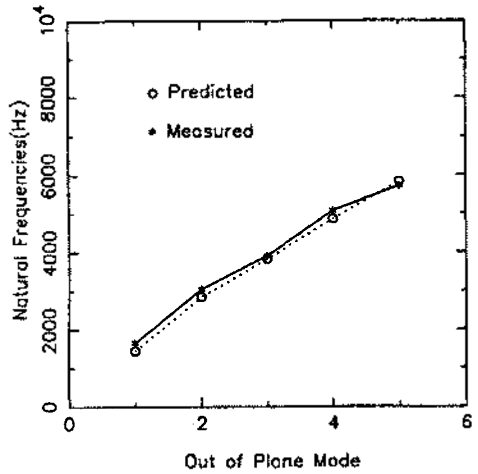

Figure 5. Out-of-plane mode natural frequencies

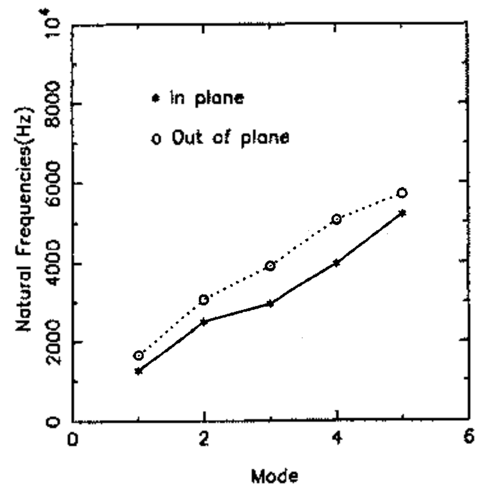

Figure 6. Natural frequencies of single cylinder engine crankshaft.

V. Conclusion

In this study, a dynamic finite element model of a high speed engine crankshaft is developed, taking the real geometry into account. For a crankshaft with wide webs, a specialized web element is developed. This study includes the effects of rotary inertia, gyroscopic moment, and shear. The eigenvalues are extracted based on the Strum sequence method. The eigenvalue solver can be used for the free vibration analysis of any type of high speed spinning structures which include symmetric mass and stiffness matrices and the skew symmetric gyroscopic matrix in the system matrices.

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