▩ 연구논문

Extended EPQ Model and Its Applications to MPS and MRP
- 확장 EPQ 모델과 MPS 및 MRP에의 응용 -

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요 지

지금까지 전통적인 EPQ(Economic Production Quantity) 모델에서는 생산율이 수요율보다 큰 경우만을 다루어 왔으나, 실제 시장에서는 그 반대로 수요율이 생산율보다 큰 경우도 종종 발생한다. 그러나 이러한 상황에 대한 EPQ 연구는 찾아보기 어려운 실정이다. 따라서 본 논문에서는 수요율이 생산율보다 큰 상황을 고려한 확장된 EPQ 모델(Extended EPQ Model)을 유도하고, 이를 생산재고의 유한보충률 상황(Finite Replenishment Rate Environment)에서의 MPS(Master production Schedule)와 MRP(Material Requirements Planning)에 적용하였다.

1. Introduction

The EOQ(Economic Order Quantity) model describes a typical situation met by wholesalers: a large delivery of an item instantaneously raises the stock level and then a series of smaller demands slowly reduces it. However, when the finished goods are stocked at the end of a production line and production rate is greater than demand rate, goods will accumulate at a finite rate while the line is operating. This gives a situation where the instantaneous replenishment of the EOQ model is replaced by a finite replenishment rate. A similar pattern is met with stocks of work in process between two machines: the first machine builds up stock at a finite rate while the second machine creates demand to reduce it.

On the other hand, in the classic EPQ(Economic Production Quantity) model it is assumed that production rate is always greater than demand rate. However, the opposite case is often occurred in any real situation and there have been no researches on the inventory models and decision analysis dealing with that case. The extended EPQ model including this idea is developed in the first part of this paper.

In addition, current lot sizing rules used in MPS(Master Production Schedule) and MRP(Material Requirement Planning) handle only instantaneous replenishment. However, when the goods are stocked at the end of a production line, they will accumulate at a finite rate while the line is operating. Therefore, the applications of the extended EPQ model to the MPS and MRP in the finite replenishment rate environment are necessary and provided in the second part of the paper.

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There have been so many research papers on the inventory models, but none of them deal with the situations and applications presented in this paper. Henig and gerchak[5] studied random yield in production. Yano and Lee[12] detailed a review in random yield studies. Ciarallo et al.[2] worked with variable capacity and Wang and Gerchak[11] considered a simultaneous incorporation of random yield and variable capacity.

Bielecki and Kumar[1] investigated the optimality of zero-inventory policies in production systems with uncertain capacity. Shogan[10], Meyer et al.[7], Posner and Berg[9], Gronevelt et al.[3,4], Moinzadeh and Aggarwal[8] studied cases of random failures, breakdowns and disruptions with lost sales. On the other hand, Kapuscinski and Tayur[6] investigated a maximum production capacity with possible backloggings. We believe that our extended EPQ model and its application to the MPS and MRP may provide the decision framework and further research opportunities for the given situation.

2. The Extended EPQ Model

2.1 Description of the Alternatives

Notation:

P = current, constant production rate in units per year

P' = increased, constant production rate in units per year

D = constant demand rate in net requirements per year

If production rate is less than demand rate, a firm may choose one of the possible reactions:

Alternative 1: Production without increasing current production rate ($P = P'$) and no subcontraction

Alternative 2: Production with increasing current production rate and no subcontraction

- Alternative 2-1: $P < P' < D$
- Alternative 2-2: $P' = D$
- Alternative 2-3: $P' > D$

Alternative 3: Subcontraction for the entire demand ($P' = 0$)

Alternative 4: Combination of the production without increasing current production rate ($P = P'$) and subcontraction for the remaining demand

Alternative 5: Combination of the production with increasing current production rate ($P < P' < D$) and subcontraction for the remaining demand

2.2 Derivation of the Optimal Lot Size

Notation:

Q_p = optimal production lot size (EPQ)

Q_s = optimal subcontraction(purchase) lot size (EOQ)

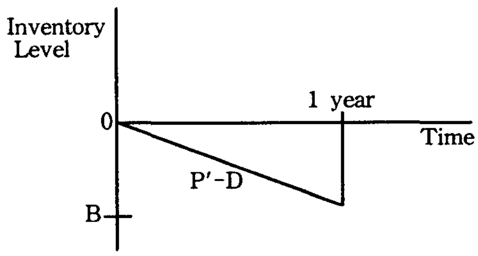
B = maximum level of backorders

I_{max} = maximum on-hand inventory level

A_p = setup cost for a production order
A_s = ordering cost for a subcontraction order
V_p = unit variable cost for production
V_s = unit variable cost for subcontraction
r = carrying cost rate (percentage of the carrying cost to the total inventory value)
V_r = carrying cost per unit per year
π = shortage cost per unit short, independent of the duration of the shortage (goodwill cost)
$\hat{\pi}$ = shortage cost per unit short per year
C_i = cost of increasing production rate by one unit
TRC = Total Relevant Cost per year

2.2.1 Alternative 1 - Production without increasing current production rate(P = P') and no subcontraction

1) Inventory Graph



2) TRC = Production-related costs + Backorder-related costs
= Setup cost + Unit cost for production + Goodwill cost + Shortage cost

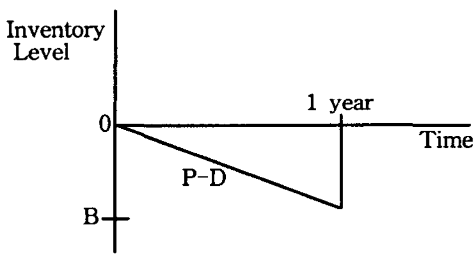
$$= \frac{A_p P}{Q_p} + PV_p + B\pi + \frac{B}{2}\hat{\pi}$$

3) Q_p = P, B = D - P

2.2.2 Alternative 2 - Production with increasing current production rate and no subcontraction

Alternative 2-1: P < P' < D

1) Inventory Graph



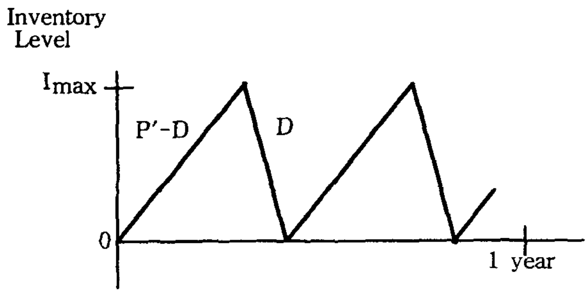
2) TRC = Production-related costs + Backorder-related costs
= Setup cost + Unit cost for production + Cost of increase + Goodwill cost + Shortage cost

$$= \frac{A_p P'}{Q_p} + P'V_p + C_i(P'-P) + B\pi + \frac{B}{2}\hat{\pi}$$

3) $Q_p = P'$, $B = D - P'$

Alternative 2-2: P′ = D

1) Inventory Graph



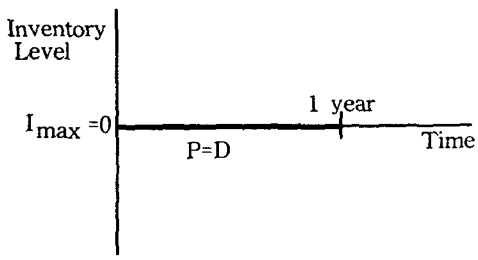
2) TRC = Production-related costs
= Setup cost + Unit cost for production + Cost of increase

$$= \frac{A_p P'}{Q_p} + P'V_p + C_i(D-P)$$

3) $Q_p = P' = D$

Alternative 2-3: P′ > D

1) Inventory Graph



2) TRC = Production-related costs
= Setup cost + Carrying cost + Unit cost for production + Cost of increase

$$= \frac{A_p D}{Q_p} + \frac{Q_p}{2} V_p r\left(1-\frac{D}{P'}\right) + DV_p + C_i(P'-P)$$

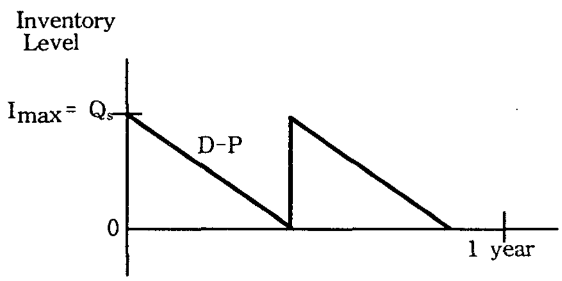
3) $\frac{dTRC}{dQ_p} = \frac{-AD}{Q_p^2} + \frac{V_p r}{2}(1-\frac{D}{P'}) = 0 \qquad \therefore Q_p = \sqrt{\frac{2A_pD}{V_p r(1-\frac{D}{P'})}}$

$\frac{d^2 TRC}{d\ Q_p^2} = \frac{2ADQ_p}{Q_p^4} > 0 \qquad \therefore Q_p$ is optimal.

$I_{max} = (P' - D)\frac{Q_p}{P'}$

2.2.3 Alternative 3 - Subcontraction for the entire demand (P' = 0)

1) Inventory Graph



2) TRC = Subcontraction (purchase)-related costs

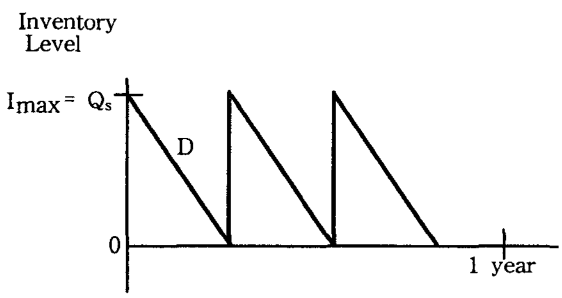
= Ordering cost + Carrying cost + Unit cost for subcontraction

$= \frac{A_sD}{Q_s} + \frac{Q_s}{2}V_s r + DV_s$

3) $Q_s = \sqrt{\frac{2A_sD}{V_s r}}$

2.2.4 Alternative 4 - Combination of the production without increasing current production rate (P = P') and subcontraction for the remaining demand

1) Inventory Graph



2) TRC = Production-related costs + Subcontraction-related costs

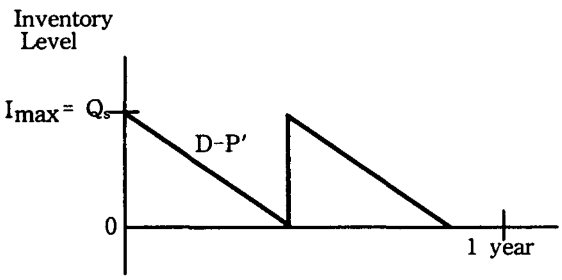
= Setup cost + Unit cost for production + Ordering cost + Carrying cost + Unit Cost for subcontraction

$$= \frac{A_pP}{Q_p} + PV_p + \frac{A_s(D-P)}{Q_s} + \frac{Q_s}{2}V_sr + (D-P)V_s$$

3) $Q_p = P,\ Q_s = \sqrt{\dfrac{2A_s(D-P)}{V_sr}}$

2.2.5 Alternative 5 - Combination of the production with increasing current production rate (P < P' < D) and subcontraction for the remaining demand

1) Inventory Graph



2) TRC = Production-related costs + Subcontraction-related costs
= Setup cost + Unit cost for production + Cost of increase + Ordering cost
+ Carrying cost + Unit cost for subcontraction

$$= \frac{A_pP}{Q_p} + C_i(P'-P) + \frac{A_s(D-P')}{Q_s} + \frac{Q_s}{2}V_sr + (D-P')V_s$$

3) $Q_p = P',\ Q_s = \sqrt{\dfrac{2A_s(D-P')}{V_sr}}$

3. Applications of the Extended EPQ Model to MPS and MRP Lot Sizing

The applications of the extended model to the time-phased schedules of MPS and MRP are divided by the following two cases:

Case 1. production rate > demand date

Case 2. production rate < demand rate.

Since the demand patterns in the MPS and MRP are discrete and lumpy, demand and production rates are converted to the average ones per time bucket for the assumption of constant demand. Hence, three more definitions are needed:

$\bar{D}$ = constant demand rate per time bucket

$\bar{P}$ = current, constant production rate per time bucket

$\bar{P}'$ = increased, constant production rate per time bucket

3.1 CASE 1 - Production Rate > Demand Rate

This is the general case where the classic EPQ model is applied. In this case, management may decide to make or buy the products based on the total cost comparison. Therefore, two alternatives can be considered.

Alternative 1. Make

$$Q_p = \sqrt{\frac{2A_p\bar{D}}{V_pr}}\sqrt{(1-\frac{\bar{D}}{P})}$$

Alternative 2. Buy

$$Q_s = \sqrt{\frac{2A_s\bar{D}}{V_sr}}$$

3.2 CASE 2 - Production Rate < Demand Rate

This is the case where the extended EPQ model is used. All the same alternatives are used with a slightly modified lot sizing formula.

Alternative 1: Production without increasing current production rate ($P = P'$) and no subcontraction ---> $Q_p = P$

Alternative 2: Production with increasing current production rate and no subcontraction

Alternative 2-1: $P < P' < D$ ---> $Q_p = P'$

Alternative 2-2: $P' = D$ ---> $Q_p = P' = D$

Alternative 2-3: $P' > D$ ---> $Q_p = \sqrt{\frac{2A_p\bar{D}}{V_pr(1-\frac{\bar{D}}{P'})}}$

Alternative 3: Subcontraction for the entire demand ($P' = 0$)

---> $Q_s = \sqrt{\frac{2A_sD}{V_sr}}$

Alternative 4: Combination of the production without increasing current production rate ($P = P'$) and subcontraction for the remaining demand

---> $Q_p = P,\ Q_s = \sqrt{\frac{2A_s(\bar{D}-\bar{P})}{V_sr}}$

Alternative 5: Combination of the production with increasing current production rate ($P < P' < D$) and subcontraction for the remaining demand

---> $Q_p = P',\ Q_s = \sqrt{\frac{2A_s(\bar{D}-\bar{P}')}{V_sr}}$

4. Solution Generation and Examples

In this section, three numerical examples for the models developed above are illustrated to help the understanding.

4.1 Example 1 - The Extended EPQ Model

The ABC Company manufactures metal window frames. The demand for a certain model is constant and known to be 20,000 units per year. The current production rate is 15,000 unit per year. The company may purchase the item from a subcontractor. For a manufactured lot, the setup cost is $150 and the unit variable cost is $1.75. For a purchased lot, the fixed cost is $25 and the unit variable cost is $2. A 20 percent annual inventory carrying cost rate is used. Shortages are backordered at the fixed costs of $0.5 per unit short and $5 per unit short per year. If the company increases production rate by one unit, it costs $1. Determine the optimal (economic) inventory policy for this model window frame. The solutions for this example are as follows:

D = 20,000 units/year, P = 15,000 units/year, A_p = \$150, A_s = \$25, V_p = \$1.75, V_s = \$2, r = 0.2, π = \$0.5/unit short, π = \$5/unit short/yr, C_i = \$1/unit

Alternative 1:

Q_p = 15,000, B = 20,000 - 15,000 = 5,000

$$TRC = \frac{(150)(18,000)}{(18,000)} + (18,000)(1.75) + (2,000)(0.5) + (\frac{2,000}{2})(5) + (1)(3,000) = \$41,400$$

Alternative 2-1:

P' = 18,000 units/year

Q_p = 18,000, B = 20,000 -18,000 = 2,000

$$TRC = \frac{(150)(18,000)}{(18,000)} + (18,000)(1.75) + (2,000)(0.5) + (\frac{2,000}{2})(5) + (1)(3,000) = \$40,650$$

Alternative 2-2:

P' = 20,000 units/year

Q_p = 20,000, B = 0

$$TRC = \frac{(150)(20,000)}{(20,000)} + (20,000)(1.75) + (1)(5,000) = \$40,150$$

Alternative 2-3:

P' = 20,000 units/year

$$Q_p = \sqrt{\frac{(2)(150)(20,000)}{(1.75)(0.2)(1-\frac{20,000}{23,000})}} = 11,464.2 \approx 11,465$$

$$TRC = \frac{(150)(20,000)}{11,465} + (20,000)(1.75) + (\frac{11,465}{2})(1.75)(0.2)(1-\frac{20,000}{23,000})$$
$$+ (1)(23,000-20,000)$$
$$= \$38,523$$

Alternative 3:

$$Q_s = \sqrt{\frac{(2)(25)(20,000)}{(2)(0.2)}} = 1,581.1 \approx 1,582$$

$$TRC = \frac{(25)(20,000)}{1,582} + (20,000)(2) + (\frac{1,582}{2})(2)(0.2)$$
$$= \$40,632$$

Alternative 4:

$$Q_p = 15,000$$

$$Q_s = \sqrt{\frac{(2)(25)(5,000)}{(2)(0.2)}} = 790.6 \approx 791$$

$$TRC = \frac{(150)(15,000)}{1,582} + (15,000)(1.75) + \frac{(25)(5,000)}{791} + (5,000)(2)$$
$$+ (\frac{791}{2})(2)(0.2)$$
$$= \$36,716$$

Alternative 5:

$$P' = 18,000 \text{ unit/year}$$
$$Q_p = 18,000$$

$$Q_s = \sqrt{\frac{(2)(25)(2,000)}{(2)(0.2)}} = 500$$

$$TRC = \frac{(150)(18,000)}{1,800} + (18,000)(1.75) + (1)(3,000) + \frac{(25)(2,000)}{500}$$
$$+ (2,000)(2) + (\frac{500}{2})(2)(0.2)$$
$$= \$38,850$$

Therefore, the best choice is the alternative 4 with the minimum TRC of $36,716 per year.

4.2 Example 2 - Application To The MPS When Production Rate Is Greater Than Demand Rate

P = 180 units/year, $\overline{P} = \frac{180}{12} = 15$ units/month

A_p = \$60, A_s = \$50, $V_p = V_s$ = \$20, r = 0.1

Purchase lead time = 0

Demand

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15

D = 5 + 5 + . . . + 15 + 15 + = 120 units/year

$\bar{D} = \frac{120}{12}$ = 10 units/month

Alternative 1. Make

$$Q_p = \sqrt{\frac{2(60)(10)}{10(0.1)}}\sqrt{(1-\frac{15}{10})} = 20$$

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15
Ending Inv.	10	10	5	0	10	10	10	0	0	0	0	0
MPS	15	5			15	5	15	5	15	15	15	15
POR	20				20		20		20	20	20	20

*POR = Planned Order Release

Setup Cost = (7)(60) = \$420

Carrying cost = $(\frac{10+10+5+10+10+10}{2})(10)(0.1)$ = \$27.5

Unit cost for production = (135)(10) = \$1,350

TRC = \$420 + \$27.5 + \$1,350 = \$1,797.5

Alternative 2. Buy

$$Q_s = \sqrt{\frac{2(50)(10)}{10(0.1)}} = 31.6 = 32$$

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15
Ending Inv.	27	22	17	12	7	2	19	4	21	6	23	8
MPS	32						32		32		32	
POR	32						32		32		32	

Ordering Cost = (4)(50) = \$200

Carrying cost = $(\frac{27+22+\cdots+23+8}{2})(10)(0.1)$ = \$84

Unit cost for subcontraction = (128)(10) = \$1,280

TRC = \$200 + \$84 + \$1,280 = \$1,564

Therefore, the best choice is the alternative 2 (Buy) with the minimum TRC of \$1,564 per year.

4.3 Example 3 – Application To The MPS When Production Rate Is Less Than Demand Rate

P = 72 units/year, $\overline{P} = \frac{72}{12}$ = 6 units/month

A_p = \$60, A_s = \$50, $V_p = V_s$ = \$20/unit, r = 0.1

π = \$0.20/unit short, $\hat{\pi}$ = \$10/unit short/year, C_i = \$2/unit

Purchase lead time = 0

Demand

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15

D = 5 + 5 + . . . + 15 + 15 = 120 units/year

$D = \frac{120}{12}$ = 10 units/month

Alternative 1: Production without increasing current production rate (P =P′) and no subcontraction

$\overline{P} = \frac{72}{12}$ = 6 units/month

Q_p = P = 72

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15
Ending Inv.	1	2	3	4	5	6	-3	-12	-21	-30	-39	-48
MPS	6	6	6	6	6	6	6	6	6	6	6	6
POR	72											

Setup Cost = (1)(60) = \$60

Carrying cost = $\left(\frac{1+2+\cdot\cdot\cdot-39-48}{2}\right)(10)(0.1)$ = \$10.5

Unit cost for production = (72)(10) = \$720

Goodwill Cost = (153)(0.20) = \$30.6

Shortage cost = $\left(\frac{153}{2}\right)(10)$ = \$765

TRC = \$60 + \$10.5 + \$720 + \$30.6 + \$765 = \$1,586.1

Alternative 2-1. Production with increasing current production rate (P<P′<D) and no subcontracion

P′ = 108 units/year

$P = \frac{108}{12}$ = 9 units/month

Q_p = P′ = 108

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15
Ending Inv.	4	8	12	16	20	24	18	12	6	0	-6	-12
MPS	9	9	9	9	9	9	9	9	9	9	9	9
POR	108											

Setup Cost = (1)(60) = \$60

Carrying cost = $(\frac{4+28+\cdot\cdot\cdot-6-12}{2}(10)(0.1)$ = \$60

Unit cost for production = (108)(10) = \$1,080

Goodwill cost = (18)(0.20) = \$3.6

Shortage cost = $(\frac{18}{2})(10)$ = \$90

Cost of increasing production rate = (108-72)(2) = \$72

TRC = \$60 + \$60 + \$1,080 + \$3.6 + \$90 = \$1,365.6

Alternative 2-2. Production with increasing current production rate (P′= D) and no subcontraction

P′ = 120 units/year

P = $\frac{120}{12}$ = 10 units/month

Q_p = P′ = D = 120

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15
Ending Inv.	5	10	15	20	25	30	25	20	15	10	5	0
MPS	10	10	10	10	10	10	10	10	10	10	10	10
POR	120											

Setup Cost = (1)(60) = \$60

Carrying cost = $(\frac{5+10+\cdot\cdot\cdot+5+0}{2})(10)(0.1)$ = \$90

Unit cost for production = (120)(10) = \$1,200

Cost of increasing production rate = (120-72)(2) = \$96

TRC = \$60 + \$90 + \$1,200 = \$1,446

Alternative 2-3. Production with increasing current production rate (P′> D) and no subcontracion

P′ = 168 units/year

$\overline{P}'$ = $\frac{168}{12}$ = 14 units/month

$$Q_p = \sqrt{\frac{2(60)(10)}{10(0.1)}}\sqrt{(1-\frac{14}{10})} = 18.5 \approx 19$$

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15
Ending Inv.	9	9	4	13	13	8	7	5	4	3	2	1
MPS	14	5		14	5		14	14	14	14	14	14
POR	19			19			19	19	19		19	19

Setup Cost = (7)(60) = $420

Carrying cost = $(\frac{9+9+\cdot\cdot\cdot+3+2}{2})(10)(0.1)$ = $41.5

Unit cost for production = (122)(10) = $1,220

Cost of increasing production rate = (168-72)(2) = $192

TRC = $420 + $41.5 + $1,220 + $192 = $1,873.5

Alternative 3. Subcontraction for the entire demand

$$Q_s = \sqrt{\frac{2(50)(10)}{10(0.1)}} = 31.6 \approx 32$$

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15
Ending Inv.	27	22	17	12	7	2	19	4	21	6	23	8
MPS	32						32		32		32	
POR	32						32		32		32	

Ordernig Cost = (4)(50) = $200

Carrying cost = $(\frac{27+22+\cdot\cdot\cdot+23+8}{2})(10)(0.1)$ = $84

Unit cost for subcontraction = (128)(10) = $1,280

TRC = $200 + $84 + $1,280 = $1,564

Alternative 4. Combination of the production without increasing current production rate (P = P′) and subcontraction for the remaining demand

P = 72 units/year

$P = \frac{72}{12}$ = 6 units/month

$Q_p = P = 72$

$$Q_s = \sqrt{\frac{2(50)(10-6)}{10(0.1)}} = 20$$

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15
Ending Inv.	1	2	3	4	5	6	17	8	19	10	1	11
MPS	6	6	6	6	6	6	26	6	26	6	6	6
POR(production)	72											
POR(subcontract)							20		20			20

Setup cost = (1)(60) = \$60

Ordering Cost = (3)(50) =\$150

Carrying cost = $(\frac{1+2+\cdot\cdot\cdot+1+11}{2})(10)(0.1)$ = \$43.5

Unit cost for production = (72)(10) = \$720

Unit cost for subcontracion = (60)(10) = \$600

TRC = \$60 + \$150 + \$43.5 + \$720 + \$600 = \$1,573.5

Alternative 5. Combination of the production with increasing current production rate (P<P'<D) and subcontraction for the remaining demand

P' = 108 units/year

$P = \frac{108}{12}$ = 9 units/month

$Q_p = P' = 108$

$Q_s = \sqrt{\frac{2(50)(10-9)}{10(0.1)}} = 10$

Month	1	2	3	4	5	6	7	8	9	10	11	12
Forecast	5	5	5	5	5	5	15	15	15	15	15	15
Ending Inv.	4	8	12	16	20	24	18	12	6	0	4	8
MPS	9	9	9	9	9	9	9	9	9	9	19	19
POR(production)	108											
POR(subcontract)											10	10

Setup cost = (1)(60) = \$60

Ordering Cost = (2)(50) = \$100

Carrying cost = $(\frac{4+8+\cdot\cdot\cdot+4+8}{2})(10)(0.1)$ = \$66

Unit cost for production = (108)(10) = \$1,080

Unit cost for subcontraction = (20)(10) = \$200

TRC = \$60 + \$100 + \$66 + \$1,080 + \$200 = \$1,506

Therefore, the best choice is the alternative 2-1 with the minimum TRC of \$1,365.6 per year.

5. Concluding Remarks

The extended EPQ model provides the solution to the situation where production rate is less than demand rate in the finite replenishment environment. Although its application to the time-phased schedules of MPS and MRP has a limitation because of the constant demand rate assumption, the model may provide the decision framework and its applications to the MPS and MRP as the EOQ model did in the MRP lot sizing rules.

The model concerns about the increased production rate and provides the bounds of production rate for each alternatives. However, it cannot answer the following question; "How much should a firm optimally increase its production rate?" Therefore, finding the optimal production rate will be necessary for further study.

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