⊠ 응용논문

Sequencing mixed-model assembly lines to minimize line length and throughput time using two-phase method
- 이단계법을 이용한 조립라인의 길이 및 완성시간의 최소화모형 -

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요 지

본 연구는 혼합모델 조립라인의 길이를 최적화하고 이 라인의 길이에서 완성시간을 최소화하는 새로운 제품투입순서를 결정하기 위하여 이단계법(two phase method)을 사용하였다. 최적화된 라인의 길이를 가지고 완성시간을 최소화하면 이 완성시간은 최적해이거나, 최적해에 가까운 근사해이고 이때 제품투입순서는 두 목적함수를 만족하는 새로운 투입순서이다.
제안된 이단계법으로 수식화한 모델들은 수치예를 통해 여러 가지 문제들을 실행한 결과 두 목적함수를 각각 실행시킨 결과보다 좋은 결과임을 증명하였다.

1. Introduction

Much of industrial production is based upon an assembly line, along which components are assembled into the finished product. In mixed model assembly lines, several models of same general products are assembled sequentially on a common line. The jobs arrived at each workstation can perform his assigned tasks on the product along paced conveyor.

Most of the proposed approaches for mixed model assembly line sequencing assume that the number of distinct products is small and that the mix of products is relatively stable. Various versions of the mixed model assembly line problem exist. These fall two broad categories; (1) line balancing aspect to spread out a particular model and models as smoothly as possible[3], [5], [8], (2) model sequencing aspect to allow optimum utilization of the assembly line operators' and/or minimum variation of the actual production from the desired production[1], [2], [4], [7].

In the model sequencing problem, Thomopolous first proposed a heuristic approach including a weighted penalty term comprising work congestion, idle time, and utility tasks. Dar-Ei and Corther suggested a heuristic which attempted to generate a sequence that minimized the overall length of the assembly line or throughput time[4], [8]. Various heuristics were suggested to determine good sequencing to optimize their proposed objective.

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Recently, Bard et al. (1992) demonstrated an analytic framework for optimal sequences to minimize two objectives, the assembly line length and throughput time, respective. In particular, they showed that for a multitude of problems, the minimum line length and minimum throughput time were always within 5% of each other parameters were held constant[1].

The purpose of this research is to determine the new sequencing of models that minimizes line length and throughput time and adopt two-phase method that addresses both objectives, sequentially. In the first phase, the objective function of the line length is minimized, and this optimal value then hooks up with the second phase's constraint. In the second phase, the optimal sequence of model can be determined while throughput time is minimized at a given line length.

This research developed four different Mixed Integer Programming LP models, based on the Bard et al.'s models(1992), that address combinations of status of work stations and operators' schedule as design parameters.

In order to demonstrate the effectiveness of the proposed algorithm, two-phase method(TPM), the same data employed by Bard et al. was used, and also various size of problem was performed.

2. Description of problem and model parameters

Mixed-model assembly lines are concerned with the progressive assembly on a simple line for several models of particular product type, e. g. home appliances, automobile, etc. Effectiveness of these lines depends on the optimal design of the line(line balancing, number of work stations, cycle time, length and speed of the line, etc.) and optimal scheduling of the line(sequencing, leveling part usages, etc.). In the ideal case, any product within a family can be assembled in any order, and that only minimal changes are required to redesign and rebalance the line when a new order arrives.

Table 1 shows the general parameters to consider in sequencing problem[1]. Assembly line consists of J work stations with a operator respectively, and links to conveyor as a material handling system moving at a constant speed v_c. A unit arrives at work station with predetermined interval called launch interval.

Table 1. Characteristics of sequencing problem.

Parameters	Characterization
Launch discipline	Fixed variable(single or multi-valued)
Station restriction	Open, close(any combination)
Operator Schedule	Early start, late start
Design objective	minimize line length, minimize throughput time

Launch interval can be defined as equi-spaced or various spaced on the line, regardless of the model. The operators can perform his assigned tasks on the product riding on the downstream (to direction of next work station) conveyor during corresponding processing time t_{jm}, that is model type m at work station j, They return upstream to the next unit at constant speed after completion.

Work stations may be defined as one of these : Closed station has boundaries which cannot be crossed by operators (e. g. spray paint booth, heat chambers, dip tank, etc.) In contrast, open station has no boundaries so that can be clossed. Operators, however, must not be permitted to interfere with each other, or to service same unit simultaneously, in any type of workstation.

Each operator of work stations can perform his job according to late start or early start. Early start implies that operators perform their job as soon as job arrives at work station. Idle time may be occurred in early start while no idle time is permitted in late start. It is always possible to set up the line and schedule the operators so that each works continuously. This is achieved with a late start schedule which indirectly assures that a sufficient amount of work-in-process inventory is available on the conveyer to avoid starvation. The result is likely to be an increase in the size of the facility. The alternative is to schedule the operators as early as possible with the realization that some loss in productive time is inevitable.

We have two objectives : to minimize line length that is important factor of facility design, and to minimize throughput time that is the typical job shop objective. A common definition of throughput time in flowshop problem is the time between when a job is released to the works or machines and when it is completed and ready for delivery. It is composed of processing time, setup time, moving (material handling) time, plus waiting (idle) time.

In mixed model assembly line problem, it can be defined as the time between when first unit of the cycle is released to the first worker of work station 1 and when the last unit of the cycle is completed by the last worker.

It is important to recognize that a tradeoff exists between these two objectives because idle time is one of the components of throughput time. Two-phase method is introduced to solve both objectives consecutively; phase I optimizes line length, then phase II determines the sequencing of models to minimize throughput time given that line length. To do this, we let optimized line length of the first phase be constraint of second phase.

3. Formulation of models

We develope four different models that address combinations of status of work stations and operators schedule. To formulate, We are given a set of M product models to be assembled on a common line consisting of J stations. Each model requires a set of tasks to be completed. Based on the line design the operator at each station j performs a subset of the tasks on each unit of the product. The product units are fixed to a conveyor belt moving at a constant speed v_c. An operator takes negligible time to move between products. Each model involves the following notation.

3.1 Notation

3.1.1 Indices

I position of a unit the sequence ; i = 1, ... , I

j work station ; j = i, ... , J

m model type ; m =1, ... , M

3.1.2 Input data

v_c velocity of conveyor

d_m demand for model type m

t_{jm} time to assemble model type m at work station j

I number of units to be sequenced ; $I = \sum_{m=1}^{M} d_m$

J number of work stations

M number of different model types

3.1.3 Computed parameters

T total work content ; $T = \sum_{j=1}^{J} \sum_{m=1}^{M} t_{jm}$

w launch interval

Π^* optimal solution(minimum line length) of phase I objective function.

3.1.4 Decision variables

X_{im} binary decision variable equal to 1 if model type m is in the ith position ; 0 otherwise.

Z_{ij} starting position of operator at work station j just before beginning the assembly of ith unit the sequence

S_{ij} accumulated starting position of operator at open work station j for the ith unit.

Y_j line length of work station j

Q_{ij} idle time for the ith unit at work station j

K_j accumulated line length up to work station j

w_i launch interval time between ith unit and (i+1)th unit

3.2 Model 1 - Closed work stations, Early start

For the early start case, a fixed zero reference point is defined for each work station so that the operators always start at or beyond this point and performs his job along the downstream conveyor with the unit. By the definition of closed station, operators must perform his job within their work stations. If the next unit hasn't arrived at the work station, operator waits at the origin starting point.

Mathematical and linear expressions that address closed stations and early start for all operators of each work station are shown below.

Phase I

$$\min \quad \Pi = \sum_{j=1}^{J} Y_j \qquad (1.1)$$

subject to

$$\sum_{m=1}^{M} X_{im} = 1 \qquad \text{for } all\ i \qquad (1.2)$$

$$\sum_{i=1}^{I} X_{im} = d_m \qquad \text{for } all\ m \qquad (1.3)$$

$$Z_{ij} + v_c\left(\sum_{m=1}^{M} X_{im} t_{jm} - w\right) \le Z_{i+1,j} \qquad i=1,\cdots,I-1,\ \text{for } all\ j \qquad (1.4)$$

$$Z_{ij} + v_c \sum_{m=1}^{M} X_{im} t_{jm} \le Y_j \qquad \text{for } all\ i,\ j \qquad (1.5)$$

$$X_{im} \in \{0,\ 1\},\quad Y_j \ge 0,\quad Z_{ij} \ge 0,\quad Z_{1j} = 0 \qquad \text{for } all\ i,\ j \qquad (1.6)$$

Phase II

$$\min \left\{\left(\sum_{j=1}^{J-1} Y_j + v_c \sum_{i=1}^{I} \sum_{m=1}^{M} X_{im} t_{Jm}\right) / v_c + \sum_{i=2}^{I} Q_{iJ}\right\} \qquad (1.7)$$

subject to

constraints (1.2), (1.3), (1.5), (1.6), and

$$Z_{ij} + v_c\left(\sum_{m=1}^{M} X_{im} t_{jm} - w\right) \le Z_{i+1,j} \qquad i=1,\cdots,I-1,\ j=1,\cdots,\ J-1 \qquad (1.8a)$$

$$Z_{iJ} + v_c\left(\sum_{m=1}^{M} X_{im} t_{jm} - w + Q_{i+1,J}\right) = Z_{i+1,J} \qquad i=1,\cdots,I-1 \qquad (1.8b)$$

$$Q_{i+1,J} \ge 0 \qquad i=1,\ \cdots,I-1 \qquad (1.9)$$

$$\sum_{j=1}^{J} Y_j = \Pi^* \qquad (1.10)$$

In the phase I, objective function (1.1) is to minimize the line length that sums the length of each work station from 1 to J. The first constraint (1.2) assures that each position in the sequence is occupied by exactly one model type. The second constraint (1.3) makes all demand of each model for the cycle satisfied. In inequalities of (1.4) and (1.5), the term $\sum_m X_{im} t_{jm}$ is processing time of the ith unit, and when multiplied by velocity of conveyor v_c , gives the operator displacement. Inequality (1.5) indicates that length of any work station j must be at least as great as the maximum displacement.

In phase II, objective function (1.7) sums the component of throughput time. The term $\sum_{j=1}^{J-1} Y$ is overall length until first unit reaches the last work station J, and when divided by v_c, gives the overall time. The second term implies that the last operator works continuously for $\sum_m d_m t_{jm}$ period. The last term of (1.7) is the sum of operator's idle time in last work station. Constraints of phase II are almost same as those of phase I except constraint (1.4).

Inequality (1.4) is separated by (1.8a) and (1.8b) contributes to the calculation of the idle time in the last work station.

Note that if w is treated as a variable, it is possible to obtain the optimal launch interval. Furthermore, if w is allowed to vary from one unit to the next, then by replacing w with w_j in (1.4), (1.8a), and (1.8b). This can be applied in general for the remaining models.

3.3 Model 2 - Closed work stations, Late start

Phase I

$$\min \quad \Pi = \sum_{j=1}^{J} Y_j \tag{2.1}$$

subject to

$$\sum_{m=1}^{M} X_{im} = 1 \qquad \text{for } all\ i \tag{2.2}$$

$$\sum_{i=1}^{I} X_{im} = d_m \qquad \text{for } all\ m \tag{2.3}$$

$$Z_{ij} + v_c\left(\sum_{m=1}^{M} X_{im}\, t_{jm} - w\right) = Z_{i+1,j} \qquad i=1, \cdots, I-1, \text{ for } all\ j \tag{2.4}$$

$$Z_{ij} + v_c \sum_{m=1}^{M} X_{im}\, t_{jm} \le Y_j \qquad \text{for } all\ i,\ j \tag{2.5}$$

$$X_{im} \in \{0,\ 1\}, \quad Y_j \ge 0, \quad Z_{ij} \ge 0 \qquad \text{for } all\ i,\ j \tag{2.6}$$

Phase II

$$\min \quad \left(\sum_{j=1}^{J-1} Y_j + v_c \sum_{i=1}^{I} \sum_{m=1}^{M} X_{im}\, t_{Jm} - Z_{11} + Z_{IJ}\right) / v_c \tag{2.7}$$

subject to

constraints (2.2), (2.3), (2.4), (2.5), (2.6), and

$$\sum_{j=1}^{J} Y_j = \Pi^* \tag{2.8}$$

In designing the facility for the late start case, no operators' idle time is allowed. As a consequence, operators are continuously busy, and first unit of each work station can be performed at any point out of zero reference point.

By equation (2.5) in phase I, operators are continuously busy. No idle time term, therefore, is necessary in objective function of phase II. The intervals until the first unit reaches to the first operator and the last operator should be excluded and included, respectively, for the exact throughput time.

3.4 Model 3 - Open work stations, Early start

Phase I

$$\min \quad \Pi = K_J \tag{3.1}$$

subject to

$$\sum_{m=1}^{M} X_{im} = 1 \qquad \text{for } all\ i \tag{3.2}$$

$$\sum_{i=1}^{I} X_{im} = d_m \qquad \text{for } all\ m \tag{3.3}$$

$$S_{ij} + v_c\left(\sum_{m=1}^{M} X_{im}\, t_{jm} - w\right) \le S_{i+1,j} \qquad i=1, \cdots, I-1, \text{ for } all\ j \tag{3.4}$$

$$S_{ij} + v_c \sum_{m=1}^{M} X_{im} t_{jm} \le K_j \quad \text{for } all\ i,\ j \tag{3.5}$$

$$S_{ij} + v_c \sum_{m=1}^{M} X_{im} t_{jm} \le S_{i,j+1} \quad \text{for } all\ i,\ j=1,\cdots,J-1 \tag{3.6}$$

$$X_{im} \in \{0,\ 1\},\quad Y_j \ge 0,\quad S_{ij} \ge 0,\ S_{11}=0 \quad \text{for } all\ i,\ j \tag{3.7}$$

Phase II

$$\min \left\{ \left(S_{1J} + v_c \sum_{i=1}^{I} \sum_{m=1}^{M} X_{im} t_{Jm} \right) / v_c + \sum_{i=2}^{I} Q_{iJ} \right\} \tag{3.8}$$

subject to

constraints (3.2), (3.3), (3.5), (3.6), (3.7), and

$$S_{ij} + v_c \left(\sum_{m=1}^{M} X_{im} t_{jm} - w \right) \le S_{i+1,j} \quad i=1,\cdots,I-1,\ j=1,\cdots,\ J-1 \tag{3.9a}$$

$$S_{iJ} + v_c \left(\sum_{m=1}^{M} X_{im} t_{Jm} - w + Q_{i+1,J} \right) = S_{i+1,J} \quad i=1,\cdots,I-1 \tag{3.9b}$$

$$Q_{i+1,J} \ge 0 \quad i=1,\cdots,I-1 \tag{3.10}$$

$$K_J = \Pi^* \tag{3.11}$$

In open stations, operators may use adjacent work space to perform their job, but may not interfere with neighbours. We now introduce the term K_j that is an accumulated line length up to the station j.

The constraint (3.6) indicates that the position of j+1st work station for the ith unit is always greater than the position of jth work station for the ith unit. This inequality contributes to the computation of accumulated work station length.

3.5 Model 4 - Open work stations, Late start

With the same manner of the case of closed work station and late start (model 2), no idle time occurs and the operator is continuously busy. The model can be formulated as follows.

Phase I

$$\min \quad \Pi = K_J \tag{4.1}$$

subject to

$$\sum_{m=1}^{M} X_{im} = 1 \quad \text{for } all\ i \tag{4.2}$$

$$\sum_{i=1}^{I} X_{im} = d_m \quad \text{for } all\ m \tag{4.3}$$

$$S_{ij} + v_c \left(\sum_{m=1}^{M} X_{im} t_{jm} - w \right) = S_{i+1,j} \quad i=1,\cdots,I-1,\ \text{for } all\ j \tag{4.4}$$

$$S_{ij} + v_c \sum_{m=1}^{M} X_{im} t_{jm} \le K_j \quad \text{for } all\ i,\ j \tag{4.5}$$

$$S_{ij} + v_c \sum_{m=1}^{M} X_{im} t_{jm} \le S_{i,j+1} \quad \text{for } all\ i,\ j=1,\cdots,J-1 \tag{4.6}$$

$$X_{im} \in \{0,\ 1\},\quad Y_j \ge 0,\quad S_{ij} \ge 0 \quad \text{for } all\ i,\ j \tag{4.7}$$

Phase II

$$\min \left(S_{1J} - S_{11} + v_c \sum_{i=1}^{I} \sum_{m=1}^{M} X_{im} t_{Jm}\right) / v_c \tag{4.8}$$

subject to

constraints (4.2), (4.3), (4.4), (4.5), (4.6), (4.7), and

$$K_J = \Pi^* \tag{4.9}$$

4. Numerical examples

In order to demonstrate the proposed algorithm, two-phase method(TPM), we used the same data that was employed by Bard et al.. All calculations were done by linear programming package LINDO running on a personal computer, Pentium 75. Data for the numerical examples are shown below.

Number of stations : J=4

Number of models : M=3

Demand of a cycle : d=(5,3,2)

Velocity of conveyor : v_c=1 (unit time)

Position in sequence : I=10

Assembly times for each model : t_{1m}=(4,8,7), t_{2m}=(6,9,4), t_{3m}=(8,6,6), t_{4m}=(4,7,5)

Total work content : T=244

Launch interval : w=6

For the first time, we examined four proposed models with above data when the launch discipline was held constant. Outcomes are listed in last four columns of Table 2 for the optimal sequence. In Table 2, the results involving minimization of line length, and throughput time by Bard et al. are also listed in first four columns and middle four columns, respectively.

Figure 1 and 2 display the operator movement diagram associated with the solution obtained for model 1 and model 3, respectively.

As we can see, the results of line length with TPM are exactly same optimal line length by Bard et al. In particular, TPM determined optimal sequencing that resulted in better throughput times in cases of closed stations (model 1, and model 2) when compared with throughput times of Bard. et al.

As previously mentioned, a trade off exists between line length and throughput time. According to Bard et al., line length is sometimes increased when throughput time is optimized. We can confirm it to check the line length of closed, early case by Bard et al. in Table 2. It may happen because objective oriented to throughput time can chase optimization of throughput time at the sacrifice of line length.

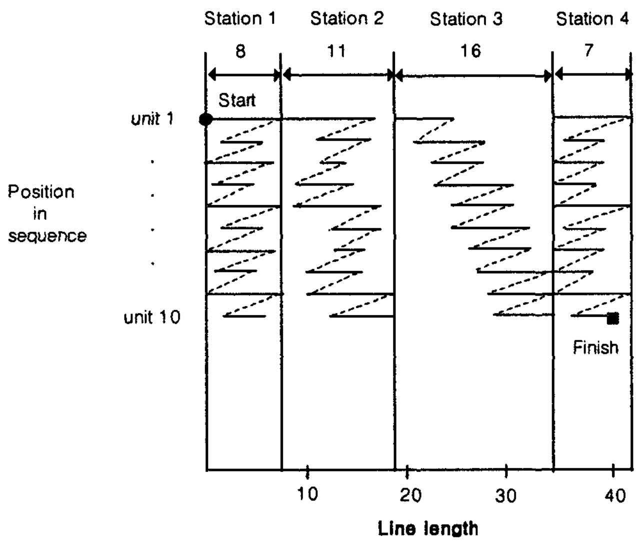

Figure 1. Optimal sequencing for Model 1

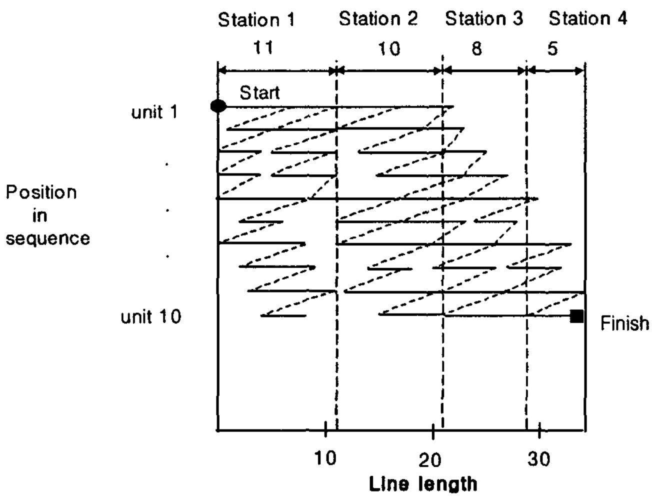

Figure 2. Optimal sequencing for Model 3

Table 2. Comparison of TPM's results with Bard et al.'s

Position	Minimize line length				Minimize throughput time				TPM			
	Early closed	Late closed	Early open	Late open	Early closed	Late closed	Early open	Late open	Early closed (Model 1)	Late closed (Model 2)	Early open (Model 3)	Late open (Model 4)
1	2	1	1	1	2	1	1	1	2	1	1	1
2	1	1	1	1	3	1	1	1	1	1	1	1
3	1	2	1	2	1	1	1	1	3	2	1	2
4	3	3	2	1	1	2	2	3	1	1	2	1
5	1	1	1	2	2	1	1	1	2	3	1	2
6	2	3	3	1	3	3	3	2	1	2	1	3
7	1	1	2	3	1	2	2	1	3	3	2	1
8	3	3	3	3	1	3	3	2	1	1	3	2
9	1	1	2	2	2	2	2	3	2	2	2	3
10	2	2	1	1	1	1	1	1	1	1	3	1
Length	**42**	**50**	**34**	**41**	43	52	34	41	**42**	**49**	**34**	**41**
Time	96	93	87	84	**95**	**93**	**87**	**84**	**94**	**92**	**87**	**84**

Table 3. Comparison of TPM with various w in case of model 1

	Minimize line length			Minimize throughput time			TPM		
position	(w=6)	(w=var)	(w_i=var)	(w=6)	(w=var)	(w_i=var)	(w=6)	(w=var)	(w_i=var)
1	2	2	1	2	1	1	2	1	1
2	1	2	1	1	2	2	1	2	2
3	3	1	2	3	1	1	3	1	1
4	1	3	3	1	3	3	1	3	3
5	2	3	2	2	2	1	2	2	1
6	1	2	2	1	1	2	1	1	2
7	1	1	3	3	3	2	3	3	1
8	3	1	1	1	2	1	1	1	3
9	1	1	1	2	1	3	2	2	2
10	2	1	1	1	1	1	1	1	1
Line length	**42**	**32**	**32**	42	34	33	**42**	**32**	**32**
Throughput time	96	113	97	**94**	**92**	**91**	**94**	**92**	**91**

For the second time, model 1 was performed with two situations that w was treated as a variable to obtain optimal launch interval and that w_i was treated as variable to obtain optimal launch interval between ith and i+1st unit. Outcomes are listed to compare with Bard. et al. in Table 3.

We can see that proposed TPM performs better in throughput times while line lengths are equal to those of Bard et al.

In order to demonstrate the effectiveness of the proposed algorithm, two-phase method(TPM), various size of problem was performed. The results showed that TPM always performed better than single objective of line length and prevented lumpy throughput time from optimal line length[7].

5. Conclusions

Bard et al. (1992) showed that for a multitude of problems, the minimum line length and minimum throughput time were always within 5% of each other when other parameters were held constant. In practical manner, however, we may be interested in a good sequence to minimize the throughput time when line length is given.

This paper develops two-phase method that guarantees optimum/near optimum of throughput time with optimum line length. In the first phase, the objective function of the line length is minimized and this optimal values then hooks up with the second phase's constraint. In the second phase, the optimal sequencing of models can be determined while throughput time is minimizing at given line length.

We developed four different models with combination of parameters of station restrictions and operator schedule. Each model was formulated with relatively simple and direct term. We examined each model using the numerical data by Bard et al.

In conclusion, proposed two-phase method determines a new sequencing to give a better throughput time when optimal line length is held constant and launch interval is treated as decision variable. Sequencing by TPM is better alternative than the sequencing that is produced by running each objective, respectively, such as line length or throughput time. It is demonstrated by various problems with different cycle size.

To apply these exact methods in the field, the heuristic methods will be remained as a further research.

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