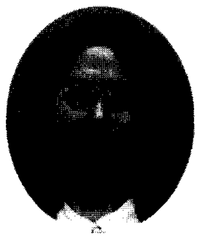
Flexural Modeling of Strengthened Reinforced Concrete Beam with Nonlinear Layered Finite Element Method

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ABSTRACT

An analytical method based on the nonlinear layered finite element method is developed to simulate an overall load-deflection behavior of strengthened beams. The developed model distinguishes itself by its capability to trace residual flexural behavior of a beam after the fracture of brittle strengthening materials at peak load.

The model, which uses a rather advanced numerical technique for iterative convergence to equilibrium, can be regarded as superior to the two models based on load control and displacement control.

The model predictions were compared with the experimental results and it was observed that there was good agreement between them.

Keywords : strengthened beam, nonlinear finite element method, arc-length method, flexural behavior

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1. INTRODUCTION

The strengthening of reinforced concrete members by externally bonded steel plates or FRP sheets has been used exclusively in recent years. Both retrofitting methods of bonding steel plates and bonding FRPs externally offer several advantages: (1) inexpensive and rapid applicability in the field with little or no disturbance; (2) keeping the original configuration of the structure; and (3) maintaining the overhead clearance. Especially external bonding of FRP sheets has become a variable alternative to bonding steel plates due to its high strength-to-weight ratio, lightweight, resistance to chemicals, good fatigue strength, non-magnetic, and non-conductive properties[8].

This paper presents an analytical model for predicting load-deflection relations, stresses, and strains of such a strengthened reinforced concrete beams. Except for analyses focusing on the micro-level localized effects, there are a few theoretical models for predicting flexural behavior of strengthened reinforced concrete beams. The previously proposed models can be divided into two groups: (1) prediction of flexural strength only[8], and (2) prediction of overall flexural load-deflections[9]. For the prediction of a flexural strength, couple methods with the assumption of compatibility are widely employed. For the prediction of overall flexural behavior of strengthened reinforced concrete beam, couple method with assumptions on overall distribution of curvatures along the beam span and nonlinear finite element methods are usually used.

Finite element methods are basically developed with layered model which is appropriate modeling technique for beams mainly governed by flexures.

Couple method requires a rather simple solution technique with one variable involved: bisection method, secant method, Regular-Falsi method, and Newton method, for example. On the other hand, most popular solution technique for nonlinear finite element method would be Newton-Raphson method for N-variables. Since its derivation assumes load control, the method, thus, has deficiencies in finding load-deflection relationship beyond peak load. In order to trace the softening behavior, displacement-controlled method can be used. According to this method displacement at particular point(s) in a beam is(are) pre-assigned and corresponding load(s) is(are) found as reaction(s). Although this method is capable of reproducing post peak flexural behavior of a beam, only statically determinate structures or indeterminate structures subject to a limited number of point loads can be modeled.

The objective of this research is to develop general flexural model which could overcome the limit of the existing models.

The model must be able to trace post peak behavior of strengthened reinforced concrete beam and is indifferent to determinacy of a structure. For this purpose nonlinear layered finite element method is used with an advanced numerical solution technique, called arc-length method.

It is shown that the developed model is capable of tracing both pre-peak and post-peak behavior and compares well with experimental results.

2. LITERATURE REVIEW

2.1 Failure Modes

From the experiments, five failure modes can be observed for the strengthened reinforced concrete beam; (1) tension failure, (2) separation, (3) rip off, (4) tension failure and separation, and (5) separation and rip off[1]. It may not be desirable for a beam to experience local failure before tension failure of FRP sheet occurs at the section of maximum moment.

2.2 Flexural Modeling Techniques

A few analytical models predicting the flexural behavior of strengthened reinforced concrete beam are available from the literature. A typical model would be the one presented by Hamid Saadatmanesh et.al[6]. They presented a model based on compatibility of deformations for predicting overall load and curvature relationships of strengthened reinforced concrete beam for rectangular and T-sections. They assumed that linear strain distribution through the full depth of a beam, no tensile strength in concrete, no shear deformation, and no slip between composite plate and concrete beam. Based on their assumptions they found iteratively the location of the neutral axis for the equilibrium of internal forces(Fig.1):

$$\alpha f_c'' bc + \sum_{i=1}^{n} f_{si} A_{si} + f_{pl} A_{pl} = 0 \qquad (1)$$

where:

α=mean stress factor;
b=beam width;
c=depth of neutral axis measured from top concrete compressive fiber;
f_{si}=stress of steel;
f_c=stress of concrete;
A_{si}=steel of area;
d_{pl}=plate of stress; and
A_{pl}=plate of area.

Once this neutral axis is found, the internal resisting moment is obtained by summing moments from all the internal forces with respect to the neutral axis:

$$M = \alpha f_c'' bc\left(\frac{h}{2} - \gamma c\right) + \sum_{i=1}^{n} f_{si} A_{si}\left(\frac{h}{2} - d_i\right) + f_{pl} A_{pl}\left(\frac{h}{2} - d_{pl}\right) = 0 \qquad (2)$$

where:

P=force in plate;
h=beam section height;
γ=centroid factor indicatiy position of compressive force in concrete;
d_i=depth measured from top concrete fiber(level of steel rebar); and
d_{pl}=depth measured from top concrete fiber(plate).

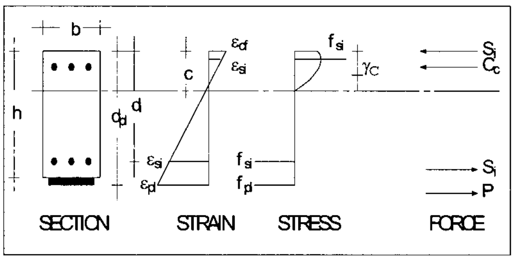


Fig.1 Strain, Stress, and Force Diagrams across the Depth of Rectangular Section[6]

Although this model compared well with experimental results it is restricted to reproducing moment (or applied load) and curvature relationships only.

Similar assumptions are made by Li-Hyung Lee et.al.[7] in predicting overall flexural load-deflection relationships. In their model, however, load versus deflection relationships are found by assuming linearly varying curvature distributions along the beam span for simply supported beam(see Fig2). They found curvatures at the locations where maximum moment occurs and at FRP sheet terminates. Curvature at support is assumed to be zero and all other curvatures between them are assumed to vary linearly. Their analytical model and test results agreed well.

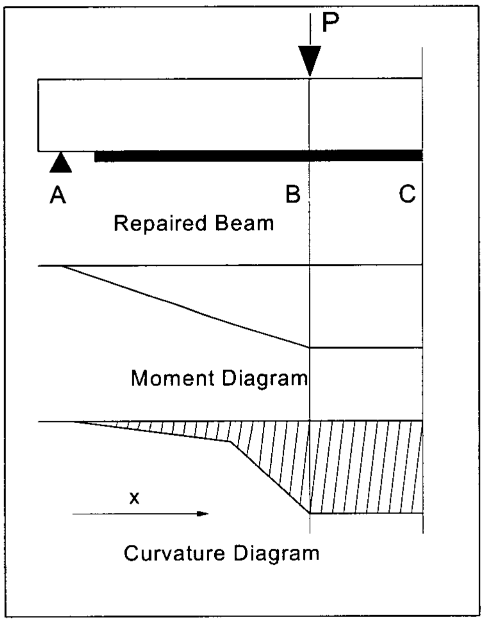


Fig. 2 Moment and Assumed Curvature Diagram[7]

Chadon Lee et.al.[8] proposed a flexural model which can be regarded as an intermediate model between couple method and nonlinear layered finite element model. They basically adopted assumptions made in couple method but improved the model by adding one additional equilibrium condition for sectional moments called 'moment equilibrium'. This condition is imposed at each pre-selected section and neutral axis and curvature at each section are obtained(Fig.3):

$$F_j^k = \sum_{i=1}^{NDB} F_{i,j}^k = 0 \quad (j = 0, 1, 2, \ldots\ldots N) \qquad (3)$$
$$M_j^k = \sum_{i=1}^{NDB} M_{i,j}^k$$

Both models are, however, limited in their applications to statically determinate structures only. As far as loading types applicable to a model are concerned, finite element method is most suitable one.

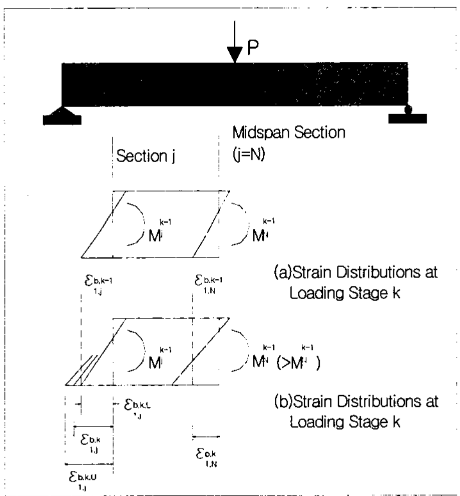


Fig.3 Extended Couple Method[8]

The appropriate model for reinforced concrete beam strengthened with brittle FRP sheets need to have the following characteristics: (1) applicability to indeterminatė structure, (2) possible inclusion of various types of externally applied loads, and (3) capability of simulating post-peak behavior after peak load due to tension fracture of FRP sheets. The model which can accomodate these characteristics are developed in this study. Nonlinear layered finite element method and arc-length method are used for this purpose.

3. DEVELOPMENT OF MODEL

3.1 Assumptions and limitation

The following assumptions are made in developing the model: 1) plane section remains plane before and after bending (Euler-Bernoulli's hypothesis): 2) among other factors, failure of strengthened beam is governed by tensile fracture of FRP sheets only.

The model assumes that there is no local failure such as separation, rip off and/or their combinations, and no shear failure occurs.

3.2 Theoretical Developments

3.2.1 Layered Finite Element Method

Approximate horizontal displacement $u_a(x)$ and vertical displacement $v_a(x)$ in Fig.4 along the reference axis of the element can be expressed using shape functions as follows[3]:

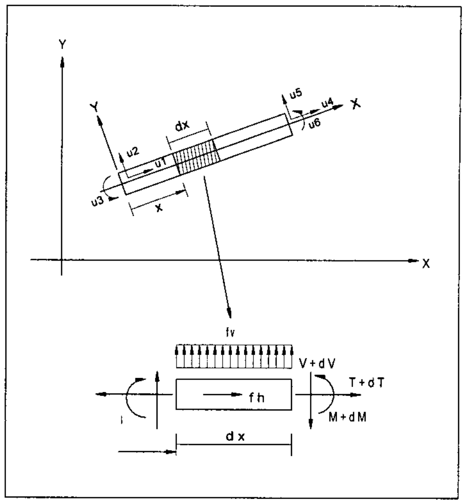


Fig. 4 Beam Column Element for Finite Formulation[3]

$$\begin{Bmatrix} u_a(x) \\ v_a(x) \end{Bmatrix} = \boldsymbol{NU} \qquad (4.a)$$

where N is the shape function given by:

$$\boldsymbol{N}=\begin{bmatrix} N_{1u}(x) & 0 & 0 & N_{2u}(x) & 0 & 0 \\ 0 & N_{1v}(x) & N_{2}v(x) & 0 & N_{3v}(x) & N_{4v}(x) \end{bmatrix} \qquad (4.b)$$

where:

$N_{1u}(x)=1-x/l$;

$N_{2u}(x)=x/l$;

$N_{1v}(x)=1-3(x/l)^2+2(x/l)^3$;

$N_{2v}(x)=x-2(x^2/l)+2(x^3/l^2)$;

$N_{3v}(x)=3(x/l)^2-2(x/l)^3$; and

$N_{4v}(x)=-(x^2/l)+(x^3/l^2)$.

The horizontal displacement $u(x,y)$ can be expressed in terms of $u_a(x)$ and $v_a(x)$ as:

$$u(x,y)=u_a(x)-y\frac{d}{dx}v_a(x) \qquad (5)$$

Then the longitudinal strain $\varepsilon(x,y)$ is given by:

$$\varepsilon(x, y) = \frac{du(x, y)}{dx} = [1 \quad -y] \begin{bmatrix} \frac{du_a(x)}{dx} \\ \frac{d^2 v_a(x)}{dx^2} \end{bmatrix} \quad (6)$$
$$= [1 \quad -y]\, \boldsymbol{BU}$$

where:

$$\boldsymbol{B} = \begin{bmatrix} \frac{dN_{1u}(x)}{dx} & 0 & 0 & d\frac{N_{2u}(x)}{dx} & 0 & 0 \\ 0 & \frac{d^2N_{1v}(x)}{dx^2} & \frac{d^2N_2v(x)}{dx^2} & 0 & \frac{d^2N_{3v}(x)}{dx^2} & \frac{d^2N_{4v}(x)}{dx^2} \end{bmatrix}$$

Let $\boldsymbol{U}_0$ be the displacement vector obtained in the previous steps, and $\triangle \boldsymbol{U}$the current incremental displacement vector to be sought. Then currently sought displacements as well as strains are:

$$\boldsymbol{U} = \boldsymbol{U}_0 + \triangle \boldsymbol{U}$$
$$\varepsilon_0(x, y) = [1 \quad -y]\, \boldsymbol{BU}_0 \quad (7)$$
$$\triangle \varepsilon_0(x, y) = [1 \quad -y]\, \boldsymbol{B} \triangle \boldsymbol{U}$$

Once strains $\varepsilon(x, y)$ are obtained, the internal axial force T(x) and the internal moment M(x) are computed as follows:

$$T(x) = T_0(x) + \triangle T(x)$$
$$M(x) = M_0(x) + \triangle M(x) \quad (8)$$

where:

$$T_0(x) = \sum_{i=1}^{n} \sigma_i(\varepsilon_0(x, y)) A_i;$$
$$\triangle T(x) = \sum_{i=1}^{n} \frac{d\sigma_i(\varepsilon_0(x, y))}{d\varepsilon} \triangle \varepsilon(x, y) A_i;$$
$$M_0(x) = \sum_{i=1}^{n} -\sigma_i(\varepsilon_0(x, y)) A_i y_i; \text{ and}$$
$$\triangle M(x) = \sum_{i=1}^{n} -\frac{d\sigma_i(\varepsilon_0(x, y))}{d\varepsilon} \triangle \varepsilon(x, y) A_i y_i.$$

The above equations can be rewritten in matrix form as:

$$\begin{Bmatrix} \triangle T(x) \\ \triangle M(x) \end{Bmatrix} = \boldsymbol{D}\, \boldsymbol{B} \triangle \boldsymbol{U} \quad (9)$$

where:

$$\boldsymbol{D} = \begin{bmatrix} \sum_{i=1}^{n} \frac{d\sigma_i(\varepsilon_0(x, y))}{d\varepsilon} A_i & -\sum_{i=1}^{n} \frac{d\sigma_i(\varepsilon_0(x, y))}{d\varepsilon} A_i y_i \\ -\sum_{i=1}^{n} \frac{d\sigma_i(\varepsilon_0(x, y))}{d\varepsilon} A_i y_i & \sum_{i=1}^{n} \frac{d\sigma_i(\varepsilon_0(x, y))}{d\varepsilon} A_i y_i^2 \end{bmatrix}$$

The application of virtual work principle and integration over the length l, lead to:

$$\int_0^l \{ \overline{\delta u_a(x)} \quad \overline{\delta v_a(x)} \} \begin{Bmatrix} -\frac{dT(x)}{dx} \\ \frac{d^2 M(x)}{dx^2} \end{Bmatrix} dx$$
$$= \int_0^l \{ \overline{\delta u_a(x)} \quad \overline{\delta v_a(x)} \} \begin{Bmatrix} f_h(x) \\ f_v(x) \end{Bmatrix} dx \quad (10)$$

This equation can be summarized as;

$$\boldsymbol{K} \triangle \boldsymbol{U} = (-\boldsymbol{F}_0 + \boldsymbol{P}_0) + \triangle \boldsymbol{P} \quad (11)$$

where:

$$\boldsymbol{K} = \int_0^l \boldsymbol{B}^T \boldsymbol{D} \boldsymbol{B}\, \boldsymbol{dx};$$
$$\boldsymbol{F}_0 = \int_0^l \boldsymbol{B}^T \begin{Bmatrix} T_0(x) \\ M_0(x) \end{Bmatrix} dx;$$
$$\boldsymbol{P}_0 = \int_0^l \boldsymbol{N}^T \begin{Bmatrix} \boldsymbol{f}_{h0}(x) \\ \boldsymbol{f}_{v0}(x) \end{Bmatrix} dx; \text{ and}$$
$$\triangle \boldsymbol{P}_0 = \int_0^l \boldsymbol{N}^T \begin{Bmatrix} \triangle \boldsymbol{f}_h(x) \\ \triangle \boldsymbol{f}_v(x) \end{Bmatrix} dx.$$

Assuming equilibrium state is reached, we get:

$$\boldsymbol{K} \triangle \boldsymbol{U} = \triangle \boldsymbol{P} \quad (12)$$

3.2.2 Arc-Length Method

In contrast to usual analysis methods, the load and displacement both are treated as variables in arc-length method. Out of balance forces, $\boldsymbol{g}(\boldsymbol{U})$,can be expressed as:

$$\boldsymbol{g}(\boldsymbol{U}) = \boldsymbol{f}(\boldsymbol{U}) - \lambda \boldsymbol{P} \quad (13)$$

where:

f= internal forces;

U=function of the current displacements; and

P= fixed total load vector.

The λ is taken as a load-controlling parameter in the above equation. Since the load level is treated as a variable, an extra governing equation is required and this is given by a constraint in the form of arc(see Fig. 5):

$$\triangle U_{i+1}^T \triangle U_{i+1} + b\triangle\lambda_{i+1}^2 P^T P = \triangle U_i^T \triangle U_i + b\triangle\lambda_{i+1}^2 P^T P = \ldots.\triangle l^2 \quad (14)$$

where,

U_i= incremental displacement after the i-1th iteration;

$\triangle\lambda_i$= incremental load change after this iteration;

$\triangle l$= incremental length to be discussed later; and

b= scalar parameter, current stiffness parameter.

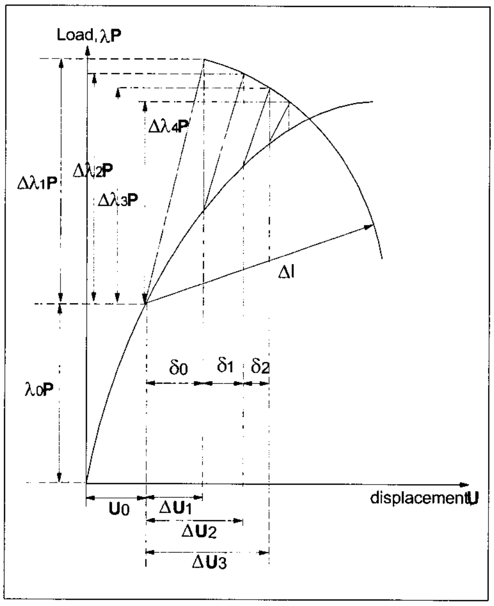


Fig.5 Basic arc-length procedure and notation for one degree-of-freedom system[2]

Taking $\triangle l=0$, we have:

$$\delta_i^T \triangle U_{i-1} + b\,\delta\lambda_{i-1} P^T P = 0 \quad (15)$$

where:

δ_i= the iterative displacement; and

$(\triangle U_{i-1}, \triangle\lambda_{i-1})$= secant change over the increment.

With perfect convergence, we will have $f_0(U_0)=\lambda_0 P$

where, λ_0=load level at end of last increment

For i=0 onwards, the iterative procedure then involves:

$$\lambda_{i+1}=\lambda_o+\triangle\lambda_{i+1}=\lambda_i+\delta\lambda_i$$
$$U_{i+1}= U_o+\triangle U_{i+1}= U_i+\eta_i\,\delta_i \quad (16)$$

where $\triangle U_{i+1} = \triangle U_i + \eta_i\,\delta_i$ is the incremental displacement and η_i is the step-length which, for the present, may be read as unity.

The modified Newton-Raphson technique is applied where the tangent stiffness matrix(K) is formed at the beginning of each increment and it is fixed for all the iterations within the increment. Using the indirect solution procedure of Crisfield and Ramm[2],

$$\delta_i = -K^{-1}(f_i(U_i)-\lambda_{i+1} P) = \overline{\delta_i} + \lambda_{i+1}\,\delta_T \quad (17)$$

where,

$\overline{\delta_i} = -K^{-1} f_i(U_i)$; and

$$\delta_T = K^{-1} P.$$

However, the iterative vector δ_i is only fully defined once λ_{i+1} is known. Substitution of equation (16) and (17) into the constraint equation (14) gives

$$a_1 \lambda_{i+1}^2 + a_2 \lambda_{i+1} + a_3 = 0 \quad (18)$$

where:

$a_1 = \eta_i \delta_T^T \delta_T$;

$a_2 = 2 \delta_T^T (\triangle U_i + \overline{\delta_i})$; and

$a_3 = \eta_i \overline{\delta_i^T} \overline{\delta_i} + 2 \overline{\delta_i^T} \triangle U_i$.

Assuming exact satisfaction of the constraint of equation at the previous iteration, the terms in square brackets in equation may be omitted. The appropriate root of Eq.19 is chosen by ensuring an acute angle θ between $\triangle U_i$ and $\triangle U_{i+1}$, i.e.:

$$\cos\theta = 1 + \frac{\eta_i}{\triangle l^2}(d_4 + \lambda_{i+1} d_1) \quad (19)$$

where,

$d_1 = \delta_T^T \triangle U_i$; and

$d_4 = \overline{\delta_i^T} \triangle U_i$.

If both values of $\cos\theta$ are positive, one may choose the appropriate λ_{i+1} as the root closest to the linear solution of

$$\lambda_{i+1, lin} = -a_3 / a_2 \quad (20)$$

The overall numerical scheme for arc-length method is given in Fig.6.

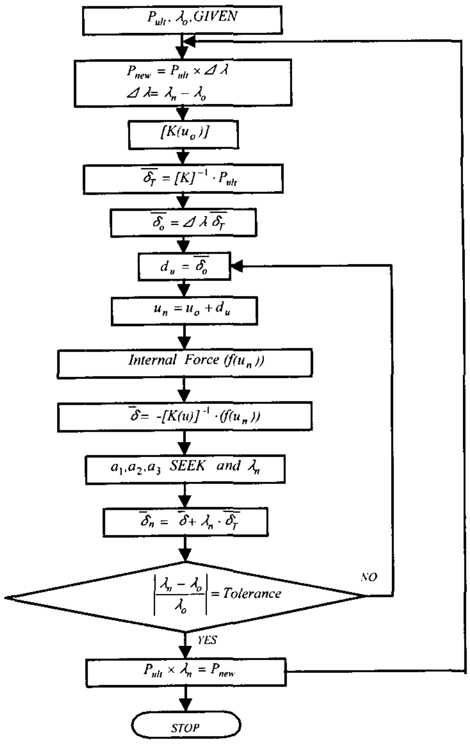


Fig.6 Flow of Solution Technique[9]

4. CONSTITUTIVE MODELS[9]

Empirical constitutive models for concrete, steel rebar, and FRP sheet are used. The models are presented in Fig.7.

The meaning of main parameters appearing in Eq.(21) through Eq.(23) are illustrated in Fig.7.

4.1 Concrete under Compression

For $0 \le \varepsilon < \varepsilon_c$

$$f_c = -\frac{f_c'}{(\varepsilon_c)^2}(\varepsilon - \varepsilon_c)^2 + f_c'$$

$$(21)$$

For $\varepsilon_c \le \varepsilon < \varepsilon_r$

$$f_c = \left(\frac{f_r - f_c'}{\varepsilon_r - \varepsilon_c}\right)(\varepsilon - \varepsilon_c) + f_c'$$

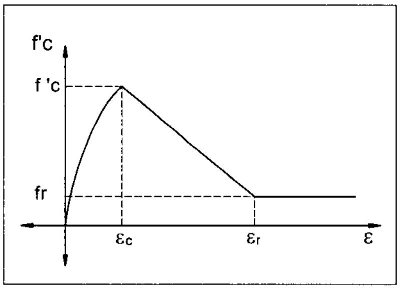
4.2 Steel Rebar under Tension and Compression

$$
\begin{aligned}
&For\ \ 0 \le \varepsilon < \varepsilon_y \\
&\quad f_s = E_y \varepsilon \\
&For\ \ \varepsilon_y \le \varepsilon < \varepsilon_h \\
&\quad f_s = f_y \\
&For\ \ \varepsilon_h \le \varepsilon < \varepsilon_u \\
&\quad f_s = \frac{(f_y - f_u)}{(\varepsilon_h - \varepsilon_u)^2}(\varepsilon - \varepsilon_u)^2 + f_u
\end{aligned}
\tag{22}
$$

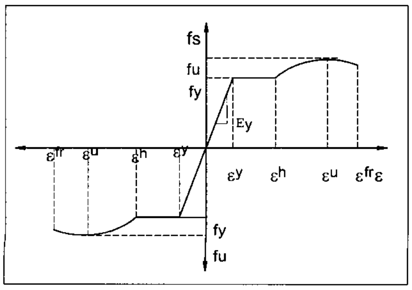
4.3 FRP sheet under Tension

Based on experimental observations the linear elastic behavior up to failure is assumed for FRP under tension.

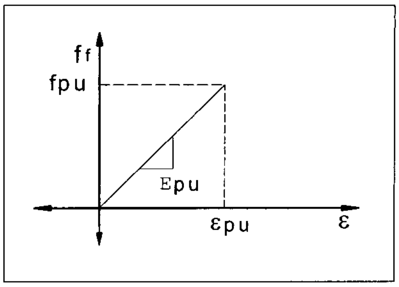
$$
\begin{aligned}
&For\ \ 0 \le \varepsilon < \varepsilon_{pu} \\
&\quad f_f = E_{pu} \cdot \varepsilon \\
&For\ \ \varepsilon_{pu} < \varepsilon \\
&\quad f_f = 0
\end{aligned}
\tag{23}
$$



(a) Concrete Compressive Stress-Strain Model



(b) Rebar Stress-Strain Model



(c) FRP Sheet Tensile Stress-Strain Model

Fig. 7 Constitutive Models

5. COMPARISONS WITH TEST RESULTS

In order to verify the validity of the developed model the model predictions are compared with experimental results reported by different researchers [1,4,5,10].

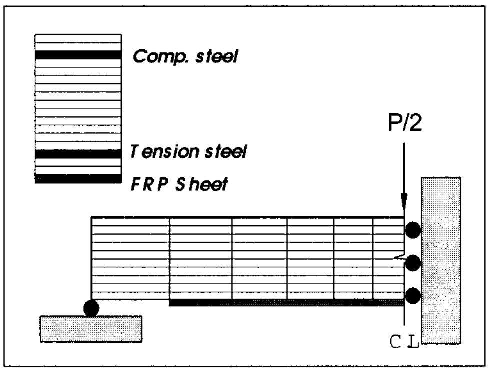


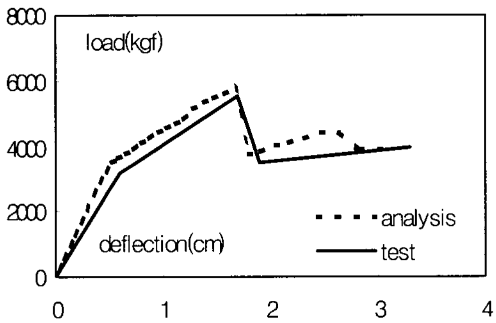
Fig. 8 Modeling by F.E.M

Table 1 summarizes main parameters used for each test and Fig.8 illustrates developed finite element modeling.

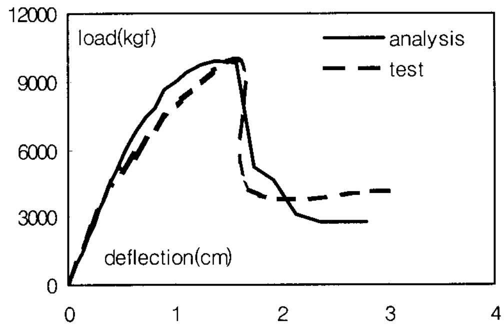
Fig. 9 shows that the developed model is reasonably reproducing prepeak flexural behavior, ultimate load, and post peak flexural behavior of strengthened reinforced concrete beam. It is wroth mentioning that the developed model is superior to other models in its capability of incorporating different type of loads, analysis of indeterminate structures, and tracing the post-peak behavior after peak load.

Table 1 Summary of Main Parameters for each Beam[1,4,5,10]

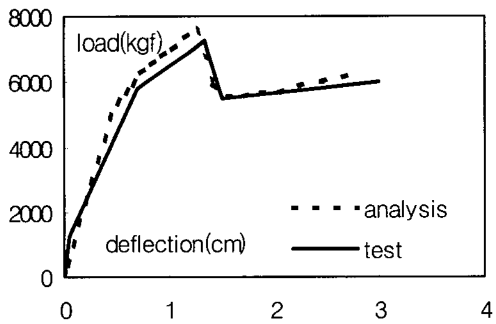
Beam	Ref.	Beam Length (cm)	b (cm)	Tensile Steel		Comp. Steel		fy (kg/㎠)	f′c (kg/㎠)	Strengthening Sheet			
				Area (㎠)	ds (cm)	Area (㎠)	ds′ (cm)			Material	fpu (kg/㎠)	Eps (kg/㎠)	Thickness (cm)
SMFC1N	1	200	15	2-D10 (1.443)	22	2-D10 (1.43)	3	4374	250	CFRP	35000	2600000	0.0165
SLFC1N	1	200	15	2-D13 (2.54)	22	2-D10 (1.43)	3	4252	250	CFRP	35000	260000	0.0165
F3-130	4	200	15	2-D13 (2.54)	22	2-D10 (1.43)	3	4520	200	CFRP	35500	235000	0.033
C0E	5	100	15	2-D13 (2.54)	12	2-D10 (1.43)	3	4060	278	GFRP	4500	227000	1EA
BCFCA	10	300	25	3-D19 (8.60)	35	2-D10 (1.43)	5	4526	250	CFRP	49935	2670000	0.011
BCFCB	10	300	25	3-D19 (8.60)	35	2-D10 (1.43)	5	4526	250	CFRP	45650	2490000	0.011



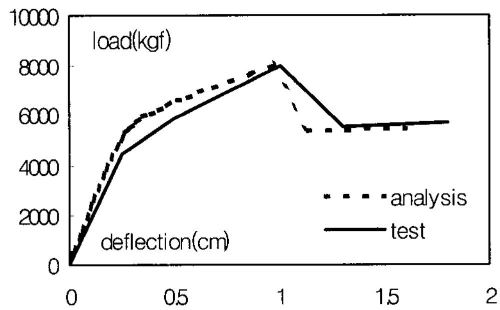
(a) Ref.[1] SMFC1N



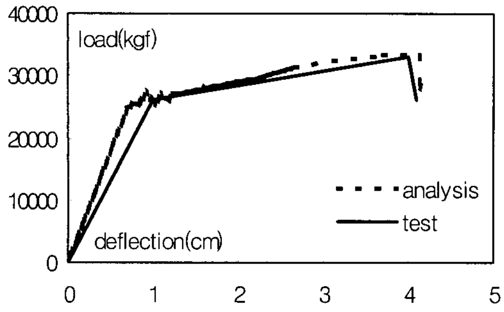
(c) Ref.[4] F3-130-1.9



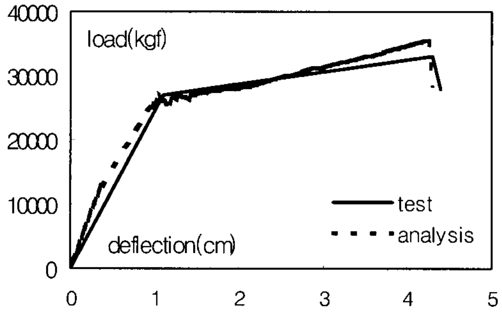
(b) Ref.[1] SLFC1N



(d) Ref.[5] C0E



(e) Ref.[10] BCFCA



(f) Ref.[10] BCFCB

Fig.9 Comparisons between Test Results and Model Predictions

6. CONCLUSIONS

In this research, various types of theoretical models and solution techniques suggested for predicting flexural behavior of strengthened reinforced concrete beam are reviewed. Their merits and demerits are discussed and based on which a refined model is developed. The developed model can be regarded as more general which are mostly one than previously suggested models based on couple method, extended couple method, load controlled finite element method and displacement controlled finite element method. The validity of the developed model is demonstrated by comparing its predictions with experimental results reported by different researchers. The developed model is found to predict experimental results reasonably well. Compared with those models suggested by other researchers, the developed model has the following advantages:

1) compared with couple method, the model calculates curvatures along the beam span without assumptions. In addition, the model is applicable to indeterminate structures;

2) compared with load-controlled finite element method, the model can predict residual flexural behavior of strengthened reinforced concrete beam after tensile fracture of brittle FRP sheets; and

3) compared with displacement - controlled finite element method, the model can be applicable to more general loading types and structures.

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