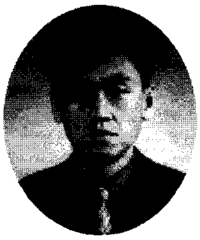
A Study on Span to Depth Ratio for Minimum Thickness of One-Way Slab



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ABSTRACT

A Computer-based iterative method is provided for the calculation of minimum thickness values for one-way slabs to satisfy the maximum permissible limits given in the ACI Building Code. An algorithm includes the effects of cracking and time-dependent effects due to creep and shrinkage. Comparison of the calculated minimum thickness values with the current ACI limits is conducted to investigate limitations of the current tabulated minimum thickness, which are constant to a range of design conditions.

keywords : a computer-based iterative method, minimum thickness, one-way slab, ACI building code

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1.INTRODUCTION

The ACI Building Code (ACI 318-95) allows the use of minimum thickness values for one-way slab to satisfy deflection deflection control requirements without the need to calculate deflections(1). The minimum thicknesses provided in Table 9.5(a) of ACI 318-95 apply only to members not supporting or attached to partitions or other construction likely to be damaged by large deflections.

Minimum thicknesses are not provided for members supporting or attached to nonstructural elements, implying that for these cases, deflections should be calculated and compared to the limits given in Table 9.5(b) of ACI 318. It appears that many designers use the minimum thickness table for members supporting non-structural elements also.

Minimum thickness as a fraction of span length is attractive to designers because of its simplicity. The objective of this study is to determine limitations of the current tabulated values for application to a range of design conditions and to develop recommendations for minimum thickness values that are consistent with specified limits on calculated deflection and also cover a wider range of design conditions than the current values. The current study is limited to one-way slab systems.

The present study is based on the general approach to deflection calculations given in the ACI Code. The effective *moment of inertia approach* is used to account for cracking, and a long-time *multiplier* is used *to account* for time-dependent effects of creep or shrinkage. While the uncertainties involved in predicting deflections of reinforced concrete members are recognized, it is emphasized that the aim here is to relate minimum thickness to code-specified deflection limits through the use of code-specified deflection calculation criteria. Deflection limits are based on values given in Table 9.5(b) of ACI 318.

2. DETERMINISTIC MODEL BASED ON ACI 318-95

The ACI Code places limits on computed deflections. For members not supporting non-structural elements likely to be damaged by large deflections, a limit is placed on immediate deflection due to live load. However for members supporting or attached to nonstructural elements, a limitation is placed on incremental deflection that occurs after installation of the nonstructural elements. *If the non-structural elements are not* likely to be damaged by large deflections, a limit of l/240 is used. However, if large deflections are likely to cause damage, a limit of l/480 is used. The deflection limits given in the Code are based on historical precedent rather than any well documented objective measure of serviceability related to deflection. An account of the historical development of deflection limit criteria is given by Warwaruk(6)

Previous studies by Thomson and Scanlon have shown that the limitations on incremental deflection, which can be induced by a sustained long-time load and a variable portion of live load, are more severe than the limitations on live load deflection(5). Since most buildings, particularly office and apartment buildings, will contain partitions that may or may not be damaged by large deflections, the limits on incremental deflection are taken to be the governing criteria in this study. In order to compute the incremental deflection after installation of non-structural components the construction sequence and load history must be known. Figure 1 shows a typical schematic load vs time history for a slab system in a multi-story structure(8). During construction, the load increases as floors above are supported temporarily on floors below. After construction, the load drops to the sustained load level. An increment in sustained load is added when non-structural components are installed. Live load is then applied intermittently during the service life time of the structure.

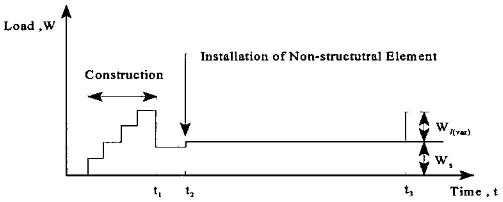

Fig. 1 Schematic Load-Time History

A simplified load vs time history is shown in Fig.2. A single instantaneous application of construction load W_{co} is applied at time t_1. Since the maximum load on the system often occurs during construction, W_{co} can be taken as equal to the service dead plus live load. The load then drops to the sustained load level and remains constant thereafter. A single application of live load is shown at time t_3. Installation of non-structural components is conservatively assumed to occur at time t_1.

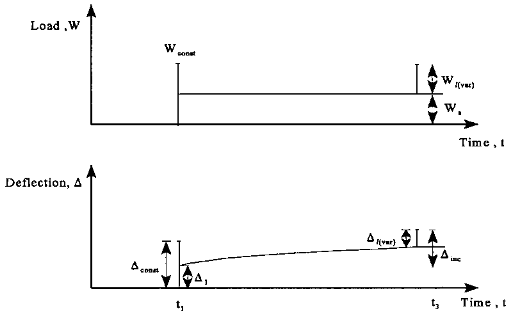

Fig.2 Simplified Load-Time History and Corresponding Deflection-Time History

Based on this simplified load history, the incremental deflection can be calculated as,

$$\Delta_{inc} = \frac{\zeta W_{lvar} l^4}{384 E_c I_e} + \lambda \frac{\zeta W_s l^4}{384 E_c I_e} \qquad (1)$$

where,

W_{lvar} = variable portion of live load

W_s = sustained load (= W_{sw} + W_{sd} + W_l)

W_{sw} = dead load of self-weight

W_{sd} = superimposed dead load

W_{lsus} = sustained portion of live load (= γW_l)

γ = fraction of sustained portion of live load
λ = long-time deflection multiplier
(Eq. 9-10, ACI 318-95)
ζ = deflection coefficient depending on support condition
= 5 for simply supported
= 1.4 for both ends continuous
= 2 for one end continuous
= 48 for cantilever
I_e = effective moment of inertia
(Eq. 9-7, ACI 318-95)
E_c = modulus of elasticity for concrete
(Section 8.5.1, ACI 318-95)
l = span length

The deflection limit is reached when the calculated deflection equals the required allowable deflection, i.e.

$$\Delta_{inc} = (\Delta_{inc})_{allow} \qquad (2)$$

Expressing I_e as αI_g where I_g is $bh^3/12$, and rearranging Eq. (1) provides the following expression for the span to depth ratio l/h in terms of the allowable deflection.

$$\frac{l}{h} = \left[\frac{(\Delta_{inc})_{allow}}{l}\,\frac{32E_c\alpha b}{\zeta(\lambda W_s + W_{lvar})}\right]^{0.333} \qquad (3)$$

A similar expression was developed by Rangan (1982) except that Rangan expressed the span to depth ratio in terms of the member effective depth, d(4). The primary advantage is that the procedure used to determine minimum thickness is consistent with the code method used to compute deflection for comparison with specified deflection limits. Also, the resulting equation for minimum thickness can be related directly to the equation for deflection calculation.

This expression cannot be used directly to calculate the minimum thickness because the right hand side contains two quantities that are not known until after a thickness has been selected and the required reinforcement determined. These quantities are the member self weight, Wsw , and the effective moment of inertia which depends on member dimensions and reinforcement. An iterative procedure was therefore developed to determine the span to depth ratio corresponding to a given superimposed dead plus live load, permissible incremental deflection, and span. The steps in the procedure are as follows:

1. Initialize h (member depth).
2. Evaluate the area of steel required for strength.
3. Check reinforcement ratio less than code maximum value ($\rho < \rho_{max}$).
4. If $\rho > \rho_{max}$, increase h and repeat steps 1 through 3 until $\rho < \rho_{max}$..
5. Calculate I_e for service dead plus live load using ACI Code expressions.
6. Calculate l/h from Equation (2) to give new depth h..
7. Repeat steps 1 through 6 until change in calculated depth is within specified tolerance.

The above procedure was incorporated in a computer program and results were also checked by hand calculation for several cases.

3. PARAMETRIC STUDY

A parametric study was performed to evaluate the effects of typical design variables on the computed span to depth ratios, and to compare values with current ACI 318 values obtained from Table 9.5(a). The variables considered were:

Span length (ft): 10, 15, 20, 25, 30, 35, 40
Live load including partition load (psf): 60, 70, 100, 200
Modulus of rupture coefficient ($k_r = f_r / \sqrt{f'c}$): 4, 5, 7.5
Support conditions: simply supported, continuous, one end fixed, cantilever

The modulus of rupture was varied to examine the sensitivity of results to cracking strength. The support conditions match the boundary conditions in ACI 318 Table 9.5(a). The range of span length and live load considered are typical of values encountered in design of building structures.

The following parameters were held constant:

Concrete compressive strength (psi): 4000
Steel yield strength (psi): 60,000
Superimposed dead load (psf): 15
Sustained live load (psf): 20
Concrete unit weight (pcf): 150
Long-time multiplier, λ: 2

Fig.3 shows the variation of calculated maximum span to depth ratios for varying live load and span length as well as the current ACI 318 values.

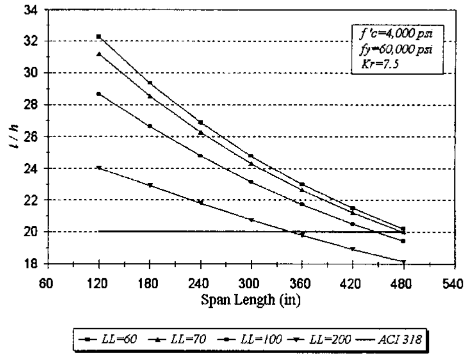

Fig.3(a) Variation of Span to Depth Ratio for Simply Supported One-Way Slab with l/240

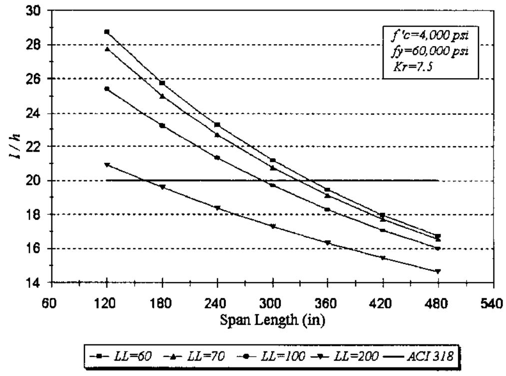

Fig.3(b) Variation of Span to Depth Ratio for Simply Supported One-Way Slab with l/480

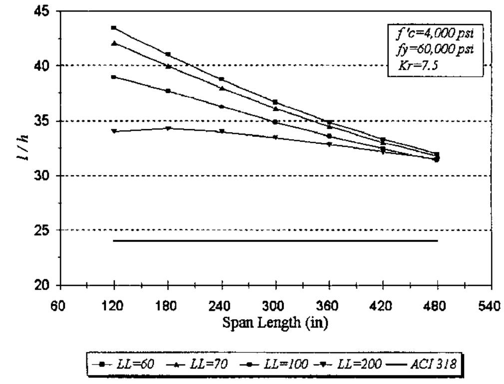

Fig.3(c) Variation of Span to Depth Ratio for One End Continuous One-Way Slab with l/240

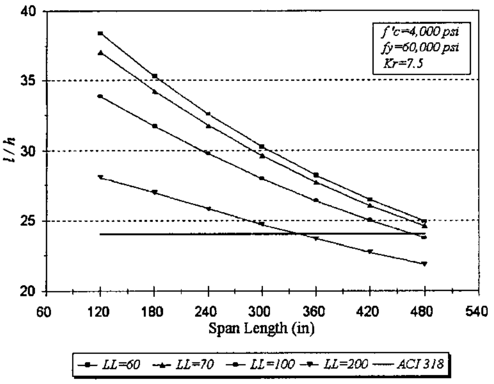

Fig.3(d) Variation of Span to Depth Ratio for One End Continuous One-Way Slab with l/480

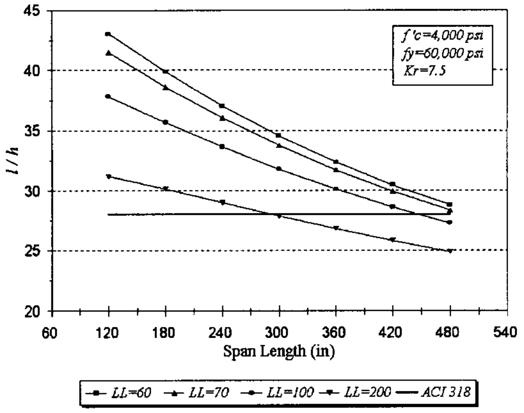

Fig.3(e) Variation of Span to Depth Ratio for Both End Continuous One-Way Slab with l/240

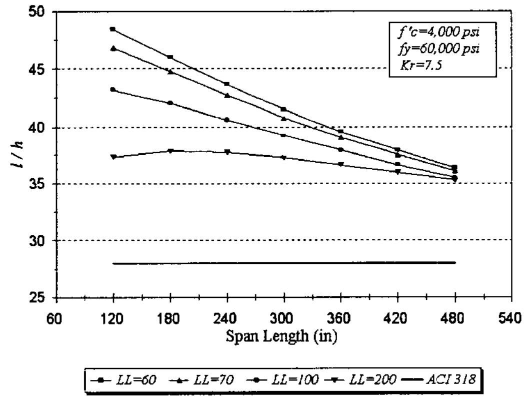

Fig.3(f) Variation of Span to Depth Ratio for Both End Continuous One-Way Slab with l/480

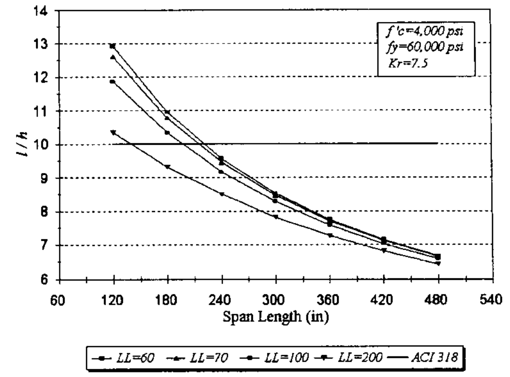

Fig.3(g) Variation of Span to Depth Ratio for Cantilevered One-Way Slab with l/240

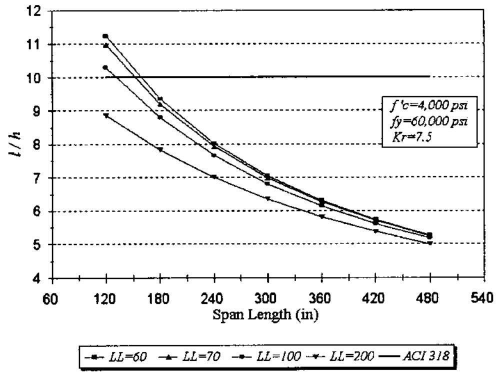

Fig.3(h) Variation of Span to Depth Ratio for Cantilevered One-Way Slab with l/480

The plots demonstrate that the minimum thickness for a specified deflection limit increases with increasing live load. The results also demonstrate that the span to depth ratio decreases as the span length increases. This is because the slab dead load increases as the span increases. The current ACI 318 values are independent of live load and the span to depth ratios are independent of span the length. Sensitivity of the results to effective cracking strength (modulus of rupture) is illustrated in Fig. 4.

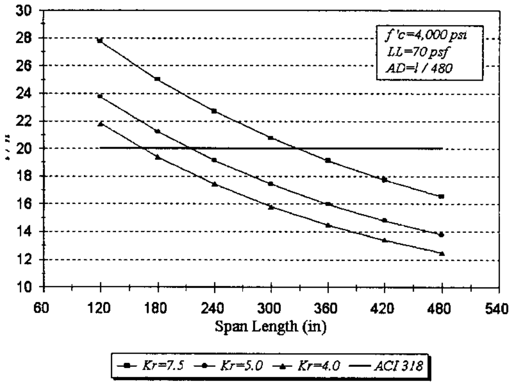

Fig.4(a) Effect of Cracking Strength on Span to Depth Ratio for Simply Supported One-Way Slabs

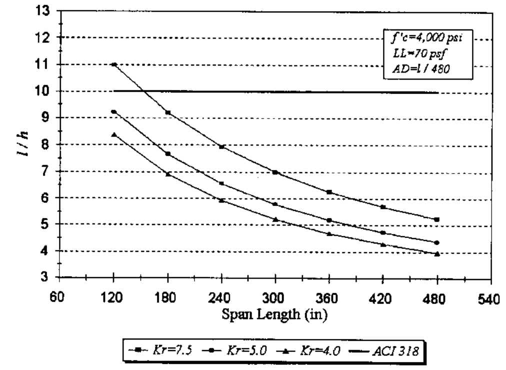

Fig.4(b) Effect of Cracking Strength on Span to Depth Ratio for One End Continuous One-Way Slabs

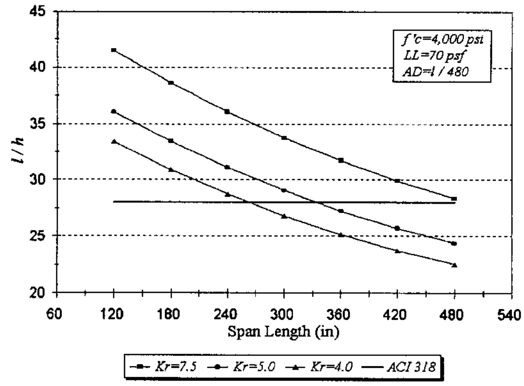

Fig.4(c) Effect of Cracking Strength on Span to Depth Ratio for Both End Continuous One-Way Slabs

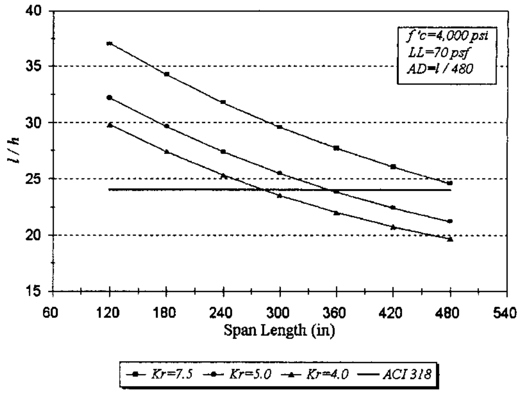

Fig.4(d) Effect of Cracking Strength on Span to Depth Ratio for Cantilevered One-Way Slabs

The results indicate that the current span to depth ratios are conservative at shorter span lengths even when applied to the deflection limit of *l*/480. However the results also indicate that there are limits on the span length for which the current values are valid.

4. CONCLUSIONS

The results presented in this study indicate that the minimum thickness values given in ACI 318-95 Table 9.5(a) are conservative for slabs not supporting non-structural elements likely to be damaged by large deflections for span lengths typically found in building structures. These values are also applicable to slabs supporting partitions for spans up to about 35 ft for continuous slabs and up to about 25 ft for simply supported slabs.

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