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Development of a Rigid-Ended Beam Element and a Simplified 3-Dimensional Analysis Method for Ship Structures

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Abstract

In this paper, a 2-dimensional novel beam element is developed and a method to replace the 3-dimensional analysis with 2-dimensional analysis is proposed. The developed novel beam element named rigid-ended beam element can consider the effect of three kinds of span points within one element, which was impossible in modeling with the ordinary beam element. Calculated results for the portal frame using the rigid-ended beam element agree with the results using membrane elements. And also, the proposed simplified 3-dimensional analysis method which includes two step analysis using influence coefficients shows good accuracy. Structural analysis using the rigid-ended beam element and the simplified 3-dimensional method is revealed to have good computing efficiency due to unnecessity of the elements corresponding to the brackets and simplification of 3-dimensional analysis.

1 Introduction

Ship structures are composed of transverse web frames and longitudinal girders as shown in Figure. 1. At the initial design stage, structural analysis of primary members such as web frames and girders is carried out by using beam element for simple and reasonable evaluation of the designed structure. However, end brackets make variable cross section, and joint areas common to connected members are considerable, so proper modeling of variable cross section and joint areas is important for reliable results. And also, as longitudinal girders support the transverse web frames, interactions between longitudinal girders and transverse web frames must be considered.

In structural analysis by beam elements, brackets and joint areas are usually represented by rigid elements to simplify the analysis. Determination of extent of the rigid ends corresponding to joint areas is simple, but determination of rigid length corresponding to brackets is not simple because of variable cross section. For accurate analysis, extent of rigid ends for brackets, which are called as span points, must be determined from the three kinds of view points, i.e., bending, shearing and axial deformation. In general, the three kinds of span points are not coincident. In analysis of ship structures using ordinary beam elements, it was impossible for one rigid element to take account of three span points, so accuracy of calculated results has often been lower. In this paper, to increase computing efficiency and accuracy in analysis of ship structures with brackets, a

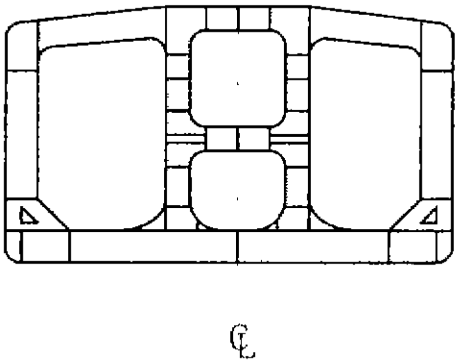

Figure 1. Transverse frame of double hull tanker.

novel beam element which can correctly consider the effect of brackets is proposed and usefulness of the element is shown. A novel beam named rigid-ended beam element includes the effect of three span points within itself, so reduction of element and prevention of computational errors such as overflow or underflow are possible.

Interactions between transverse members and longitudinal girders can be considered by 3-dimensional structural analysis for the whole frames. In this paper, to consider the interaction effect correctly and at the same time increase computing efficiency, a simplified 3-dimensional analysis method is proposed. The method does not conduct 3-dimensional structural analysis, but considers 3-dimensional effect by calculation of infuence coefficients of members at each crossing point between transverse members and longitudinal girders and solution of compatibility equations.

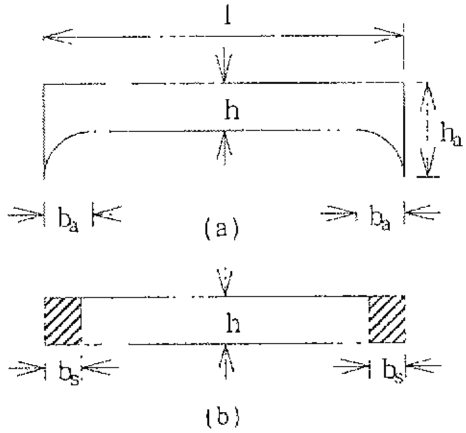

Figure 2. Definition of span point.

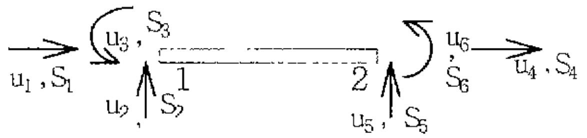

Figure 3. Nodal displacements and forces.

2 Definition of Span Point

Extent of the span point shown in Figure 2 is determined from the condition that end deformations are the same between (a) and (b) in Figure 2. From this condition, three kinds of span points are defined, that is to say, axial span point, bending span point and shearing span point. For exact determination of locations of the span points, calculation of end deformations according to loading conditions is necessary, based on the above condition. But, for convenient determination of the span points, empirical formulas based on experiments are often used. Each span point can be calculated using the proposed formulas[Yamaguchi, 1958 & Jang, 1992].

3 Stiffness Matrix of Rigid-Ended Beam Element

The rigid-ended beam element is the combined element of a uniform beam and rigid ends. That is to say, one element includes a uniform beam and rigid elements of different lengths corresponding to three kinds of span points. Nodal displacements and forces of the rigid-ended beam element are defined in Figure 3. The stiffness matrix of the rigid-ended beam element are derived from the differential equations. Solutions of the differential equations for the rigid-ended beams are obtained by integration of the differential equation and substitution of boundary condition[Seo et al., 1997].

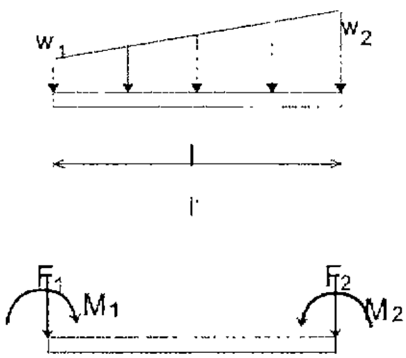

Figure 4. Equivalent nodal forces.

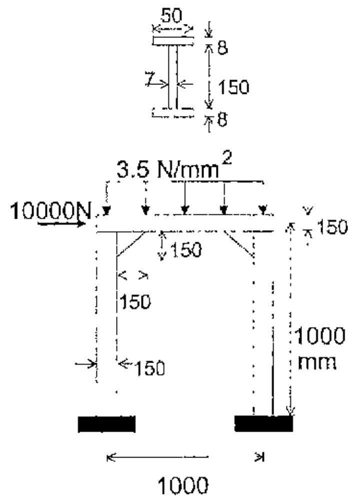

Figure 5. Portal frame with brackets (Young's modulus = 210,000 N/m^2, Poisson's ratio = 0.3).

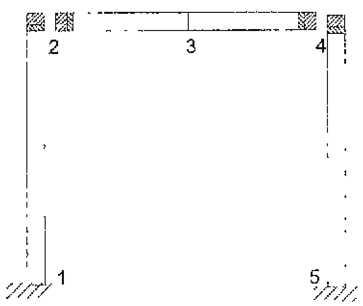

Figure 6. Finite element model of the portal frame using rigid-ended beam elements.

4 Derivation of Equivalent Nodal Forces of Rigid-Ended Beam Element

Equivalent nodal forces resulting from the distributed load within the element can be derived from the displacement function of the beam. By substituting boundary conditions to the displacement function, the equivalent nodal forces can be determined. Detailed expressions are presented in Appendix 2.

5 Calculated Results Using Rigid-Ended Beam Element

In this section, the accuracy and efficiency of the rigid-ended beam element is confirmed. As an example, a portal frame structure with brackets shown in Figure 5 is selected. Finite element analyses using membrane and truss elements and ordinary beam elements[Swanson Analysis System

Table 1. Calculated results for the portal frame.

Item		Rigid-ended beam element	Ordinary beam element with different sections	Membrane & truss element
Deformation at Point 2 (*mm*)	X-dir	3.4294E-1	3.0050E-1	3.6989E-1
	Y-dir	-1.1612E-2	-1.3920E-2	-1.1591E-2
Deformation at Point 3 (*mm*)	X-dir	3.3576E-1	2.9285E-1	3.5866E-1
	Y-dir	-1.2639E-1	-1.1195E-1	-1.3030E-1
Deformation at Point 4 (*mm*)	X-dir	3.2858E-1	2.8521E-1	3.5727E-1
	Y-dir	-3.9220E-2	-4.0326E-2	-4.3233E-2
No. of node		5	21	205
No. of element		4	20	160 / 72
No. of degrees-of-freedom		9	57	390
CPU Time ratio		1	98	4369

Inc., 1994] are carried out, and analysis using the rigid-ended beam elements is also carried out. Calculated results are shown in Table 1. The results show good accordance, but reveal differences in computing efficiency.

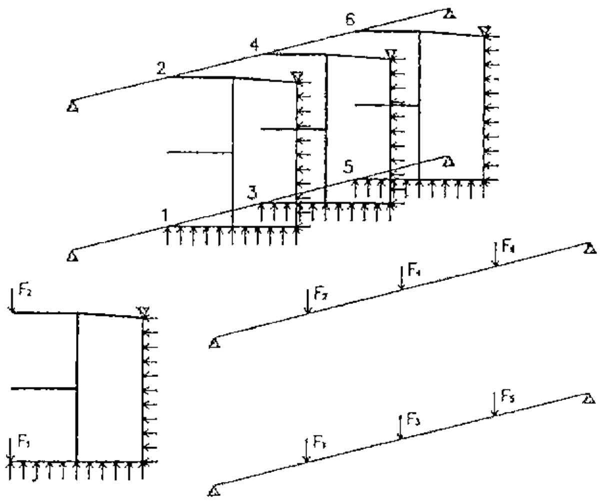

Figure 7. 3-dimensional frame under distributed loads.

6 Simplified 3-Dimensional Structural Analysis

Within the midship region, typical transverse web frames are repeated and longitudinal girders support the frames. Interaction effects between the web frames and the girders are considerable, so to obtain accurate results, 3-dimensional analysis for the whole structure is necessary. However, if it is possible to utilize repetitive characteristics of the primary ship structure, the analysis for the whole structure can be much simplified. The simplified 3-dimensional structural analysis method utilize the same structural characteristics of transverse web frames. The method extends 2-dimensional frame analysis to 3-dimensional structural analysis by introducing influence coefficients and the compatibility condition.

In this method, torsional rigidity of girders is assumed to be zero, because most primary structural members have shapes of T or L type and torsional rigidity of these open sections is negligible, compared with bending rigidity of the section. Using this assumption, the effect of interaction between girders and web frames can be included in reaction forces as shown in Figure 7.

The following simultaneous equation can be obtained by the condition that deflections of the web frames and girder are compatible at the intersection points.

$$(e_{ik} + d_{ik})F_k = v_i \qquad (1)$$

where

d_{ik} = influence coefficient of the web frame

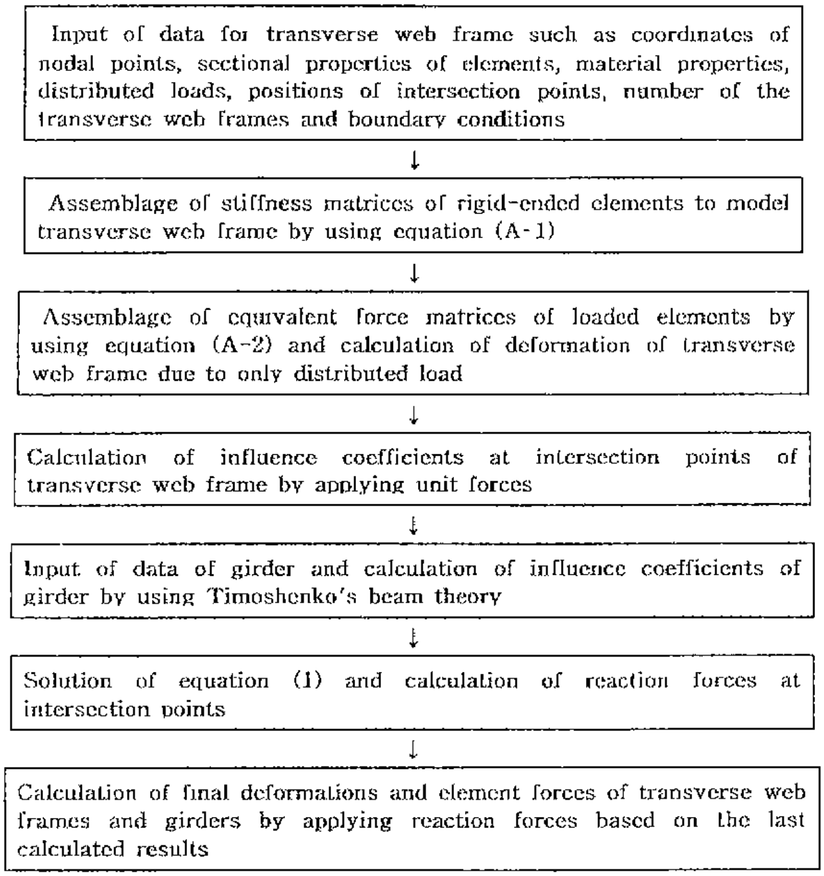

Figure 8. Procedure for simplified 3-dimensional structural analysis.

Table 2. Calculated results for the model shown in Figure 9(Node Numbers are shown in Figure 7).

Node	Vertical Deformation (mm)	Reaction Force of Girders (N)	Vertical Deformation(ANSYS) (mm)	Reaction Force of Girders(ANSYS) (N)
1	4.8824	1.8400E+5	4.8838	1.8402E+5
2	3.3312	4.3902E+4	3.3321	4.3900E+4
3	6.7588	1.2757E+5	6.7607	1.2761E+5
4	4.6660	4.1433E+4	4.6672	4.1434E+4
5	4.8824	1.8400E+5	4.8838	1.8402E+5
6	3.3312	4.3902E+4	3.3321	4.3900E+4

e_{ik} = influence coefficient of girder
F_k = reaction force at intersection i
v_i = deflection of the web frame at intersection i when only uniform load is applied

Influence coefficients of web frame and girder can be calculated by using Timoshenko's beam theory[Dym et al., 1973]. By solving equation (1), reaction forces and deflections can be calculated, and internal bending moments and shear forces can also be calculated using equilibrium condition.

The procedure of the simplified 3-dimensional analysis method can be summarized as Figure 8.

Calculated results for the example shown in Figure 9 are presented in Table 2. In Figure 9, uniform load of 70 N/mm is applied at the bottom and uniform load of 50 N/mm is applied at the shell. Calculated results show good accordance, but nodal points to be needed in solution are much different. In the 3-dimensional frame analysis, 70 nodal points with 6 degrees-of-freedom are needed, but in the simplified 3-dimensional analysis, only 9 nodal points with 3 degrees-of-freedom are needed and another 6 degrees-of-freedom corresponding to intersecting points are needed after 2-dimensional analysis. Table 2 supports the validity of the proposed simplified analysis method.

7 Conclusions

In this paper, for efficient and reasonable analysis of bracketed ship strucutures, a rigid-ended beam element which could consider the effect of rigid ends extended to the span points was developed and the simplified 3-dimensional structural analysis method was proposed. The stiffness coefficients of the rigid-ended beam element were derived from the differential equations and integration, and the equivalent nodal forces were derived from the displacement function considering rigid ends. The developed rigid-ended beam element could consider the effect of three kinds of span points within one element, which resulted in much reduction of computing time. Calculated results using the rigid-ended beam element agreed with the results by the finite element analysis using membrane elements and ordinary beam elements. The proposed simplified 3-dimensional structural analysis method also utilize characteristics of the repeated transverse web frames and

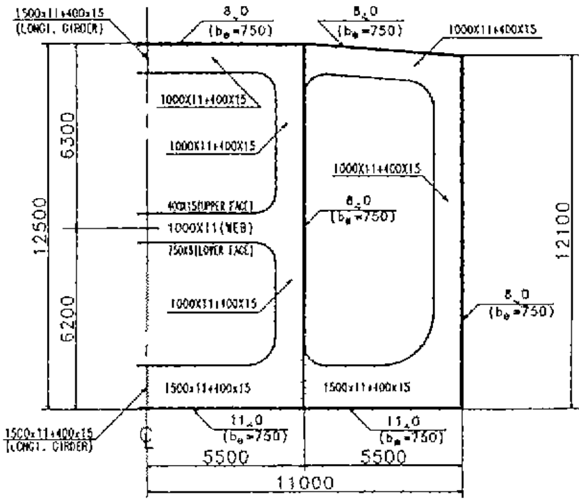

Figure 9. Sectional view of model for calculation (Web Frame Space = 3 m).

the straight longitudinal girders. Interaction effects between the transverse members and the longitudinal girders was considered by influence coefficients and the compatibility condition. As, following the proposed method, only the 2-dimensional analysis was carried out, computational efficiency could be much raised without accuracy being lowered. As structural analysis using the rigid-ended beam element and the simplified 3-dimensional method was revealed to have good computing efficiency and reasonable accuracy, the proposed method can be used properly in the structural optimization process which accompanies repeated calculation and requires efficient solution procedures.

References

1. Yamaguchi, I., 1958, On the Tests of Fixed Beam with Brackets, Journal of the Society of Naval Architects of Japan, Vol.102, pp.199-207
2. Jang, C.D., Na, S.S., 1992, On the Development of Generalized Slope Deflection Method for the Analysis and Design of Ship Structures, Journal of the Society of Naval Architects of Korea, Vol.29, No.4, pp.202-213
3. Seo, S.I., Lim, S.J., 1997, Development of a rigid-ended beam element and its application to simplify 3-dimensional analysis of bracketed frame structures, Trans. of the Society of Naval Arch. of Korea, Vol.34, No.3
4. Swanson Analysis Systems Inc, 1994, Introduction of ANSYS for Revision 5.0, Vol.1
5. Dym, C.L., Shames, I.H., 1973, Solid Mechanics - A Variational Approach, McGraw-Hill

Appendix 1 Stiffness Coefficients of Rigid-Ended Beam Element

(A-1) Stiffness coefficients, **[K]** =

$\frac{E}{R_1}$					
0	$E\frac{P_2}{D_e}$			SYM	
0	$E\frac{P_1}{D_e}$	$E\frac{-J_1+\frac{E}{G}Q_1+P_1 l}{D_e}$			
$-\frac{E}{R_1}$	0	0	$\frac{E}{R_1}$		
0	$-E\frac{P_2}{D_e}$	$-E\frac{P_1}{D_e}$	0	$E\frac{P_2}{D_e}$	
0	$E\frac{J_2}{D_e}$	$E\frac{J_1-\frac{E}{G}Q_1}{D_e}$	0	$-E\frac{J_2}{D_e}$	$E\frac{J_2 l-J_1+\frac{E}{G}Q_1}{D_e}$

where

A = cross sectional area of beam

A_s = effective shear area of uniform cross section of beam

b_{sa1}, b_{sa2} = axial span point

b_{sb1}, b_{sb2} = bending span point

b_{ss1}, b_{ss2} = shearing span point

$D_e = -J_1 P_2 + P_1 J_2 + \frac{E}{G} Q_1 P_2$

E = Young's modulus

G = shear modulus

I = moment of inertia of uniform cross section of beam

$J_1 = -\frac{1}{3I}(l_4^3 - l_3^3) + \frac{1}{2I} l (l_4^2 - l_3^2)$

$J_2 = \frac{1}{2I}(l_4 - l_3)^2 + \frac{1}{I}(l_4 - l_3)(l - l_4)$

l = length of beam

$$l_1 = b_{sa1}$$

$$l_2 = l - b_{sa2}$$

$$l_3 = l - b_{sb1}$$

$$l_4 = l - b_{sb2}$$

$$l_5 = b_{ss1}$$

$$l_6 = l - b_{ss2}$$

$$P_1 = \frac{1}{2I}(l_4^2 - l_3^2)$$

$$P_2 = \frac{1}{I}(l_4 - l_3)$$

$$Q_1 = \frac{1}{A_s}(l_6 - l_5)$$

$$R_1 = \frac{l_2 - l_1}{A}$$

Appendix 2 Displacement Function and Equivalent Nodal Forces

Lateral displacement function is derived from the integration of the differential equation. Resulting displacement function is as follows.

$$v(x) = v_2(x)u_2 + v_3(x)u_3 + v_5(x)u_5 + v_6(x)u_6 \tag{A-2}$$

where

$$v_2(x) = a_2\left\{J_1(x) - \frac{E}{G}Q_1(x)\right\} - J_2(x)d_2 + 1$$

$$v_3(x) = a_3\left\{J_1(x) - \frac{E}{G}Q_1(x)\right\} - J_2(x)d_3 + x$$

$$v_5(x) = a_5\left\{J_1(x) - \frac{E}{G}Q_1(x)\right\} - J_2(x)d_5$$

$$v_6(x) = a_6\left\{J_1(x) - \frac{E}{G}Q_1(x)\right\} - J_2(x)$$

$$J_1(x) = \int_0^x \int_0^u \frac{x}{I(t)}dtdu$$

$$J_1(x) = \int_0^x \int_0^u \frac{1}{I(t)}dtdu$$

$$Q_1(x) = \int_0^x \frac{1}{A_s(u)}du$$

$$P_1(x) = \int_0^x \frac{u}{I(u)}du$$

$$P_2(x) = \int_0^x \frac{1}{I(u)}du$$

$$a_2 = \frac{P_2}{-J_1P_2 + P_1J_2 + \frac{E}{G}Q_1}$$

$$a_3 = \frac{J_2+P_2 l}{-J_1P_2+P_1J_2+\frac{E}{G}Q_1P_2}$$

$$a_5 = \frac{-P_2}{-J_1P_2+P_1J_2+\frac{E}{G}Q_1P_2}$$

$$a_6 = \frac{J_2}{-J_1P_2+P_1J_2+\frac{E}{G}Q_1P_2}$$

$$d_2 = \frac{P_1}{-J_1P_2+P_1J_2+\frac{E}{G}Q_1P_2}$$

$$d_3 = \frac{-J_1+\frac{E}{G}Q_1+P_1 l}{-J_1P_2+P_1J_2+\frac{E}{G}Q_1P_2}$$

$$d_5 = \frac{-P_1}{-J_1P_2+P_1J_2+\frac{E}{G}Q_1P_2}$$

$$d_6 = \frac{J_1-\frac{E}{G}Q_1}{-J_1P_2+P_1J_2+\frac{E}{G}Q_1P_2}$$

For the linearly varying distributed load shown in Figure 6, equivalent nodal forces are calculated as follows.

$$\{F\} = \{F_1, M_1, F_2, M_2\}^T \tag{A-3}$$

where

$$\begin{aligned} F_1 = {} & a_2\left(\tfrac{w_2-w_1}{l}H_1 + w_1H_2 - \tfrac{E}{G}\tfrac{w_2-w_1}{l}L_1 - \tfrac{E}{G}w_1L_2\right) \\ & -d_2\left(\tfrac{w_2-w_1}{l}H_3 + w_1H_4\right) + \tfrac{w_1+w_2}{2}l \end{aligned}$$

$$\begin{aligned} M_1 = {} & a_3\left(\tfrac{w_2-w_1}{l}H_1 + w_1H_2 - \tfrac{E}{G}\tfrac{w_2-w_1}{l}L_1 - \tfrac{E}{G}w_1L_2\right) \\ & -d_3\left(\tfrac{w_2-w_1}{l}H_3 + w_1H_4\right) + \tfrac{w_1+2w_2}{6}l^2 \end{aligned}$$

$$\begin{aligned} F_2 = {} & a_5\left(\tfrac{w_2-w_1}{l}H_1 + w_1H_2 - \tfrac{E}{G}\tfrac{w_2-w_1}{l}L_1 - \tfrac{E}{G}w_1L_2\right) \\ & -d_5\left(\tfrac{w_2-w_1}{l}H_3 + w_1H_4\right) \end{aligned}$$

$$\begin{aligned} M_2 = {} & a_6\left(\tfrac{w_2-w_1}{l}H_1 + w_1H_2 - \tfrac{E}{G}\tfrac{w_2-w_1}{l}L_1 - \tfrac{E}{G}w_1L_2\right) \\ & -d_6\left(\tfrac{w_2-w_1}{l}H_3 + w_1H_4\right) \end{aligned}$$

$$\begin{aligned} H_1 = {} & \tfrac{1}{30I}(l_4^5 - l_3^5) - \tfrac{l_3^2}{6I}(l_4^3 - l_3^3) + \tfrac{l_3^3}{6I}(l_4^2 - l_3^2) \\ & -\tfrac{1}{6I}(l_4^3 - l_3^3)(l^2 - l_4^2) + \tfrac{1}{6I}(l_4^2 - l_3^2)(l^3 - l_4^3) \end{aligned}$$

$$\begin{aligned} H_2 = {} & \tfrac{1}{24I}(l_4^4 - l_3^4) - \tfrac{l_3^2}{4I}(l_4^2 - l_3^2) + \tfrac{l_3^3}{3I}(l_4 - l_3) \\ & -\tfrac{1}{3I}(l_4^3 - l_3^3)(l - l_4) + \tfrac{1}{4I}(l_4^2 - l_3^2)(l^2 - l_4^2) \end{aligned}$$

$$H_3 = \frac{1}{8I}(l_4^4 - l_3^4) - \frac{l_3}{3I}(l_4^3 - l_3^3) + \frac{l_3^2}{4I}(l_4^2 - l_3^2)$$
$$+\frac{1}{3I}(l_4 - l_3)(l^3 - l_4^3) + \frac{1}{4I}(l_4^2 - l_3^2)(l^2 - l_4^2)$$

$$H_4 = \frac{1}{6I}(l_4 - l_3)^3 - \frac{l}{2I}(l_4^2 - l_3^2)(l - l_2) + \frac{l}{2I}(l_4 - l_3)(l^2 - l_4^2)$$

$$L_1 = \frac{1}{3A_s}(l_6^3 - l_5^3) - \frac{l_5}{2A_s}(l_6^2 - l_5^2) + \frac{1}{2A_s}(l_6 - l_5)(l^2 - l_6^2)$$

$$L_2 = \int_0^l Q_1(x)dx = \frac{1}{2A_s}(l_6 - l_5)^2 - \frac{l}{A_s}(l_6 - l_5)(l - l_6)$$