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Design of an Adaptive Fuzzy Logic Controller Using Sliding Mode Scheme

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ABSTRACT

Using a sole input variable simplifies the design process for the fuzzy logic controller(FLC). This is called *single-input fuzzy logic controller* (SFLC). However, it is still deficient in the capability of adapting to the varying operating conditions. We here design a single-input adaptive fuzzy logic controller(AFLC) using a switching function of the sliding mode control. The AFLC can directly incorporate linguistic fuzzy control rules into the controller. Hence, some parameters of the membership functions characterizing the linguistic terms of the fuzzy rules can be adjusted by an adaptive law. In the proposed AFLC, center values of fuzzy sets are directly adjusted by a fuzzy logic system. We prove that 1) its closed-loop system is globally stable in the sense that all signals involved are bounded and 2) its tracking error converges to zero asymptotically. We perform computer simulation using a nonlinear plant.

1. Introduction

Nowadays, the controlled plants become more complex and large-scaled, and this tendency requires the development of more intelligent control schemes in the control field. To cope with this tendency of the industry, an adaptive scheme is a good alternative. The adaptive controller automatically adjusts some control parameters so that the desired control performance is obtained.

Most FLCs use the error and the change-of-error as fuzzy input variables regardless of the complexity of controlled plants [1,2]. However, these FLCs are suitable for simple lower order plants. That is, in case of complex higher order plants, all process states are commonly required as fuzzy input variables for a good performance. However, it needs a huge number of control rules, membership functions, and scaling factors. In order to simplify the design process of the FLCs, the SFLC was proposed in [3-5]. That is, the proposed SFLC uses only a single input variable for the FLC of an arbitrary controlled plant with the minimum phase property. However, it is still deficient in the capability of adapting to the variation of operating conditions. It can be improved by adding an adaptive scheme.

Compared to conventional adaptive controllers, the AFLC has some advantages. It is capable of incorporating linguistic fuzzy information from experienced human operators into a closed-loop control system. This is especially useful to the complex systems with high nonlinearities and uncertainties. It also can improve the control performance.

Some researches for AFLCs were performed by Wang [6-8]. He proposed various AFLCs using direct or indirect scheme. In his AFLCs, the excessive control actions could be derived in the supervisory control input, which was introduced to guarantee the stability of the closed-loop system. Furthermore some tuning parameters for the desired control performance were required in his researches.

We propose a new AFLC that requires only a sole fuzzy input variable, and uses the sliding mode control scheme instead of a stable error dynamics.

This paper is organized as follows. Firstly we briefly explain the design method for a SFLC in Section II. In Section III, an AFLC incorporating the scheme of the SFLC is designed by utilizing a switching function of the sliding mode control. In Sections IV and V, we represent computer simulations and concluding remarks, respectively.

2. Single-input FLC (SFLC)

Most controlled plants with the minimum phase property have skew-symmetric type rule tables as they were controlled by a FLC using two input variables of error and change-of-error [1, 3]. This property allows us to design a SFLC.

Let the controlled process be a system with n-th order (linear or nonlinear) state equation:

$$x^{(n)} = f(x, t) + b(x, t)u(t) + d(t), \quad y = x, \tag{1}$$

with

$$\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^T = [x, \dot{x}, \cdots, x^{(n-1)}]^T, \tag{2}$$

where $f(x, t)$ and $b(x, t)$ are partially known continuous functions, $d(t)$ is the unknown external disturbance, and $u(t) \in R$ and $y(t) \in R$ are the input and output of the system, respectively. $\boldsymbol{x}(t) \in R^n$ is the process state vector.

The control problem is to force $y(t)$ to follow a given bounded reference input signal $x_d(t)$. Let $e(t)$ be the tracking error vector as follows:

$$\boldsymbol{e}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_d(t) = [e, \dot{e}, \cdots, e^{(n-1)}]^T. \tag{3}$$

Now a SFLC can be designed as follows [3]:
The control rule form for a SFLC is as follows.

R^k: If D_s is LD^k then u is LU^k,

where $k = 1, 2, \cdots, N$, and LD and LU are the linguistic values taken by the general signed distance D_s and the control input u, respectively. And the general signed distance is defined as follows [3]:

$$D_s = \frac{e^{(n-1)} + \lambda_{n-1}e^{(n-2)} + \ldots \lambda_2 \dot{e} + \lambda_1 e}{\sqrt{1 + \lambda_{n-1}^2 + \ldots \lambda_2^2 + \lambda_1^2}}, \tag{4}$$

where $\lambda_i > 0$, $i = 1, 2, \cdots, n\text{-}1$, is a positive constant that determines the slope of a switching hyperplane. Then control rules is simply established on an one-dimensional space like Table 1.

In Table 1, NB, NS, ZR, PS, and PB represents Negative Big, Negative Small, ZeRo, Positive Small, and Positive Big, respectively. This is called *single-input fuzzy logic controller*(SFLC).

The proposed SFLC has many advantages: It needs only one input variable regardless of complexity of the controlled plants. Furthermore, as expressed in Eq. (4), the single input variable has knowledge of all state variables of a controlled plant. Next, the number of tuning parameters for the FLC is greatly decreased since the control rule table is constructed on an one-dimensional space. Hence, tuning of rules, membership functions, and scaling factors is much easier than the case of conventional FLCs using two or more input variables. The SFLC is also equivalent to the sliding mode control with a boundary layer [3-5]. The fact implies that the closed-loop system with a SFLC is stable.

Table 1. Rule table for a SFLC

D_s	NB	NS	ZR	PS	PB
u	PB	PS	ZR	NS	NB

As a result, the SFLC provides a simpler method for the design of conventional FLCsand gives the desired control performance.

3. Design of Adaptive FLC

An adaptive controller is one in which some control parameters are updated by a learning rule. It can improve the overall control performance in the presence of large uncertainties or unknown variations in plant dynamics.

Now we design an AFLC that are fuzzy logic systems equipped with an adaptation algorithm. Compared to conventional adaptive controllers, the AFLC has some advantages: It is capable of incorporating linguistic fuzzy information from experienced human operators into a closed-loop control system. It also improves the control performance.

Fuzzy IF-THEN rules can directly be expressed by a rigorous mathematical equation. This fact allows us to design an AFLC that adjusts some control parameters using the scheme of the SFLC.

The following two Lemmas are derived from the SFLC explained in Section 2.

Lemma 1. The SFLC with the singleton fuzzifier, product inference, and the height defuzzifier is summarized as the following form:

$$u(D_s) = \frac{\sum_{k=1}^{N} \bar{u}^k(\mu_{LD^k}(D_s))}{\sum_{k=1}^{N} (\mu_{LD^k}(D_s))}, \tag{5}$$

where $\bar{u}^k$ is the point in R at which μ_{LU^k} achieves its maximum value (assume that $\mu_{LU^k}(\bar{u}^k) = 1$), and N is the number of one-dimensional control rules.

Lemma 2. Consider the following fuzzy basis function(FBF):

$$\xi^k(D_s) = \frac{\mu_{LD^k}(D_s)}{\sum_{k=1}^{N}(\mu_{LD^k}(D_s))}. \quad (6)$$

Then a fuzzy logic system (5) can be rewritten as Eq. (7).

$$u(D_s) = \Theta_u^T \Xi_u(D_s), \quad (7)$$

where $\Theta_u = [\bar{u}^1, \bar{u}^2, \cdots, \bar{u}^N]^T$ is an adjustable parameter vector, and $\Xi_u(D_s) = [\xi^1(D_s), \xi^2(D_s), \cdots, \xi^N(D_s)]^T$ is a regressive vector composed of FBFs of Eq. (6).

Proofs of two Lemmas are omitted since they can easily be proved from [8].

The fuzzy logic systems in the form of Eq. (5) or (7) are universal approximators [8]. Thus, the fuzzy logic system (7) is qualified as a building block of an AFLC for a nonlinear system. We also see from Eq. (7) that the linguistic information from experienced human operators in the form of the fuzzy IF-THEN rules can directly be incorporated into the controller. That is, the linguistic information from experienced human operators can directly be expressed by rigorous mathematical formula in an AFLC.

The control purpose is to determine a feedback control input

$$u = u_f(D_s \mid \Theta_u) + u_a \quad (8)$$

such that the tracking error should be as small as possible under some constraints, where u_f is an AFLC and u_a is an auxiliary control input to ensure the closed-loop stability.

Consider a switching function $S_l = 0$ that was used in Eq. (4).

$$\begin{aligned} S_l &= 0 \\ &= e^{(n-1)} + \lambda_{n-1}e^{(n-2)} + \cdots + \lambda_2\dot{e} + \lambda_1 e. \end{aligned} \quad (9)$$

We determine the control law u^* when the functions f, b, and d of the controlled plant (1) are known. Here two cases must independently be considered: $S_l = 0$ and $S_l \neq 0$.

In the case of $S_l = 0$, the control law is easily determined as the following equation.

$$u^* = b^{-1}\left(-f - d + x_d^{(n)} - \sum_{i=1}^{n-1}\lambda_i e^{(i)}\right). \quad (10)$$

When $S_l \neq 0$, the control law can be derived from the concept of the sliding mode control. That is, it can be determined by the following sliding condition:

$$S_l\dot{S}_l \leq -\eta \mid S_l \mid, \quad (11)$$

where η is a positive constant. From Eq. (9),

$$\begin{aligned} \dot{S}_l &= e^{(n)} + \lambda_{n-1}e^{(n-1)} + \cdots \lambda_1\dot{e} \\ &= e^{(n)} + \sum_{i=1}^{n-1}\lambda_i e^{(i)} \\ &= f + bu + d - x_d^{(n)} + \sum_{i=1}^{n-1}\lambda_i e^{(i)}. \end{aligned} \quad (12)$$

Multiplying both sides of Eq. (12) by S_l,

$$\begin{aligned} S_l\dot{S}_l &= S_l\left(f + bu + d - x_d^{(n)} + \sum_{i=1}^{n-1}\lambda_i e^{(i)}\right) \\ &\leq -\eta \mid S_l \mid. \end{aligned} \quad (13)$$

From Eq. (13), we can get the following control law:

$$u^* \begin{cases} \leq b^{-1}\left(-f - d + x_d^{(n)} - \sum_{i=1}^{n-1}\lambda_i e^{(i)} - \eta\right) \text{for } (S_l > 0) \\ \geq b^{-1}\left(-f - d + x_d^{(n)} - \sum_{i=1}^{n-1}\lambda_i e^{(i)} + \eta\right) \text{for } (S_l < 0) \end{cases} \quad (14)$$

Combining Eq. (10) and (14), we obtain the following closed form for the control law.

$$u^* = b^{-1}\left(-f - d + x_d^{(n)} - \sum_{i=1}^{n-1}\lambda_i e^{(i)} - \rho\, sgn(S_l)\eta_m\right), \quad (15)$$

where

$$\rho = \begin{cases} 1 \text{ for } S_l \neq 0 \\ 0 \text{ for } S_l = 0 \end{cases} \quad (16)$$

and

$$\eta_m \geq \eta. \quad (17)$$

Commonly, we don't know exact information about the controlled plant (1) except for the sign of $b(x, t)$. Adding and subtracting bu^* in the right side of Eq. (12),

$$\begin{aligned} \dot{S}_l &= f + b(u_f + u_a) + d - x_d^{(n)} + \sum_{i=1}^{n-1}\lambda_i e^{(i)} + bu^* - bu^* \quad (18) \\ &= b(u_f - u^*) + bu_a - \rho sgn(S_l)\eta_m. \end{aligned}$$

Consider an optimal parameter Θ_u^* and minimum approximation error, ε_u, related to the control input:

$$\Theta_u^* = \arg\min_{\Theta_u}[\sup_x \mid u_f(D_s \mid \Theta_u) - u^* \mid], \quad (19)$$

and

$$\varepsilon_u = u_f^* - u^*, \tag{20}$$

where $u_f^* = u_f(D_s | \Theta_u^*)$. ε_u has a very small value due to the universal approximating property of the fuzzy logic system [8]. That is,

$$|\varepsilon_u| = |u_f^* - u^*| \le \varepsilon, \tag{21}$$

where $\varepsilon > 0$ is a small value. And Eq. (18) can be rewritten as

$$\begin{aligned}\dot{S}_l &= b(u_f - u_f^*) + b\varepsilon_u + bu_a - \rho\, sgn(S_l)\eta_m \\ &= b\Phi_u^T \Xi_u + b\varepsilon_u + bu_a - \rho\, sgn(S_l)\eta_m,\end{aligned} \tag{22}$$

where $\Phi_u = \Theta_u - \Theta_u^*$ and Ξ_u is the FBF that is defined by the Lemma 2.

Now we replace the $u_f(D_s | \Theta_u)$ by a fuzzy logic system (7) and develop an adaptive law to update the parameter vector Θ_u.

Theorem. Consider the control law (15) and switching function (9). If we choose the auxiliary control input u_a as

$$u_a \le -sgn(bS_l)\,|\varepsilon_u|. \tag{23}$$

then the proposed system is stable in the sense of the Lyapunov and the parameter adaptation law is given as

$$\dot{\Theta}_u = -sgn(b)\gamma S_l \Xi_u, \tag{24}$$

where γ is a positive constant that determines a kind of learning rate.

Proof. Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}S_l^2 + \frac{|b|}{2\gamma}\Phi_u^T\Phi_u. \tag{25}$$

Then,

$$\dot{V} = S_l(b\varepsilon_u + bu_a - sgn(S_l)\eta_m) + \frac{|b|}{\gamma}\Phi_u^T(\dot{\Phi}_u + sgn(b)\gamma S_l \Xi_u). \tag{26}$$

From Eq. (26), we can easily obtain the parameter adaptation law (24) because $\dot{\Phi}_u = \dot{\Theta}_u$. Also if we choose the auxiliary control input such that the given condition (23) is satisfied, then Eq. (26) is summarized as follows:

$$\dot{V} \le -\rho\eta_m. \tag{27}$$

Thus, the proposed AFLC is stable in the sense of the Lyapunov. □

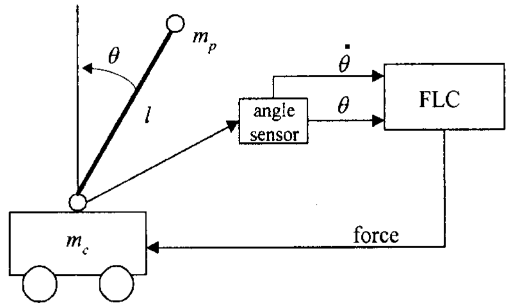


Fig. 1. The inverted pendulum control system

4. Simulation Example

Now we reveal the performance of the proposed AFLC via computer simulation. We consider a tracking problem for the inverted pendulum system. Fig. 1 shows the plant composed of a pole and a cart. The cart moves on the rail tracks in horizontal direction.

The control objective is to balance the pole starting from an arbitrary condition by supplying a suitable force to the cart. For simplicity, we do not consider the position of the cart. The plant dynamics is then expressed as:

$$\ddot{\theta} = \frac{g\sin\theta + a\cos\theta - u_p w^2 l\cos\theta\sin\theta}{l(4/3 - \mu_p\cos^2\theta)} \tag{28}$$

$$\mu_p = \frac{m_p}{m_p + m_c} \tag{29}$$

$$a = \frac{F}{m_p + m_c}, \tag{30}$$

where g is an acceleration due to gravity(= 9.8 m/sec^2), and F is the applied force. m_c(= 1.0 kg) and m_p(= 0.1 kg) are masses and l(= 0.5 m) is the pole length.

Fig. 2 represents the fuzzy sets for the control input and signed distance. In the case of the SFLC, we used

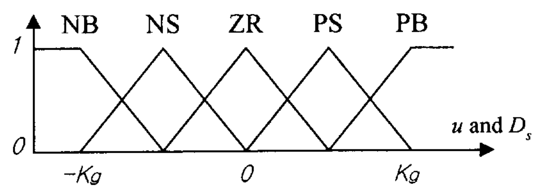
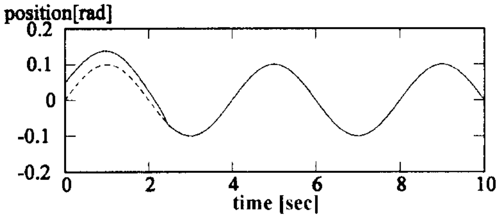
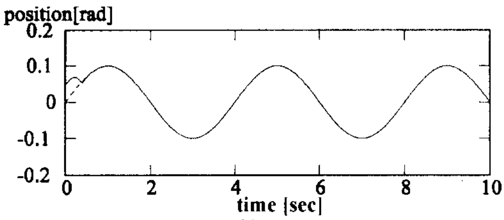


Fig. 2. The fuzzy sets for simulations

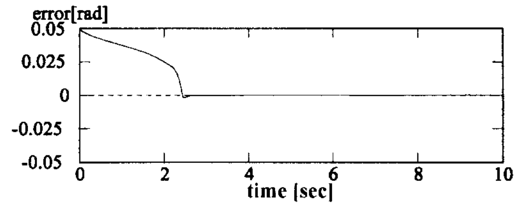


(a) SFLC.

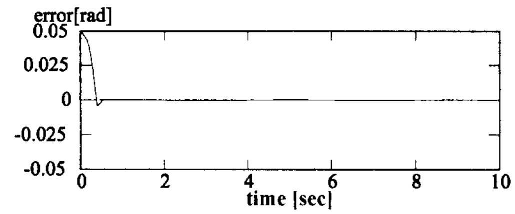


(b) AFLC.

Fig. 3. Comparison of tracking performances

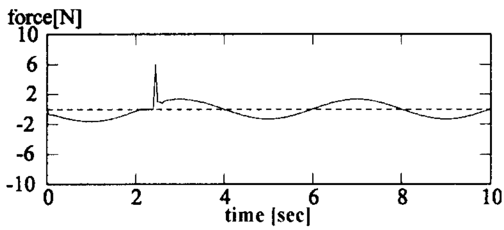


(a) SFLC.

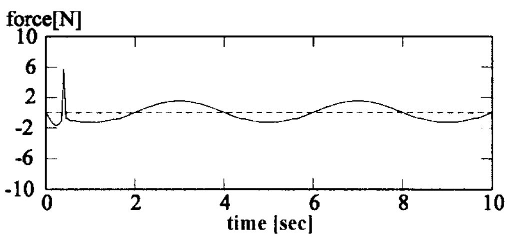


(b) AFLC.

Fig. 5. Comparison of tracking errors

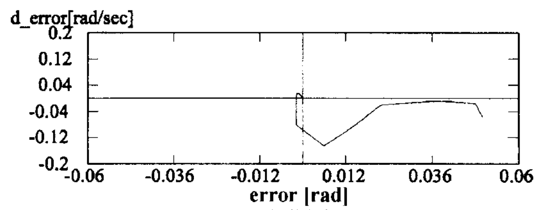


(a) SFLC.

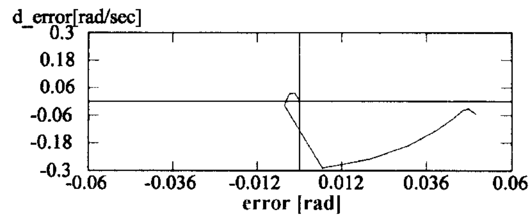


(b) AFLC.

Fig. 4. Comparison of control inputs



(a) SFLC.



(b) AFLC.

Fig. 6. Comparison of phase portraits

6 and 50 for K_g of s and D_s, respectively. The center values of membership functions for the control input are automatically adjusted by an adaptation law in the AFLC. Simulation condition of the AFLC is equally set to the case of the SFLC except for the addition of the parameter adaptation law. We use the product inference and the height defuzzification.

Figures 3, 4, 5, and 6 show the simulation results of tracking performances, control inputs, tracking errors, and phase portraits, respectively. Here (a) and (b) are the cases of the SFLC and the proposed AFLC, respectively. As shown in figures, the control performance of the proposed AFLC is better than that of the SFLC. As a results, an adaptive scheme can improve the performance of the conventional case without any adaptive scheme.

5. Concluding Remarks

Conventional FLCs had many tuning parameters such as rules, membership functions, scaling factors, and so on. We first explained the SFLC using a sole input variable. It had many advantages. Particularly, it could greatly decrease the difficulty on the design of the conventional FLCs. And then we proposed an AFLC using the scheme of the SFLC. The concept simplified the design of the AFLC as well as decreased the number of tuning parameters for the FLC.

The proposed AFLC was based on a switching function of the sliding mode control. It had no excessive control actions as well as small number of tuning parameters. If the rule base is constructed with sufficiently many rules, the role of the auxiliary control input for the closed loop stability will disappear from the universal approximating property of a fuzzy logic system. And the closed-loop stability of the proposed AFLC was ensured in the sense of the Lyapunov. Finally, we performed a computer simulation using the inverted pendulum system. Here we showed that the AFLC can improve the control performance of the SFLC.

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