한국 퍼지 및 지능시스템학회 논문지 1999, Vol. 9, No. 4. (1999)384~388

Modularized Gain Scheduled Fuzzy Logic Control with Application to Nonlinear Magnetic Bearings

Sung Kyung Hong

Flight Dynamics and Control Team (3-3-5), Agency for Defense Development

ABSTRACT

This paper describes an approach for synthesizing a modularized gain scheduled PD type fuzzy logic controller (FLC) of a nonlinear magnetic bearing system, where the gains of FLC are on-line adapted according to the operating point. Specifically, the systematic procedure via root locus technique is carried out for the selection of the gains of FLC. Simulation results demonstrate that the proposed gain scheduled fuzzy logic controller yields not only maximization of stability boundary but also better control performance than a single operating point (*without gain scheduling*) fuzzy controller.

Key word: Nonlinear magnetic bearing, Fuzzy control, Gain scheduling

1. Introduction

The active magnetic bearing (AMB), as an alternative to the conventional mechanical bearing, levitates a rotor by magnetic forces without any contact. Because of their frictionless nature and lubrication free operation, magnetic bearings have distinct advantages over conventional mechanical bearings. Despite great potentials, the highly nonlinear and inherently open-loop unstable dynamics of AMBs, coupled with rotor dynamics, present unique closed-loop control challenges that hamper widespread commercial use. Much of the literature concentrates on linear control techniques by linearizing the dynamics of magnetic bearing system about a nominal equilibrium point. This approach, however, is generally effective only in a limited small region of the nominal design condition. Nonlinear control technique such as sliding mode and feedback linearization has also been proposed [1][2]. These approaches, however, tend to result in rather complicated algorithms and are sensitive to modeling error. On the other hand, fuzzy logic control (FLC) has been considered as a potentially useful means of designing controller for nonlinear systems. In [3], [6], and [7] fuzzy logic has been successfully applied to the design of a gain scheduled controller to compensate for plant nonlinearity. In [4], by applying fuzzy logic to adjust the signal of a PID controller in such a way that nonlinear effects of magnetic bearing are compensated, operating range has been extended. However, most of these approaches tune scaling *factors*[1] (for normalization and/ or denormalization) or fuzzy rules recursively until they reach the desired level of performance. This has been viewed as lacking systematic analysis. In this paper, the systematic procedure via root locus technique, motivated by the equivalence between PD type FLC and conventional PD controller, is carried out for the selection of the gains of FLC. In addition, to maximize the stable boundary and obtain the desired uniform performance for the entire clearance of AMB, two modularized fuzzy logic based gain schedulers are augmented to the pure FLC, where the gains are online adapted according to the operating points. As a result, the one of uniqueness of this methodology lies in the compensation of the limited capability of FLC with fuzzy logic gain scheduling.

2. Dynamic Model of A Magnetic Bearing

The magnetic bearing system employed in this research is a 2-axis controlled vertical shaft magnetic bearing with symmetric structure. To simplify, we assume that the rotor acts as a rigid floating mass, and that the horizontal and vertical dynamics are uncoupled. Therefore, in this paper, the system can be divided into two identical sub systems (x and y directions), which means that each gap displacement can be controlled individually. Thus, without loss of generality, we will focus our analysis strictly on the x direction motion. The nonlinear equation of motion can be expressed

[1]It will be called, hereafter, as *gains* for the similarity with gain coefficients of conventional PID controllers

approximately as [5]:

$$m\ddot{x} = k\left[\frac{(i_b+i_p)^2}{(G-\beta x)^2} - \frac{(i_b-i_p)^2}{(G+\beta x)^2}\right] \qquad (1)$$

where, k is force constant, i_p is the control current, x is rotor displacement, G is the maximum air gap, i_b is the bias current, and β is the sensitivity of the air gap to rotor displacement. The physical parameters of this setup are given as follows [6]: $k = 0.0186$ ($lb\ in^2/amp^2$), $\beta = 0.974$, $i_b = 0.3$ (amp), $G = 0.02$ ($inch$), $m = 0.0126$ ($lb \cdot sec^2/in$). By taking the Taylors series expansion for three equilibrium points as shown in Fig. 1, the nonlinear equations of motion (1) can be represented by the piecewise linear state space equations as shown in Table 1.

3. Gain Scheduled fuzzy PD Controller

The structure of the proposed FLC is modularized into a fuzzy feedback loop and two fuzzy gain scheduling modules as shown in Fig. 2. A fuzzy rule base is used to determine a feedback control law and the others schedule fuzzy gains with respect to operating point. Detail design considerations are given in the following subsections.

3.1 PD type fuzzy feedback controller

The functional relationship represented by the feedback module can be expressed as

$$u(t) = f(e,\ \Delta e;\ K_e,\ K_{de}) \cdot K_u \qquad (2)$$

where, K_e, K_{de}, and K_u are gains which weight the input and output variables, and f is a nonlinear function of the fuzzy logic feedback controller. The control rules and the membership functions of fuzzy sets for e, de, and u, which are used in this paper are given in Fig. 3. and Fig. 4, respectively. In the design point of view, one of the main problems that arises in designing FLC is the lack of systematic procedures for tuning gains. Motivated by the equivalence between PD type FLC and conventional PD controller, we attempt to present a systematic procedure for tuning gains via conventional root locus techniques.

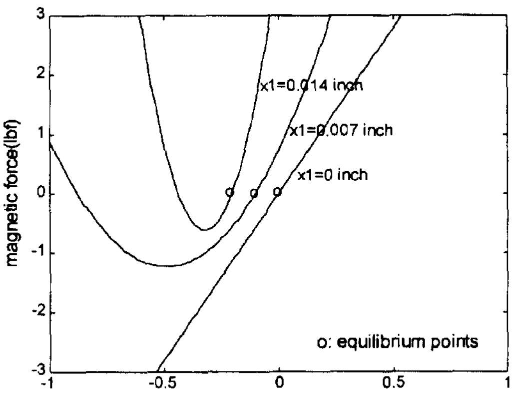


Fig. 1. Characteristics of Magnetic Force.

Table 1. Piecewise Linear System Model

Equilibrum Point	Piecewise Linearized Model
$x_1 = 0.0000\ i_p = 0.0$	$\dot{x} = \begin{bmatrix} 0 & 1 \\ 6489.7 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 444.2 \end{bmatrix} u$
$x_2 = 0.007\ i_p = 0.1$	$\dot{x} = \begin{bmatrix} 0 & 1 \\ 7357.3 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 503.0 \end{bmatrix} u$
$x_1 = 0.014\ i_p = 0.2$	$\dot{x} = \begin{bmatrix} 0 & 1 \\ 12053.0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 827.5 \end{bmatrix} u$

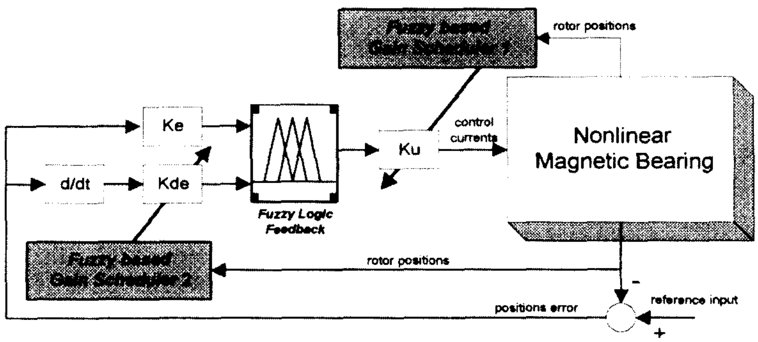


Fig. 2. Block diagram of AMB fuzzy gain scheduled control system.

de \ e	NB	NS	ZE	PS	PB
NB	NB	NB	NB	NS	ZE
NS	NB	NB	NS	ZE	PS
ZE	NB	NS	ZE	PS	PB
PS	NS	ZE	PS	PB	PB
PB	ZE	PS	PB	PB	PB

Fig. 3. Control rules.

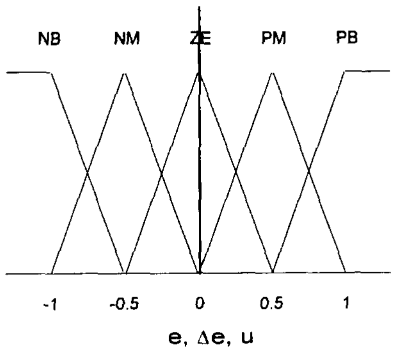


Fig. 4. Membership functions.

Equivalence between FLC-PD and conventional PD controller.

In fuzzy PD type controller, the relationship between input variables and output control action can be approximated by linear functions (i.e. equivalent of conventional PD controller) as Eqn. (3) when the membership functions are defined in simple form (triangular or trapezoidal) but of symmetrical form from their center [3].

$$u(t) = (K_u/K_e)e(t)+(K_u/K_{de})(de(t)dt) \tag{3}$$

Comparing Eqn.(3) and conventional PD controller, we can obtain following relations:

$$K_p = (K_u/K_e),\ K_D=(K_u/K_{de}) \tag{4}$$

As a qualitative proof for this equivalence of Eqn.(4), we can verify that tuning of the fuzzy variable gains gives direct influence on the controller performance and stability. Thus tuning of the gains of fuzzy variables has been shown to be the most crucial part of FLC design [3].

Tuning the gain of fuzzy variables via Root Locus Method.

The open loop transfer function of the magnetic bearing at the nominal equilibrium point (x_1=0, i_p=0) can be obtained from the piecewise linearized model in Table I. With the PD-like fuzzy controller (3), the characteristic equation can be conditioned as follows:

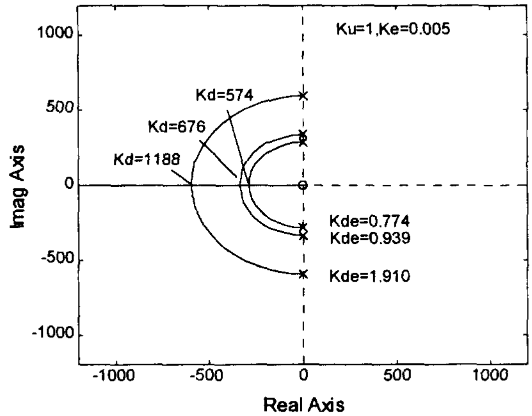


Fig. 5. Root locus for piecewise linearized model.

$$1 + G_{eq}(s) = 1 + \frac{444.2(K_u/K_{de})s}{s^2 + 444.2(K_u/K_e) - 6489.7} = 0 \tag{5}$$

The root locus of Eqn.(5), with both K_u and K_e being constants, can be constructed based on the pole-zero configuration of $G_{eq}(s)$. By same procedure, we can build root loci for the other piecewise linearized models. A family of three root loci for each equilibrium point are shown in Fig. 5 with pre-fixed gains of K_e=0.005 and K_u=1. Using the root loci, the gains for change of error (K_{de}) for each equilibrium point are obtained so as to have critical damping for the given parameters.

3.2 Fuzzy gain scheduling

Maximize the stability boundary via Ku-gain scheduling

Although fuzzy logic feedback control module improves performance beyond linear control via forming a linguistic based nonlinear control strategy, it cannot fully differentiate in which region the rotor operates. From the Fig. 1, we can notice that at the each operating point of rotor position, its extreme point appears differently. This means that without being dynamically limited by the magnitude of control output with respect to the rotor position, the system cannot maintain stability by one control law that covers only one part from dual sense dynamics [6]. Motivated by the fact that the output denormalization gain (K_u) can directly influence the magnitude of control output, we scheduled K_u to limit the magnitude of control output

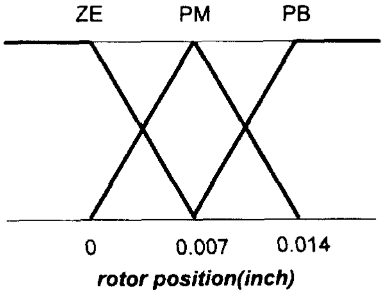


Fig. 6. Membership functions for rotor position.

according to the rotor position. The membership function and rules used for K_u-gain scheduling are given in Fig. 6 and Eqn. (6), respectively. However, it is not still guaranteed that the FLC with K_u-gain scheduler has the uniform performance for the entire operating region.

$$\begin{array}{l} \text{Rule 1 :if } x_1 \text{ is ZE then } K_u^1 = 1 \\ \text{Rule 2 :if } x_1 \text{ is PM then } K_u^2 = 0.5 \\ \text{Rule 3 :if } x_1 \text{ is PB then } K_u^3 = 0.3 \end{array} \quad (6)$$

Improve the performance via Kde-gain scheduler. Motivated by the fact that the normalization gain (K_{de}) has direct influence on controller performance, we utilized the fuzzy rules and reasoning to dynamically tune K_{de} of the feedback fuzzy controller. Each nominal gain for change of error (K_{de}) in fuzzy controller can be tuned for each operating point via root locus method as in the previous sections. To achieve desired control performance, fuzzy interpolated gain is used when the operating point deviates from nominal design conditions. The rules used for K_{de}-gain scheduling are given in (7).

$$\begin{array}{l} \text{Rule 1 : if } x_1 \text{ is ZE then } K_{de}^1 = 0.774 \\ \text{Rule 2 : if } x_1 \text{ is PM then } K_{de}^2 = 0.939 \\ \text{Rule 3 : if } x_1 \text{ is PB then } K_{de}^3 = 1.910 \end{array} \quad (7)$$

4. Simulation Results

Fig. 7 shows the responses of the closed-loop simulations of both conventional linear PD controllerand Fuzzy PD type controller. The conventional PD linear controller performs well when the rotor position is close to the designed operating point, but their effectiveness deteriorates quickly outside of the small region of stability boundary ($x_1 \leq 0.0062$ *inch*). As much as 19% extended stability boundary ($x_1 \leq 0.0073$ *inch*) is achieved by pure fuzzy logic controller. Although fuzzy controller achieves wider stability boundary than conventional PD controller's, it still can not fully overcome the nonlinearity of the AMB system for the entire operating regions. In fact, it should be noted that a fuzzy controller is simply a nonlinear version of conventional controllers. Therefore the pure fuzzy controller which uses only control error and change of error as input variables is not able to detect the operating point and make a control move accordingly. By applying K_u-gain scheduler to the pure fuzzy controller, maximized stability boundary ($x_1 \leq$ 15.00 *inches*) is achieved as shown in Fig. 8(a). Fig. 8(b) shows that fuzzy controller augmented with both K_u and K_{de} gain scheduling obtains improved response regardless of any operating points.

5. Conclusion

Nonlinear fuzzy control of an AMB by modularized fuzzy gain scheduling has been described in this paper.

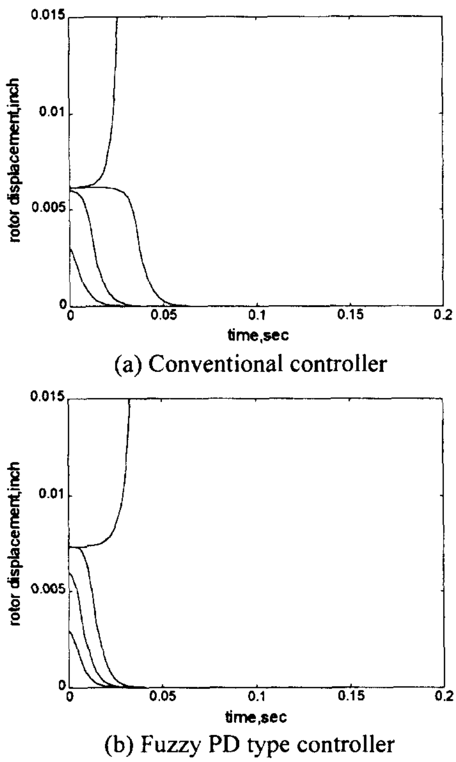
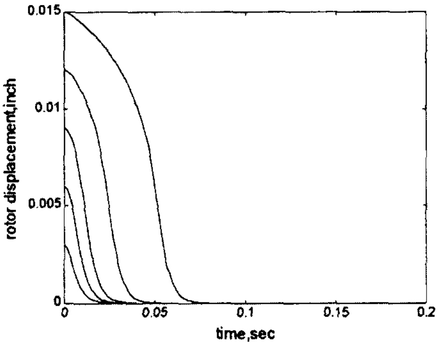
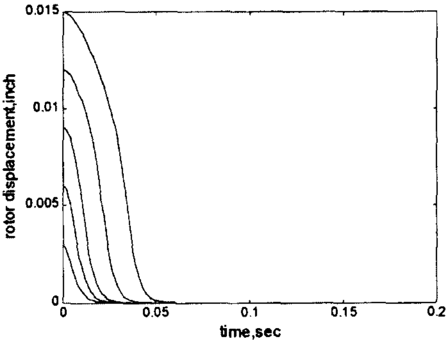


Fig. 7. Comparison of responses of the conventional PD and Fuzzy PD type controller.

[2] The conventional linear PD controller was designed by the same procedure of root locus technique which is illustrted in section 2



(a) FLC with Ku-scheduling



(b) FLC with both Ku and Kde-scheduling

Fig. 8. Responses of fuzzy gain scheduled FLC.

First, several piecewise linearized mathematical models for the AMB system were derived. Specifically, the selection of the gains of FLC has been carried out by systematic procedure via conventional root locus technique. To achieve maximum stability boundary and improved performance, we augmented K_u-gain scheduler and K_{de}-gain scheduler to the pure FLC. Through simulations it can be seen that the gain scheduled fuzzy logic controller proposed in this paper yields not only maximized stability boundary but also better control performance than a single operating point FLC.

REFERENCES

[1] R. D. Smith, and W. F. Weldon, "Nonlinear Control of a Rigid Rotor Magnetic Bearing System: Modeling and Simulation with Full State Feedback,", *IEEE Trans. on Magnetics,* vol. 31, No. 2, (1995), pp. 973-980.

[2] J. Y. Hung, "Nonlinear Control of Electromagnetic Systems," in Proc. 17th Annu. Conf. *IEEE Ind. Electron. Soc.*, Kobe, Japan, (Nov. 1991), pp. 268-273.

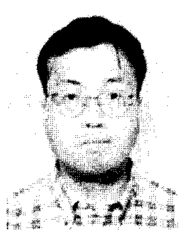
[3] S. K. Hong, and R. Langari, "Gain Scheduled Fuzzy Control with Application to Aircraft Flight Control," *Advances in Systems Science and Applications,* Vol. 3, No. 1,(1997), pp. 393-401, Special Issues

[4] J. Y. Hung, "Magnetic Bearing Control Using Fuzzy Logic," *IEEE Trans. on Industry Applications,* vol. 31, No. 6, (1995), pp. 1492-1497.

[5] E. H. Maslen, P. Hermann, M. Scott and R. R. Humphris, "Practical Limits to the Performance of Magnetic Bearings: Peak Force, Slew Rate and Displacement Sensitivity," *ASME Journal of Tribology,* vol. 111, (1989), pp. 331-336.

[6] S. K. Hong, and R. Langari, "Fuzzy Modeling and Control of a Nonlinear Magnetic Bearing System," To appear *in Journal of Intelligent & Fuzzy Systems,* 1999

[7] S. K. Hong and R. Langari, Robust Fuzzy Control of a Magnetic Bearing System Subject to Harmonic Disturbances, To appear in *IEEE Trans. on Control System Technology,* 1999.



홍 성 경 (Sung-Kyung Hong)
1987년 2월 : 연세대학교 기계공학과 졸업(석사)
1989년 2월 : 연세대학교 대학원 기계공학과 졸업(공학석사)
1998년 8월 : 미 Texas A&M University 기계공학과 졸업(공학박사)
1989년 3월~현재 : 국방과학연구소 선임연구원
주관심분야 : 퍼지제어, LMI-based 제어, 비선형 강건제어