Asymptotic Performance of ML Sequence Estimator Using an Array of Antennas for Coded Synchronous Multiuser DS-CDMA Systems

Sang G. Kim, Byung K. Yi, and Raymond Pickholtz

Abstract: **The optimal joint maximum-likelihood sequence estimator using an array of antennas is derived for synchronous direct sequence-code division multiple access (DS-CDMA) systems. Each user employs a rate 1/n convolutional code for channel coding for the additive white Gaussian noise (AWGN) channel. The array receiver structure is composed of beamformers in the users' directions followed by a bank of matched filters. The decoder is implemented using a Viterbi algorithm whose states depend on the number of users and the constraint length of the convolutional code. The asymptotic array multiuser coding gain (AAMCG) is defined to encompass the asymptotic multiuser coding gain and the spatial information on users' locations in the system. We derive the upper and lower bounds of the AAMCG. As an example, the upper and lower bounds of AAMCG are obtained for the two user case where each user employes the maximum free distance convolutional code with rate 1/2. The near-far resistance property is also investigated considering the number of antenna elements and user separations in the space.**

Index Terms: **Channel coding, jointly maximum-likelihood sequence estimation, multiuser detection, asymptotic array multiuser coding gain.**

I. INTRODUCTION

Direct-sequence code division multiple access (DS-CDMA) scheme has drawn much attention in cellular and personal communications systems due to its capacity increase, robustness to multipath environments, and soft handoff. Conventional detection strategy treats the signals from other users as noise and is not optimum in terms of bit error rate. Verdu [1] proposed and analyzed the optimum maximum likelihood sequence estimator (MLSE) for asynchronous Gaussian multiple-access channels. Since the MLSE has high computational complexity that grows exponentially with the number of users, most of the work in this research field has been focused on finding suboptimum detectors with linear complexity [2]–[9]. While multiuser detection (MUD) provides improved performance and capacity, relatively little work has been done on employing it combined with error correction coding. Only recent works have been focused on the coded MUD [10]–[12]. The optimum jointly maximum likelihood estimation (JMLSE) and decoding was initially proposed and analyzed by Giallorenzi and Wilson for convolutionally coded system with rate 1/2 over a single-path AWGN channel. Shama took a different approach for JMLSE of coded asynchronous multiuser systems with code rate $1/n$. All of the works cited above assumed signal reception using single antenna element.

In recent years, the application of an array of antennas has been suggested for mobile communication systems to overcome the problem of limited channel bandwidth, to satisfy an ever growing demand for a large number of mobiles on communications channels [13]. The application of an array of antennas to uncoded MUD problem has been reported from many authors [14]–[16]. Miller and Schwartz extended Verdu's work and derived the optimum multi-element sequence detector. Lee and Pickholtz obtained the exact performance of maximum likelihood array detector for synchronous DS-CDMA systems and showed that a significant improvement can be achieved. Brown and Kaveh proposed a receiver structure that is a multichannel extension of the coherent decorrelating detector.

In this paper, we derive the optimum joint MLSE using an array of antennas for synchronous DS-CDMA systems in which each user uses rate $1/n$ convolutional code to improve its performance on a single-path AWGN channel. This paper is organized as follows. In Section II, we present the system model and clarify the notations used. In Section III, we derive the optimum JMLSE. In Section IV, we derive the expression for the upper and lower bounds on the probability of error and analyze the performance in terms of asymptotic behavior and near-far resistance. Numerical examples are given in Section V. Concluding remarks are given in Section VI.

II. SYSTEM MODEL

We consider a one-cell network where all users transmit to the base station equipped with an array of antennas. Each user first encodes L_u bits of uncoded block via an encoder that produces L_c bits of coded block. Since the encoder generates n coded bits for each information bit, $L_c = nL_u$. Without loss of generality, we rely on the following assumptions.

- The array is composed of N sensors taken to be omnidirectional.
- A uniform linear array is employed where N sensors are

Manuscript received March 24, 1999; approved for publication by Dong In Kim, Division II Editor, September 20, 1999.

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uniformly spaced with displacement Δ.

- The sources are far enough that all incoming signals are plane waves.
- The array response to each source is so slowly varying that it remains constant over L_c.

Following the formulations given in [12], [14], we express the received signal as

$$\mathbf{r}(t) = \sum_{k=1}^{K}\sum_{i=0}^{L_u-1}\sum_{j=0}^{n-1}\sqrt{E_k(i,j)}d_k(i,j)s_k[t-(i\cdot n+j)T_c]\mathbf{a}_k+\mathbf{n}(t), \quad (1)$$

where $E_k(i,j)$ is the received energy of the jth coded bit of the kth user at time index i, $d_k(i,j) \in \{1,-1\}$ is the corresponding coded bit value, T_c is the coded bit duration, $s_k(t)$ is the time-limited, real, and unit energy user-specific signature sequence, such that $s_k(t)=0$ if $t \notin [0,T_c]$ and $\int_0^{T_c} {s_k}^2(t)dt=1$, and $\mathbf{a}_k$ is the array response vector of the kth user. We denote the crosscorrelation between signature sequences as $\rho_{i,j} = \int_0^{T_c} s_i(t)s_j(t)dt$. Then the array response vector can be written as

$$\mathbf{a}_k = [1, e^{-j\theta_{k1}} \cdots e^{-j\theta_{kN-1}}]^T, \quad (2)$$

where $\theta_{ki} = 2\pi i\left(\frac{\Delta}{\lambda}\right)\sin\theta_k,\ i=0,1,\cdots,N-1,\ \lambda$ is the wavelength of the carrier, and θ_k is the direction-of-arrival (DOA) of the kth user signal. The directions matrix $\mathbf{A} \in Z^{N\times K}$ is given by $(\mathbf{a}_1 \cdots \mathbf{a}_k)$ where Z denotes the complex numbers. $\mathbf{n}(t)$ is an $(N\times 1)$ vector of temporally and spatially white, complex, zero-mean, and independent Gaussian random noise with its real and imaginary power spectral density $\sigma^2 = \frac{N_o}{2}$. It is not difficult to show that the set of matched filter outputs given by

$$y_k(i,j) = \mathrm{Re}\left\{\hat{\mathbf{a}}_k^H \int_{(i\cdot n+j)T_c}^{(i\cdot n+j+1)T_c} \mathbf{r}(t)s_k[t-(i\cdot n+j)T_c]dt\right\} \quad (3)$$

forms a set of sufficient statistics [14]. $(\cdot)^H$ denotes complex-conjugate transpose and Re denotes real part and $\hat{\mathbf{a}}_k$ denotes the estimated array response vector for user k. Then, the composite vector of matched filter outputs

$$\mathbf{y} = [y_1(0,0) \cdots y_K(0,0) \cdots y_1(L_u-1,n-1) \cdots y_K(L_u-1,n-1)]^T \quad (4)$$

may be represented by

$$\mathbf{y} = \mathbf{HEd}+\mathbf{n}, \quad (5)$$

where $\mathbf{H}$ is a block diagonal matrix with dimension $KL_un \times KL_un$

$$\mathbf{H} = \begin{bmatrix} \mathbf{M} & 0 & \cdots & 0 \\ 0 & \mathbf{M} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{M} \end{bmatrix}, \quad (6)$$

and $\mathbf{E}$, $\mathbf{d}$, $\mathbf{n}$, $\mathbf{M}$ are shown in Eq. (7)–(10) at the bottom of this page, where $\otimes$ denotes the Kronecker product and $\Re$ denotes the correlation matrix. $\mathbf{n}$ is a zero mean Gaussian random vector with covariance matrix $N_o\hat{\mathbf{H}}$. $\hat{\mathbf{H}}$ is the same form of matrix given in (6) except $\hat{\mathbf{M}}$ instead of $\mathbf{M}$ where (i,j)th component of is given by $\mathrm{Re}(\hat{\mathbf{a}}_i^H\hat{\mathbf{a}}_j)\rho_{i,j}$. In this paper, we assume that we have perfect estimates of the DOA's. Hence, $\hat{\mathbf{H}} = \mathbf{H}$. Although this assumption is idealistic for practical situations, it can serve as a best reference model and sets upper bounds on the achievable performance.

III. OPTIMUM JOINT ML SEQUENCE ESTIMATOR

Conditioned on $\mathbf{d}$, $\mathbf{y}$ is a multivariate Gaussian random vector with the conditional *pdf*

$$f_{\mathbf{Y}}(\mathbf{y}|\mathbf{d}) = \frac{1}{(2\pi N_o)^{(KLn/2)}(\det[\mathbf{H}])^{1/2}} \times \exp\left[-\frac{1}{2N_o}(\mathbf{y}-\mathbf{HEd})^T\mathbf{H}^{-1}(\mathbf{y}-\mathbf{HEd})\right]. \quad (11)$$

$$\mathbf{E} = diag\left[\sqrt{E_1(0,0)}\cdots\sqrt{E_K(0,0)}\cdots\sqrt{E_1(L_u-1,n-1)}\cdots\sqrt{E_K(L_u-1,n-1)}\right] \quad (7)$$

$$\mathbf{d} = [d_1(0,0) \cdots d_K(0,0) \cdots d_1(L_u-1,n-1) \cdots d_K(L_u-1,n-1)]^T \quad (8)$$

$$\mathbf{n} = [n_1(0,0) \cdots n_K(0,0) \cdots n_1(L_u-1,n-1) \cdots n_K(L_u-1,n-1)]^T \quad (9)$$

$$\mathbf{M} = \mathrm{Re}\left(\hat{\mathbf{A}}^H\mathbf{A}\right)\otimes\Re = \begin{bmatrix} \mathrm{Re}(\hat{\mathbf{a}}_1^H\mathbf{a}_1) & \mathrm{Re}(\hat{\mathbf{a}}_1^H\mathbf{a}_2)\rho_{1,2} & \cdots & \mathrm{Re}(\hat{\mathbf{a}}_1^H\mathbf{a}_K)\rho_{1,K} \\ \mathrm{Re}(\hat{\mathbf{a}}_2^H\mathbf{a}_1)\rho_{2,1} & \mathrm{Re}(\hat{\mathbf{a}}_2^H\mathbf{a}_2) & \cdots & \mathrm{Re}(\hat{\mathbf{a}}_2^H\mathbf{a}_K)\rho_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Re}(\hat{\mathbf{a}}_K^H\mathbf{a}_1)\rho_{K,1} & \mathrm{Re}(\hat{\mathbf{a}}_K^H\mathbf{a}_2)\rho_{K,2} & \cdots & \mathrm{Re}(\hat{\mathbf{a}}_K^H\mathbf{a}_K) \end{bmatrix}_{K\times K} \quad (10)$$

Note that $\mathbf{M} = \mathbf{M}^T$, so $\mathbf{H} = \mathbf{H}^T$. Then, after some manipulation, we have

$$f_{\mathbf{Y}}(\mathbf{y}|\mathbf{d}) = K \cdot \exp\left[-\frac{1}{2N_o}\Omega(\mathbf{d})\right], \tag{12}$$

where

$$K = \frac{1}{(2\pi N_o)^{(KLn/2)}(\det[\mathbf{H}])^{1/2}} \exp\left(-\frac{1}{2N_o}\mathbf{y}^T\mathbf{H}^{-1}\mathbf{y}\right) \tag{13}$$

and

$$\begin{aligned}\Omega(\mathbf{d}) &= \mathbf{d}^T\mathbf{E}(\mathbf{HEd} - 2\mathbf{y}) \\ &= \sum_{i=0}^{L_u-1} \mathbf{d}^T(i)\mathbf{E}(i)\left[\mathbf{ME}(i)\mathbf{d}(i) - 2\mathbf{y}(i)\right] \\ &= \sum_{i=0}^{L_u-1} \omega_i\left[\mathbf{d}(i-1), \mathbf{d}(i)\right]\end{aligned} \tag{14}$$

is the path metric and

$$\mathbf{d}(i) = [d_1(i,0) \cdots d_K(i,0) \cdots d_1(i,n-1) \cdots d_K(i,n-1)]^T, \tag{15}$$

$$\mathbf{E}(i) = diag\left[\sqrt{E_1(i,0)} \cdots \sqrt{E_K(i,0)} \cdots \sqrt{E_1(i,n-1)} \cdots \sqrt{E_K(i,n-1)}\right]. \tag{16}$$

Since $f_{\mathbf{Y}}(\mathbf{y})$ is a constant for a given realization $\mathbf{y}$ and we assume that all sequences are equiprobable, the *a posteriori* probability is given by

$$\Pr(\mathbf{d}|\mathbf{y}) = K'' \cdot \exp\left[-\frac{1}{2N_o}\Omega(\mathbf{d})\right]. \tag{17}$$

The maximum-likelihood detector selects the information sequences that, when encoded, maximize the log-likelihood function of (17) or *minimize* the path metric given in (14). This may be implemented using a Viterbi algorithm whose states depend on the number of users and the constraint length of the convolutional code. The following definitions are used to describe the JMLSE for coded multiuser detection [12]:

1. *State*: A state $\mathbf{x}_i$ at time index i is defined as $\mathbf{x}_i = [b_1(i) \cdots b_1(i-m+1) \cdots b_K(i) \cdots b_K(i-m+1)] = \mathbf{b}^T(i)$, where m is the constraint length of the encoder and the coded symbols are generated using the encoding function F_{enc}; $\mathbf{d}(i) = F_{enc}[\mathbf{b}(i)]$ where $d_k(i,j) = f_j[b_k(i) \cdots b_k(i-m+1)]$ is the jth coded bit of the kth user at time index i and is the generator for the jth coded bit.
2. *Branch*: A branch $\mathbf{B}_i$ at time index i is defined as $\mathbf{B}_i = (\mathbf{x}_{i-1}, \mathbf{x}_i)$.

The formulation of the algorithm can be summarized as follow:

For each state $\mathbf{x}_i$

1. Encode state $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$: $\mathbf{d}(i-1) = F_{enc}[\mathbf{b}(i-1)]$, $\mathbf{d}(i) = F_{enc}[\mathbf{b}(i)]$.
2. Compute all branch metrics $\omega_i[\mathbf{d}(i-1), \mathbf{d}(i)]$ entering that state, according to (14).
3. Add each branch metric to the corresponding survivor accumulated metric at state $\mathbf{x}_{i-1}$.
4. Find the minimum accumulated metric.
5. Store the minimum accumulated metric and the corresponding survivor.

The estimated sequence is the survivor sequence that corresponds to the minimum accumulated metric. The above algorithm requires $2^K 2^{K(m-1)}$ metric calculations for each stage. Hence, the decoding complexity is exponential in the product of the number of users and the constraint length of the code.

IV. PERFORMANCE ANALYSIS

To derive some performance bounds for JMLSE, we will closely follow the analysis in [1] and [10]. We define the following sets:

$$\begin{aligned}D = \{\mathbf{d} = \{d_k(i,j) \in \{-1,1\},\ i = 0,1,\cdots,L_u-1, \\ j = 0,1,\cdots,n-1,\ k = 1,2,\cdots,K\}\},\end{aligned} \tag{18}$$

$$E = \left\{\mathbf{e} = \left\{\begin{array}{r} \varepsilon_k(i,j) \in \{-1,0,1\}, \\ i = 0,1,\cdots,L_u-1, \\ j = 0,1,\cdots,n-1, \\ k = 1,2,\cdots,K; \\ \varepsilon_k(i,j) \neq 0, \text{ for some } i,j \end{array}\right\}\right\}, \tag{19}$$

$$A(\mathbf{d}) = \{\mathbf{e} : \mathbf{e} \in E, \mathbf{d} + 2\mathbf{e} \in C\}, \tag{20}$$

$$C = \{\mathbf{d} : \mathbf{d} \in h(\mathbf{b})\}, \tag{21}$$

where D is the set of transmitted coded sequence, E is the set of (non-zero) error sequences, $A(\mathbf{d})$ is the set of admissible error sequences conditioned on $\mathbf{d}$, and $h(\cdot)$ is the encoding rule by the code from information sequence $\mathbf{b}$ to coded sequence $\mathbf{d}$. If we follow the definition as in [10], then the information error sequence can be represented as

$$\mathbf{I} = h^{-1}(\mathbf{d} + 2\mathbf{e}) - \mathbf{b}. \tag{22}$$

The admissible error sequences that affect the ith bit of the kth user of $\mathbf{d}$ are

$$A_k^{L_u}(\mathbf{d}, i) = \{\mathbf{e} : \mathbf{e} \in A(\mathbf{d}),\ I_k(i) \neq 0\}. \tag{23}$$

Since we only have interest in the asymptotic behavior of JMLSE, we confine ourselves to the set of the simple error sequences, $SS_K^{L_u}(\mathbf{d}, i)$ which is the subset of $A_k^{L_u}(\mathbf{d}, i)$. Then, the upper bound on the probability of bit error is given by

$$P_k^{L_u}(i) \leq \sum_{\mathbf{d} \in C} \sum_{\mathbf{e} \in SS_k^{L_u}(\mathbf{d}, i)} \Pr\left[\Omega(\mathbf{d} + 2\mathbf{e}) \leq \Omega(\mathbf{d})\,|\mathbf{d}\right] \cdot \Pr(\mathbf{d}). \tag{24}$$

The event $\Omega(\mathbf{d}+2\mathbf{e}) \leq \Omega(\mathbf{d})$ can equivalently be expressed as

$$\mathbf{e}^T\mathbf{E}\mathbf{n} \geq \mathbf{e}^T\mathbf{EMEe}. \tag{25}$$

If we define the left-hand side of (25) as δ and the right-hand side as γ, then δ is a Gaussian random variable with zero mean and variance $N_o\gamma$. As it is done in [10], we define $\eta_k^{L_u}(\mathbf{e}) = \frac{\gamma}{nE_k} = \frac{\gamma}{E_{bk}}$ where $E_{bk} = nE_k$ is the uncoded bit energy for user k. Then, (24) is rewritten as

$$P_k^{L_u}(i) \leq \sum_{\mathbf{d}\in C} \sum_{\mathbf{e}\in SS_k^{L_u}(\mathbf{d},i)} Q\left(\sqrt{\frac{nE_k}{N_o}\cdot\eta_k^{L_u}(\mathbf{e})}\right)\cdot \Pr(\mathbf{d}). \tag{26}$$

In the high signal-to-noise ratio (SNR) region, the terms in (26) with the minimum efficiency will dominate the asymptotic behavior of JMLSE. Define the minimum efficiency as

$$\eta_{k,\min}^{L_u} = \inf_{\mathbf{d}\in C} \inf_{\mathbf{e}\in SS_k^{L_u}(\mathbf{d},i)} \eta_k^{L_u}(\mathbf{e}). \tag{27}$$

Analogous to [10], we define $\eta_{k,\min}^{L_u}$ as the asymptotic array multiuser coding gain (AAMCG) which combines the asymptotic multiuser coding gain and the spatial information of users' locations in the system. The AAMCG is an efficiency parameter which is a measure of the energy gain or loss of the receiver with an array of antennas relative to an uncoded BPSK system with a single antenna element. In the limiting cases where there is only one user in the system, or users employ perfectly orthogonal signature sequences, or users' locations are spatially orthogonal, then $\eta_{k,\min}^{L_u}$ is the asymptotic array coding gain (AACG). To obtain $\eta_{k,\min}^{L_u}$ we need to compute all error sequences in $SS_k^{L_u}(\mathbf{d},i)$. Instead of doing that we will upper and lower bound the worst case efficiency for the given DOA's. As in [10], we can obtain the upper bound by selecting a specific valid error sequence in $SS_k^{L_u}(\mathbf{d},i)$ since the minimum over all valid error sequences in is not larger than the $\eta_k^{L_u}(\mathbf{e})$ for this specific sequence. As an example, two users employ the maximum free distance convolutional code with rate 1/2 with the constraint length 3 and the generators $\mathbf{g}_1 = [1\ 0\ 1]$, $\mathbf{g}_2 = [1\ 1\ 1]$. If user 1 sends all zero sequence, and user 2 sends all zero sequence except for stage s_0, where one is sent, then a valid simple error sequence is

$$\mathbf{e} = [-1\ \ 1\ -1\ \ 1\ \ 0\ \ 1\ -1\ \ 1\ \ 1\ \ 0\ \ 1\ -1]^T. \tag{28}$$

If the user's energies are dissimilar, then the effective efficiency for this particular error sequence is

$$\begin{aligned}\eta_1^{L_u}(\mathbf{e}) =& \frac{5}{2}\cdot\mathrm{Re}\left(\mathbf{a}_1^H\mathbf{a}_1\right) + \frac{5}{2}\frac{E_2}{E_1}\cdot\mathrm{Re}\left(\mathbf{a}_2^H\mathbf{a}_2\right)\\ &- 4\sqrt{\frac{E_2}{E_1}}\cdot\mathrm{Re}\left(\mathbf{a}_1^H\mathbf{a}_2\right)\cdot\rho_{1,2}.\end{aligned} \tag{29}$$

The error events can be classified as a single-user and multiuser cases in a broad-sense. Suppose that for every $\mathbf{e}\in SS_k^{L_u}(\mathbf{d},i)$, every nonzero element of $\mathbf{e}$ is related to user k. Then, we make the following observation.

Proposition 1: *If we use an array of antennas with N elements and assume perfect estimation of DOA's, then* $\eta_k^{L_u}(\mathbf{e}) \geq n\cdot\frac{d_{free}}{n}$ *for every* $\mathbf{e}\in SS_k^{L_u}(\mathbf{d},i)$, *such that every nonzero element of* $\mathbf{e}$ *is contained in user k's subsequences (single-user error).*

Proof:

$$\begin{aligned}\eta_k^{L_u}(\mathbf{e}) &= \frac{1}{nE_k}\sum_{i=0}^{L_u-1}\mathbf{e}^T(i)\mathbf{E}(i)\mathbf{ME}(i)\mathbf{e}(i)\\ &= \frac{1}{nE_k}\cdot E_k\cdot\mathrm{Re}\left(\mathbf{a}_k^H\mathbf{a}_k\right)\cdot we_k(\mathbf{e})\\ &= \frac{1}{n}\cdot\mathrm{Re}\left(\mathbf{a}_k^H\mathbf{a}_k\right)\cdot we_k(\mathbf{e}),\end{aligned} \tag{30}$$

where $we_k(\mathbf{e})$ is the number of nonzero elements of user k's contained in $\mathbf{e}$. Since we assume perfect estimation of user k's DOA, we have

$$\min_{\substack{\mathbf{e}\in SS_k^{L_u}(\mathbf{d},i)\\ \mathbf{d}\in C}} \frac{1}{n}\cdot\mathrm{Re}\left(\mathbf{a}_k^H\mathbf{a}_k\right)\cdot we_k(\mathbf{e}) = \frac{1}{n}\cdot N\cdot d_{free}. \tag{31}$$

□

From the result of proposition 1, we know that if single-user error events happen, JMLSE *always* achieves AACG of a single-user system. By inspecting the matrix M, we obtain a lower bound of $\eta_k^{L_u}(\mathbf{e})$. We make the following observation.

Proposition 2: *If the number of antenna elements is N, the code rate is 1/n, and we assume perfect DOA's for two user system, the lower bound on* $\eta_1^{L_u}(\mathbf{e})$ *for user 1 is then given by*

$$\begin{aligned}&\eta_{1,\min}^{L_u}(\mathbf{e})\\ &\geq \min\left[f\left\{\sqrt{\frac{E_2}{E_1}}, d_{free}, \mathbf{a}_1, \mathbf{a}_2, N, \rho_{12},\right\}, N\cdot\frac{d_{free}}{n}\right],\end{aligned} \tag{32}$$

where

$$\begin{aligned}&f\left\{\sqrt{\frac{E_2}{E_1}}, d_{free}, \mathbf{a}_1, \mathbf{a}_2, N, \rho_{12}\right\}\\ &\quad= \frac{N}{n}\cdot\left[d_{free} + \left(\frac{E_2}{E_1}\right)d_{free}\right.\\ &\quad\left. - \sqrt{\frac{E_2}{E_1}}\cdot\frac{1}{N}\cdot 2d_{free}\cdot\mathrm{Re}\left(\mathbf{a}_1^H\mathbf{a}_2\right)\cdot\rho_{12}\right].\end{aligned} \tag{33}$$

Proof: Following the method used in [10], we obtain somewhat loose lower bound on the efficiency for user 1 given by

$$\begin{aligned}\eta_1^{L_u}(\mathbf{e}) \geq \min&\left[g\left\{\sqrt{\frac{E_2}{E_1}}, we_1(\mathbf{e}), we_2(\mathbf{e}), \mathbf{a}_1, \mathbf{a}_2, N, \rho_{12}\right\},\right.\\ &\left. N\cdot\frac{d_{free}}{n}\right],\end{aligned} \tag{34}$$

where

$$g\left\{\sqrt{\frac{E_2}{E_1}}, we_1(\mathbf{e}), we_2(\mathbf{e}), \mathbf{a}_1, \mathbf{a}_2, N, \rho_{12}\right\} = \frac{1}{n}\cdot\left[\mathrm{Re}\left(\mathbf{a}_1^H\mathbf{a}_1\right)\cdot we_1(\mathbf{e}) + \left(\frac{E_2}{E_1}\right)\cdot\mathrm{Re}\left(\mathbf{a}_2^H\mathbf{a}_2\right)\cdot we_2(\mathbf{e}) - \sqrt{\frac{E_2}{E_1}}\cdot 2\cdot\min\left\{we_1(\mathbf{e}), we_2(\mathbf{e})\right\}\cdot\mathrm{Re}\left(\mathbf{a}_1^H\mathbf{a}_2\right)\cdot\rho_{12}\right]. \quad (35)$$

Since we assume perfect estimation of DOA's, it is not difficult to show that

$$\min_{we_1(\mathbf{e}), we_2(\mathbf{e})} g\left\{\sqrt{\frac{E_2}{E_1}}, we_1(\mathbf{e}), we_2(\mathbf{e}), \mathbf{a}_1, \mathbf{a}_2, N, \rho_{12}\right\} = f\left\{\sqrt{\frac{E_2}{E_1}}, d_{free}, \mathbf{a}_1, \mathbf{a}_2, N, \rho_{12}\right\}. \quad (36)$$

Hence, we obtain the lower bound of the minimum asymptotic efficiency given in (32). □

From proposition 2, we know that when user's locations are such that the array response vectors are orthogonal to each other, JMLSE *always* achieves AACG of a single-user system as we expected. On the other hand, when user's locations are such that the array response vectors are aligned with each other, the minimum asymptotic efficiency is bounded away from the worst-case value $\frac{N}{n}\cdot d_{free}\left[1+\left(\frac{E_2}{E_1}\right)-\sqrt{\frac{E_2}{E_1}}\cdot 2\rho_{12}\right]$. Another important property of JMLSE can be obtained from the lower bound given in proposition 2. *The near-far resistance* is defined as the worst case asymptotic efficiency over all possible energy distributions. We make the following observation.

Proposition 3: *If the number of antenna elements is N, the code rate is 1/n, and we assume perfect estimation of DOA's for two-user system, then the lower bound on the near-far resistance of the JMLSE is given by*

$$\bar{\eta}_{1,\min}^{L_u}(\mathbf{e}) \geq \frac{N}{n}\cdot d_{free}\left[1-\left\{\frac{1}{N}\cdot\mathrm{Re}\left(\mathbf{a}_1^H\mathbf{a}_2\right)\cdot\rho_{12}\right\}^2\right]. \quad (37)$$

Proof: The near-far resistance is defined as $\bar{\eta}_{1,\min}^{L_u}(\mathbf{e}) = \inf_{\sqrt{E_2/E_1}\in[0,\infty)} \eta_{1,\min}^{L_u}(\mathbf{e})$ [2]. Using the bound given in proposition 2, we obtain the lower bound on the near-far resistance of the JMLSE as (38) at the bottom of this page. Since the function $f(\cdot)$ is convex in terms of $\sqrt{E_2/E_1}$, Eq. (33) has minimum at $\sqrt{E_2/E_1} = \frac{1}{N}\cdot\mathrm{Re}(\mathbf{a}_1^H\mathbf{a}_2)\cdot\rho_{12}$. If we substitute this value into (38) and take infimum with respect to $\sqrt{E_2/E_1}$, then we obtain the desired result. □

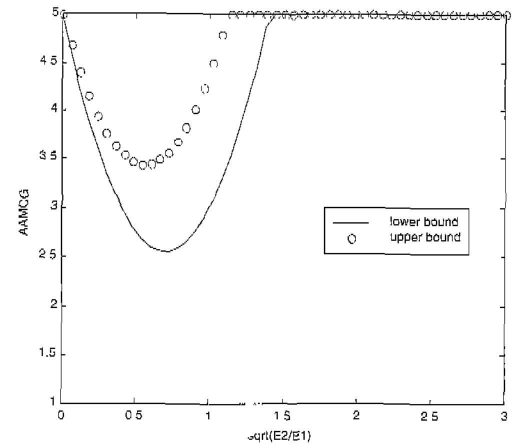

Fig. 1. Plot of lower bound on $\eta_{1,\min}^{L_u}(\mathbf{e})$ for $\rho_{12} = 0.7$ and the same array response vectors. Also shown is the actual $\eta_1(\mathbf{e})$ for the specific error sequence given in (29).

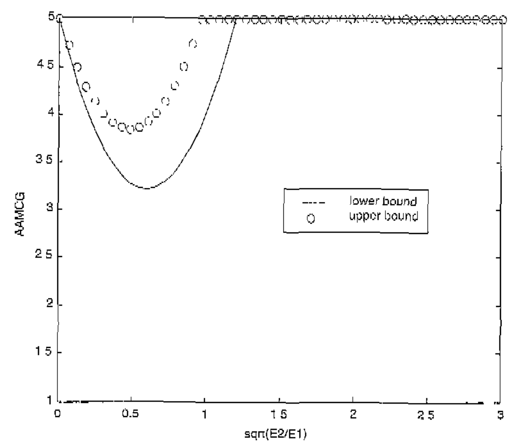

Fig. 2. Plot of lower bound on $\eta_{1,\min}^{L_u}(\mathbf{e})$ for $\rho_{12} = 0.7, \theta_1 = 10°$, and $\theta_2 = 25°$. Also shown is the actual $\eta_1(\mathbf{e})$ for the specific error sequence given in (29).

V. NUMERICAL EXAMPLES

We assume that there are two users in the system, the number of antenna elements is two, and each user employs the same maximum free distance convolutional code with rate 1/2 with generators $\mathbf{g}_1 = [1\ 0\ 1]$ and $\mathbf{g}_2 = [1\ 1\ 1]$.

Fig. 1 is plot of lower bound on given in (32) for $\rho = 0.7$ and the same array response vectors. This is the worst-case lower bound on $\eta_{1\min}^{L_u}(\mathbf{e})$, since there is no spatial discrimination between users. Also shown is the actual for the specific error sequence given in (29). Fig. 2 is the same plot except and $\theta_1 = 10°$ and $\theta_2 = 25°$. As the spatial separation between users increases, the combined effect of space division multiple access

$$\bar{\eta}_{1,\min}^{L_u}(\mathbf{e}) \geq \inf_{\sqrt{E_2/E_1}\in[0,\infty)} \min\left[f\left\{\sqrt{\frac{E_2}{E_1}}, d_{free}, \mathbf{a}_1, \mathbf{a}_2, N, \rho_{12}\right\}, N\cdot\frac{d_{free}}{n}\right] \quad (38)$$

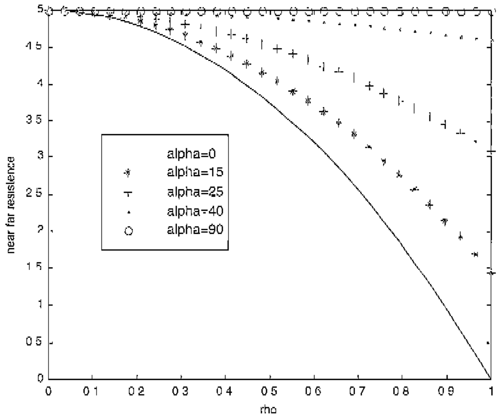

Fig. 3. Plot of the lower bound on the near-far resistance of the JMLSE as the crosscorrelation and angle separation. Alpha denotes the angle separation of user 2 relative to user 1.

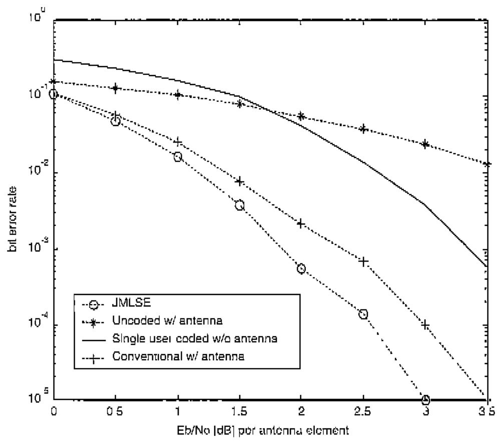

Fig. 4. Performance curves of the JMLSE for a two-user channel with $\theta_1 = 10°, \theta_2 = 35°$, and $\rho_{12} = 0.4$. Also shown are the performances of a single user system without MUI, uncoded system with antenna elements, and coded conventional matched filter detection combined with antenna elements.

(SDMA) and code division multiple access (CDMA) is evident. As the interfering signal energy, E_2 becomes small relative to E_1, the performance of the JMLSE approaches the AACG of the single-user system. Also, as E_2 becomes large relative to E_1, the same performance can be achieved. It is interesting to note that if each user experiences the error by amount of d_{free}, the upper and lower bounds coincide with each other for this particular example. In Fig. 3 we plot the lower bound on the near-far resistance of the JMLSE as the crosscorrelation and angle separation vary. Alpha denotes the angle separation of user 2 relative to user 1. We observe the benefit of using an array of antennas. The near-far resistance is bounded away from zero, even though the users employ the same code sequences provided the angular separation is sufficient. Also, we note that the JMLSE combined with an array of antennas provides high bandwidth efficiency, since it is strongly near-far resistance for the crosscorrelation values of interest. Fig. 4 shows the performance curves of the JMLSE for a two-user channel with $\theta_1 = 10°, \theta_2 = 35°$, and $\rho_{12} = 0.4$. Also shown are the performances of a single user system without MUI, an uncoded system with antenna elements, and coded conventional matched filter detection combined with antenna elements. Fig. 5 shows the performance curves of the JMLSE for a two-user channel with $\theta_1 = 10°, \theta_2 = 25°$, and $\rho_{12} = 0.6$. Also shown are the performances of a single user system without MUI, uncoded system with antenna elements, and coded conventional matched filter detection combined with antenna elements. The JMLSE outperforms a single user coded performance, but single user coded performance outperforms the conventional matched filter detection with antenna elements when the system is spatially and temporally highly correlated. Fig. 6 shows the near-far ratio performance curves of the JMLSE on a two-user channel with $\theta_1 = 10°, \theta_2 = 25°, \rho_{12} = 0.6$, and $E_{b1}/N_0 = 1dB$. Also shown are the near-far performances of a single user system without MUI, uncoded system with an-

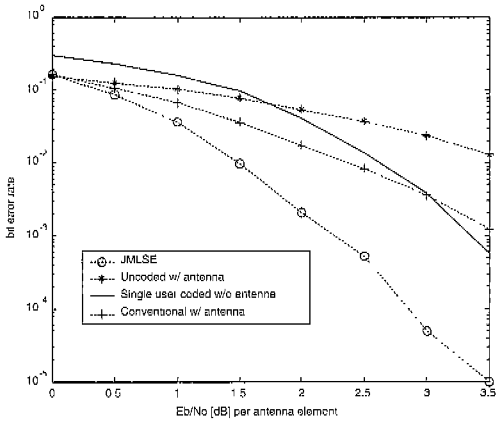

Fig. 5. Performance curves of the JMLSE for a two-user channel with $\theta_1 = 10°, \theta_2 = 25°$, and $\rho_{12} = 0.6$. Also shown are the performances of a single user system without MUI, uncoded system with antenna elements, and coded conventional matched filter detection combined with antenna elements.

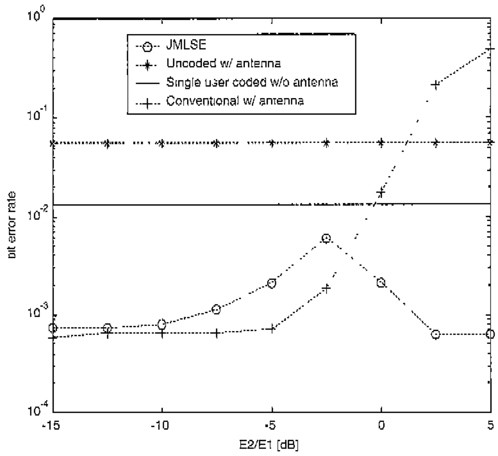

Fig. 6. Near-far ratio performance curves of the JMLSE on a two-user channel with $\theta_1 = 10°, \theta_2 = 25°, \rho_{12} = 0.6$, and $E_{b1}/N_o = 1dB$. Also shown are the near-far performances of a single user system without MUI, uncoded system with antenna elements, and coded conventional matched filter detection combined with antenna elements.

tenna elements, and coded conventional matched filter detection combined with antenna elements. Since the JMLSE jointly processes the matched filter outputs from both users, it shows a slightly inferior performance when the energy ratio is relatively small.

VI. CONCLUSION

In this paper, the JMLSE combined with an array of antennas was formulated for synchronous CDMA systems where each user employs a rate $1/n$ convolutional code to improve its performance. We obtained the upper and lower bounds on the effective efficiency. Also, we investigated the near-far property of the JMLSE and it showed strong near-far resistance due to the use of coding and an array of antennas. We assumed that we have a perfect knowledge of the system, i.e., number of active users, signature waveforms, direction-of-arrivals (DOA's), array characteristics, and the individual user's energy. Although these assumptions are not realistic, they can serve as a best possible reference model. Finally, we presented numerical results for the two user, two element array and showed that the combined MUD and array processing will outperform either alone. It is especially true when the signals are highly correlated in time or space but not both. While the small dimensional case presented for numerical illustration needs to be expanded to more realistic sizes, the results are highly suggestive of the value of the proposed scheme and provides verification of the theoretical bounds provided in this paper.

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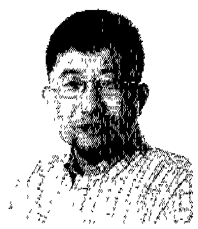

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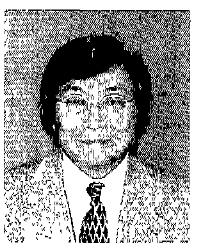

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