The Modified Similarity Theory of Movable-Bed River Model

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Abstract: A relaxed similarity theory which can be applied to river models with movable beds is established by modifying existing theory by Einstein and Chien(1954). Experimental data collected from river models with movable beds were used to evaluate the applicability of the proposed theory. Effects of similarity of flow, $\Delta F \Delta M$, and similarity of sediment movement, ΔF_s, were examined by analyzing the behaviour of total river-bed change. The results show that the smaller $\Delta F \Delta M$ or ΔF_s is, respectively, the larger total sedimentation is. The modified similarity theory established in this study would be useful and practical whenever it is impossible or very difficult to satisfy strict theoretical requirements concerning the river model experiments with movable beds.

1. Introduction

Though the movable-bed river model should be built to satisfy the similarity law of flow, sediment transport, sediment transport rate, and bed formation, it is practically impossible to satisfy strict requirements of existing similarity theories such as the theory by Einstein and Chien(1954); because the size and specific gravity of bed material cannot be selected independently. Many modified similarity theories based on experimental methods have been developed to relax existing similarity theories and build movable-bed river model reproducing flow and bed formation of prototype.

Einstein and Chien(1954) proposed a general theory including similarity laws of flow, incipient sediment transport, and bed formation. In their study, the similarity of the Manning's equation and the Froude number are used as the similarity law of flow; the similarity of the shear intensity, the intensity of sediment, and the turbulent boundary layer are also used as the similarity law of sediment transport; and the similarity of the sediment transport rate and the sediment time scale are also used as the similarity law of bed formation. It is impossible to satisfy all their similarity laws. Henderson(1966) proposed simplified method considering only essential similarity laws of the Einstein and Chien(1954). Novak and Čábelka(1981) proposed empirical theories including the relaxed similarity law of particle Reynolds number by laboratory experiments.

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The Ministry of Construction(1983) built a movable-bed river model satisfying the similarity laws of flow, the sediment transport rate, and bed formation where the critical tractive force, the settling velocity, and the sediment time scale were made the same in the river model as in the prototype, and performed laboratory experiments for the study of sediment transport and flood protection at lower reaches of the Han River in Korea. The Seoul Metropolitan Government(1990) built a movable-bed river model applying the similarity theories where the sediment transport rate and the shear velocity were the same in the river model as in the prototype, and performed laboratory experiments for the study of sediment transport and flood protection at lower reaches of the Han River in Korea. The Hyundai Engineering Cooperation, LTD.(1993) built movable-bed river model satisfying the Henderson's theory, and performed laboratory experiments to examine the suitability of basic designs for the intake structure and the descending basin at the Modi Khola River in Nepal. Analyzing and comparing previous study, the Ministry of Construction(1997) proposed relaxed theories in which scales of river model, the particle size, and the specific gravity of sediment were determined by satisfying similarity laws of flow and the sediment transport, and the sediment transport rate and the sediment time scale were determined by satisfying similarity law of bed formation. Applying its relaxed theories, the Ministry of Construction built the movable-bed river models and performed laboratory experiments for the study of sediment transport and flood protection at reaches of the South Han River in Korea.

In this study, a relaxed theory which can be applied to large movable-bed river models is proposed by modifying existing theories based on Einstein and Chien(1954) and Novak and Čábelka(1981). The proposed theory encompasses the similarity law of flow, which Froude number in the river model should be the same as in the prototype and Manning's equation should be equally applied both in model and prototype; the similarity law of sediment transport which requires Shields' entrainment function be the same in model and prototype within certain restriction of particle Reynolds number; the similarity of the bed formation based on the sediment time scale, which is in turn derived from the similarity of sediment transport rate and the continuity equation for the sediment transport rate. The hydraulic and river-bed change data sets collected from the South Han River model experiments were applied to evaluate the relaxed theory proposed in this study.

2. Development of Similarity Law of Movable-Bed River Model

2.1 Similarity Law of Flow

Because gravity is one of the most important force in open channel flow, the Froude number defined by the ratio of inertia force to gravity should be the same in river model as in the prototype, but this similarity condition is sometimes difficult to satisfy. Henderson(1966) proposed that degree of freedom in selecting scales of a river model could be increased by selecting different values of Froude number in model and prototype provided the depth is large, that is Froude number is small. He propose that if Froude number is much smaller than one and consequently Froude law is not considered important, then Froude law can be relaxed as following equation

$$\Delta F = U_r H_r^{-1/2} \tag{1}$$

in which F is Froude number, U_r is the velocity ratio, H_r is vertical length ratio, and subscript r indicates ratio of prototype quantity (subscript p) to river model quantity (subscript m). Throughout this paper quantities with the subscript r will indicate values in prototype divided by corresponding values in model. In Eq. (1), if the value of ΔF is one, it means that the Froude number is exactly the same in model and prototype. If Froude number in the prototype is much smaller than one, Froude law could be relaxed by allowing ΔF to be other than one.

In a movable-bed river model, the influence of viscosity should be considered besides the influence of gravity. The Reynolds number should be the same in model and prototype to satisfy the similarity of viscosity, but it is practically impossible to satisfy this Reynolds law and Froude law simultaneously. To solve this problem, different fluids could be used in model and prototype, but it is not realistic when river model is large. If form drag is a significant factor and flows both in model and prototype maintain turbulent flow, drag coefficient should be the same in model and prototype. So, the similarity of viscosity is satisfied by maintaining turbulence flow in the river model. However, when the friction drag is the dominant factor such as in a river with long reach and small width and depth, the similarity of viscous force is assumed to be satisfied if the roughness coefficient n is adjusted such that Manning's equation is applicable both in model and prototype. In this case, the flow on river model should be maintained turbulent and Allen(1947) propose that the flow in the model is turbulent flow when Reynolds number is larger than 1,400.

From the ratio of Manning equations, the equation which satisfies the similarity of viscous force is given as

$$\Delta M = U_r^{-1} n_r^{-1} R_r^{2/3} S_r^{1/2} \tag{2}$$

in which S_r is the slope ratio which is the same as H_r / L_r, R_r is the ratio of hydraulic radius, L_r is the longitudinal length ratio, and n_r is the ratio of roughness coefficient of Manning's equation. Using the relationship suggested by Strickler, n_r is given as

$$n_r = D_r^{1/6} \tag{3}$$

in which D_r is the ratio of grain diameter. In Eq. (2), the similarity of viscous force is exactly satisfied when the value of ΔM is one. The similarity of viscous force can be relaxed by letting the value of ΔM be other than one. Novak and Čábelka(1981) propose that Manning's equation and Strickler's equation are precise only in the special roughness range, which may be interpreted to support that ΔM could have the value other than one.

To satisfy the similarity law of flow, the similarity of Froude number and Manning's equation, which are defined in Eq. (1) and Eq. (2) respectively should be satisfied. If river width is much larger than depth, H_r equals R_r, then the following similarity law of flow is derived from Eq. (1)-(3) as

$$\Delta F \Delta M = H_r^{2/3} D_r^{-1/6} L_r^{-1/2} \tag{4}$$

in which similarity law of flow is exactly satisfied when the value of $\Delta F \Delta M$ is one. The degree of freedom in determining the particle size of model bed material can be increased by relaxed similarity condition that the value of $\Delta F \Delta M$ could be other than one.

2.2 Similarity Law of Sediment Transport

The sediment transport and the bed formation are known to be determined by the position on the Shields' diagram. To satisfy the similarity law of sediment transport, the entrainment function[Eq. (5)] and the particle Reynolds number[Eq. (6)] should be exactly the same in model and prototype.

$$F_s = \frac{1}{\Psi} = \frac{\tau_o}{\gamma(S_s - 1)D} = \frac{u^{*2}}{g(S_s - 1)D} \tag{5}$$

$$R_e^* = \frac{u^* D}{\nu} = \frac{\sqrt{\tau_o/\rho}\, D}{\nu} \tag{6}$$

in which F_s is the Shields entrainment function, Ψ is the shear intensity, τ_o is the shear stress (= γRS), S_s is the specific gravity of particle, R_e^* is the particle Reynolds number, u^* is the shear velocity, and ν is the coefficient of dynamic viscosity.

Bogardi(1959) proposes to relax somewhat the condition that the particle Reynolds number has to be exactly the same in model and prototype. He points out that the flow around the particles becomes fully turbulent when the value of R_e^* is larger than 100 on the Shields diagram, and it is unnecessary to satisfy the similarity of particle Reynolds number. Komura(1962) proposes the similarity of particle Reynolds number is unnecessary when the particle size is larger than 0.6mm. Chauvin(1962) presents that it is unnecessary to satisfy the similarity of particle Reynolds number provided that the value of particle Reynolds number is larger than 60. Novak and Čábelka(1981) argues that the forces acting on the particle on the bed in the prototype and the river model are caused mainly by its frontal resistance, so the influence of the viscous force can be considered negligible. This condition is fulfilled, according to experimental results, when the value of particle Reynolds number is greater than 3.5. But, in this condition the bed formation in model and prototype may be different, and only the threshold of motion of bed material are considered to be similar in model and prototype. To satisfy the similarity law of sediment transport, in the case of neglecting the similarity of particle Reynolds number, only the similarity of Shields' entrainment function is required to be satisfied;

$$\Delta F_s = (S_s - 1)_r^{-1} D_r^{-1} H_r^2 L_r^{-1} \tag{7}$$

2.3 Similarity of Bed Formation

The similarity laws of sediment transport and bed formation must be satisfied in river models with movable beds. Especially when the similarity law of sediment transport can not be satisfied, it is practical to perform experiments which satisfy the similarity of bed formation by adjusting the total transport rate and duration time. The continuity equation for the bed formation analogous to unsteady flow in the open channel (Henderson,1966) is useful and the equation is given as

$$\frac{\partial z}{\partial t} \pm \frac{1}{(1-\lambda)} \frac{\partial q_s}{\partial x} = 0 \tag{8}$$

in which z is the height of the bed above datum, λ is the void ratio, q_s is the volumetric rate of bed formation per unit width, which includes only sediment excluding voids. When the geometric similarity is satisfied between model and prototype, z_r equals H_r, and the similarity law of bed formation is derived from Eq. (8). The equation is given as

$$T_{2r} = q_{sr}^{-1}(1-\lambda)_r L_r H_r \tag{9}$$

in which T_{2_r} is the sediment time scale and it is necessary time to fill up certain volume with q_s. The sediment time scale is different from the hydraulic time scale[Eq. (10)] used in fixed-bed river models and has to be used in reproducing hydrograph of prototype in the model.

$$T_{1r} = L_r U_r^{-1} = L_r H_r^{-1/2} \tag{10}$$

Among many sediment transport rate equations developed by many researchers, Brown's equation is used to determine sediment transport rate in this study. The equation derived Brown(1950) is given as

$$\frac{q_s}{u^* D} = 10 F_s^2 \tag{11}$$

The Ministry of Construction(1983) and the Seoul Metropolitan Government(1990) reported that the Brown's results approximate measured sediment transport rate in the Han River. If the Brown's equation is used to calculate the sediment transport rate, the equation for the similarity of bed formation is given as

$$q_{sr} = (S_s - 1)_r^{-2} D_r^{-1} L_r^{-5/2} H_r^5 \tag{12}$$

Table 1. Comparisons of Similarity Theories for Movable-Bed River Models

Similarite Theories		Einstein and Chien(1954)	Henderson(1966)	This Study
Flow	Manning's Equation	$SM = U_r^2 S_r^{-1} H_r^{-1-2m} D_r^{2m} C_r^{-2}$	$SM = U_r^{-1} D_r^{-1/6} S_r^{1/2} H_r^{2/3}$	$\Delta M = U_r^{-1} D_r^{-1/6} S_r^{1/2} H_r^{2/3}$
	Froude Number	$SF = U_r H_r^{-1/2}$	$SF = U_r H_r^{-1/2}$	$\Delta F = U_r H_r^{-1/2}$
Sediment	Shear Intensity	$S\Psi = (\rho_s - \rho_f)_r D_r \eta_r^{-1} H_r^{-1} S_r^{-1}$		
Transport	Intensity of Bed Load	$S\Phi = q_{sr}(\rho_s - \rho_f)_r^{-3/2} D_r^{-3/2}$		
	Turbulent Boundary Layer	$S\delta = D_r \eta_r S_r^{1/2} H_r^{1/2}$		
	Shields' Entrainment Function		$SF_s = (S_s - 1)_r^{-1} D_r^{-1} H_r^2 L_r^{-1}$	$\Delta F_s = (S_s - 1)_r^{-1} D_r^{-1} H_r^2 L_r^{-1}$
	Particle Reynolds Number		$SR_e^* = H_r D_r L_r^{-1/2} \nu_r^{-1}$	
Bed	Sediment Transport Rate	$SQ_s = q_{sr}(\rho_s - \rho_f)_r^{-3/2} D_r^{-3/2}$	$SQ_s = q_{sr}(S_s - 1)_r^2 D_r L_r^{5/2} H_r^{-5}$	$q_{sr} = (S_s - 1)_r^{-2} D_r^{-1} L_r^{-5/2} H_r^5$
Change	Hydraulic Time Scale	$ST_{1r} = T_{1r} U_r L_r^{-1}$	$ST_{1r} = T_{1r} U_r L_r^{-1}$	$T_{1r} = U_r^{-1} L_r$
	Sediment Time Scale	$ST_{2r} = T_{2r} q_{sr}(\rho_s - \rho_f)_r^{-1} L_r^{-1} H_r^{-1}$	$ST_{2r} = T_{2r} q_{sr}(1-\lambda)_r^{-1} L_r^{-1} H_r^{-1}$	$T_{2r} = q_{sr}^{-1}(1-\lambda)_r L_r H_r$
	Slope	$SN = S_r L_r H_r^{-1}$	$SN = S_r L_r H_r^{-1}$	$S_r = L_r^{-1} H_r$
	Scale Distortion		$SD_s = D_{sr} L_r^{-1} H_r$	$D_{sr} = L_r H_r^{-1}$
	Flow Discharge	$SQ = Q_r L_r^{-1} H_r^{-1} U_r^{-1}$	$SQ = Q_r L_r^{-1} H_r^{-1} U_r^{-1}$	$Q_r = L_r H_r U_r$

in which $\frac{U}{\sqrt{R_t S g}} = C\left(\frac{R_t}{D}\right)^m$, R_t is total hydraulic radius, g is the gravitational acceleration, $S_s = \frac{\rho_s}{\rho_f}$, $\eta = \frac{R_b^{'}}{R_t}$, $R_t = R_b^{'} + R_b^{''} + \frac{R_w P_w}{P_b}$, $R_b^{'}$ is the hydraulic radius due to intrinsic roughness, $R_b^{''}$ is the hydraulic radius due to form roughness, R_w is the hydraulic radius due to stream bank, P_b is the wetted perimeter due to stream bed, and P_w is the wetted perimeter due to stream bank.

2.4 Summary and Comparison

The theories of Einstein and Chien(1954) and Henderson(1966) are compared with the similarity theories proposed in this study and they are listed in Table 1. In the theory proposed in this study, the values of L_r, H_r, D_r, and S_{sr} are to be determined such that the values of $\Delta F \Delta M$ and ΔF_s are one. In this case, because parameters to be determined are four and similarity laws are two, degree of freedom becomes two. Like Einstein and Chien's theory or Novak and Čábelka's theory, the sediment transport rate, the sediment time scale, fall velocity, and other conditions can be added. But, since the increased formulas to be satisfied induce the same number of parameters the degree of freedom doesn't change.

Comparing with the theory derived in the study, Henderson's theory requires that the values of L_r, H_r, D_r, and S_{sr} be determined under the condition that $\Delta F \Delta M$, ΔF_s, and the ratio of particle Reynolds numbers are one. In this case, since parameters to be determined are four and formulas to be satisfied are three, the degree of freedom becomes one. Because the Henderson theory has the problem that not only particle size and specific gravity of model sediment but also vertical and longitudinal length scale can not be determined independently, it is very difficult to satisfy the Henderson's similarity theory. Also, unless the specific gravities of bed materials are the same in model and prototype, it turns out to be impossible to use Henderson's similarity theory in a undistorted movable bed river model.

In practice, because of the restriction of laboratory space length scales(L_r, H_r) may be determined in the first place in building river models with movable beds. In this case, two parameters(D_r, S_{sr}) are automatically determined by two equations[Eq. (4) and Eq. (7)] and there is no degree of freedom left at all. So, based on the previous similarity theories and empirical results, the degree of freedom can be increased by one within a certain limit by allowing $\Delta F \Delta M$ to deviate from one while keeping ΔF_s still one. Should length scales(L_r, H_r) be determined in the first place as usual, the parameters related to model bed materials, D_r, S_{sr}, could be determined by relaxed similarity laws of flow($\Delta F \Delta M \neq 1$) and strict similarity law of sediment transport($\Delta F_s = 1$); and then q_s and T_2 in the model could be determined by the similarities of sediment transport rate and the sediment time scale. Of cause, it should be kept in mind that the flow should be turbulent and the particle Reynolds number is grater than 3.5 in the model.

3. Model Experiments

3.1 Movable Bed River Model

Applicability of similarity law proposed in this study is examined by analyzing the behavior of total river-bed change collected in the South Han River movable-bed river model(The Ministry of Construction, 1997). Hydraulic and total river-bed change data sets for the movable beds and the fixed beds were collected in the South Han River model built for the South Han

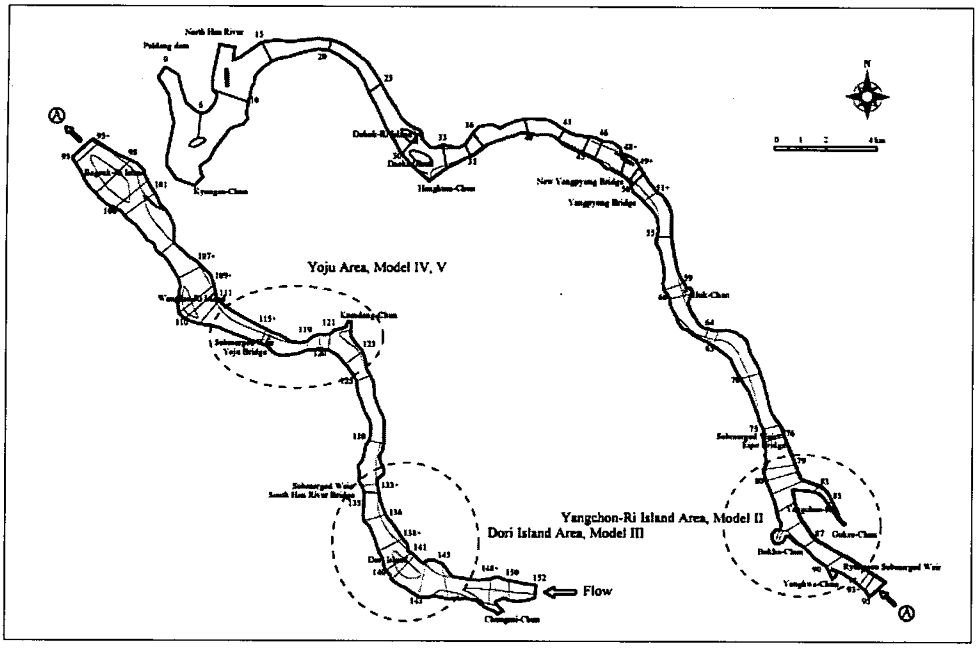

Fig. 1. The Reaches of South Han River Model

River as shown in Fig. 1(Paldang dam~Sumgang confluence; L=69km). In this study, the South Han River is divided into five model reaches to examine the effects of similarity laws proposed in this study. Model I is the reach of the South Han River, model II is the reache of Yangchon-Ri island(Yongdam-Chun confluence~Sinle-Chun confluence; L=8.0km), model III is the reach of Dori island(the South Han River bridge~Chungmi-Chun confluence; L=6.0km), and model IV and model V are the reach of Yoju(Soyang-Chun confluence~Yoju intake works). Model II~IV are sub-models built in different length scales from model I. Model IV and V are built with different model bed material but same length scales. Because the space for very long reaches of the South Han River model is not enough, all models except model IV and V are distorted river model. Model IV and V are built as undistorted river models, because the similarity of vertical flow components has to be satisfied to properly observe the flow around the submerged weir built in reaches of Yoju. In this study, the distortion scale of distorted river models is not greater than five.

In movable-bed river model, duration is determined by specific gravity and particle diameter of bed material. Because the duration has to be the same at each measuring section in the model reach, it is practical to use the same particle diameter at all measuring sections of the same model. In the South Han River movable-bed river model(Ministry of Construction, 1997) project, median diameter of bed material in each sub-model was obtained from the reports of Kyonggi-Do(1997) and Ministry of Construction(1992) and median diameter used in each model is mean values of median diameters obtained from measuring points in each models reaches. And then, the most practical model bed material in each model was determined by testing if

Table 2. Scales and Bed Materials of Models

River Model		Scale		Model Bed Materials		
		L_r	H_r	Class	S_{sm}	D_m (mm)
Case 1	Model I	220	80	Sand	2.59	0.35
	Model II	100	50	Sand	2.59	0.40
Case 2	Model I	220	80	Sand	2.59	0.35
	Model III	100	50	Sand	2.59	0.40
Case 3	Model I	220	80	Sand	2.59	0.35
	Model IV	100	100	Anthracite	1.47	0.07
	Model V	100	100	Quartz Sand	2.00	0.26

selected model bed material satisfied the similarity of sediment transport. The longitudinal length scale, vertical length scale, and particle diameter and specific gravity of model sediment used in each models are listed in Table 2.

3.2 Experiment Conditions

To satisfy the similarity of bed formation, the duration in the model, which is calculated from the duration of a flood hydrograph in prototype divided by the sediment time scale(T_{2_r}), has to be reproduced. To reproduce duration in the model, the hydrograph and the sediment time scale in prototype have to be determined in the first place. In the South Han River movable-bed river model, the flood hydrograph(Fig. 2) measured at Yoju gauge station in 1995 is used because experiments are aimed for the study of sediment transport and flood protection in as large a flood as the design flood. Because it is practically impossible to reproduce continuous change of hydrograph of prototype in the river model, the discrete hydrograph, represented by five discrete hydrograph shown as dotted lines in Fig. 2, is derived.

The discharge in each step in the model experiments is determined by the ratio of the model inflow to the peak flow of the flood hydrograph at Yoju gauge station. And the duration in the

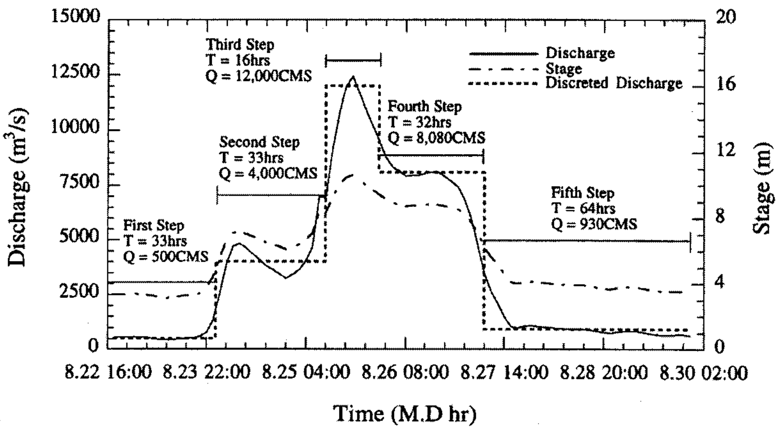

Fig. 2. Flood Hydrograph at Yoju Gauge Station in August, 1995

model is determined from the duration of each step in prototype divided by T_{2_r} defined in Eq. (9). The inflow discharges of branches listed in the report(Ministry of Construction, 1997) are assumed in steady uniform flow. At the time of the flood in 1995, the hydrograph at the North Han River is not the same as at the South Han River, but the same shape of hydrograph is used in the South Han River model under the assumption that bed formation on upstream reach of the South Han River would not be affected by the variation of hydrograph in the North Han River. During the flood in 1995, the outflow discharge at Chungpyeong dam proportional to the design flood scale is 19,300 m^3/s.

4. Analysis of Results of Laboratory Experiments

4.1 Movable-Bed River Model

Once longitudinal and vertical length scales of movable-bed river model have been determined, grain diameter and specific gravity of bed material should be selected by similarity equations of $\Delta F \Delta M$ and ΔF_s. In this case, no degree of freedom is left in selecting bed material, because these two parameters are determined by two similarity equations. Therefore, this similarity condition is difficult to apply in a large movable-bed river model such as the South Han River model, because model material is not of sufficient variety and finding acceptable bed material is not easy in korea. So, it is necessary to increase the degree of freedom in selecting the sediment by relaxing a similarity requirement. In this case it is known through theoretic and experimental experiences that relaxing the condition of $\Delta F \Delta M$ rather than ΔF_s is desirable. Since Henderson(1966) proposed the degree of freedom could be increased by allowing different Froude numbers in model and prototype in the case of deep water flow or small Froude number, and the sediment transport and bed formation are the main aspect of movable-bed river model experiments, bed material is selected such that the grain size and specific gravity of that material satisfy the condition, $\Delta F_s = 1$, as close as while the condition, $\Delta F \Delta M = 1$, somewhat loosely. Once d_m and S_{s_m} are selected, hydraulic quantities in the model can be calculated readily. Comparisons of similarity conditions of each model are listed in Table 3. In which the range of $\Delta F \Delta M$ values is 0.74-1.57 and that of

Table 3. Comparisons of Similarity Condition of Models

Models		D_p (mm)	D_r	$(S_s - 1)_r$	$\Delta F \Delta M$	ΔF_s
Case 1	Model I	8.3	23.7	1.038	0.74	1.18
	Model II	8.3	20.8	1.038	0.82	1.16
Case 2	Model I	6.2	17.7	1.038	0.78	1.58
	Model III	6.2	15.5	1.038	0.86	1.55
Case 3	Model I	1.7	4.86	1.038	0.96	5.77
	Model IV	1.7	24.3	3.510	1.27	1.17
	Model V	1.7	6.54	1.650	1.57	8.91

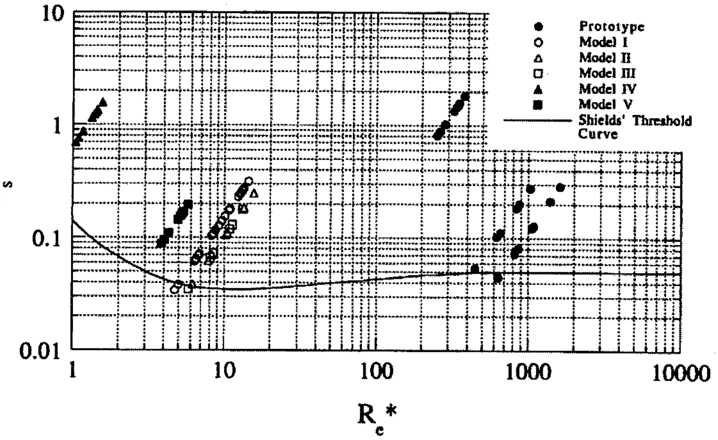

Fig. 3. Behavior of Sediment Transport According to Similarity Conditions

ΔF_s values is 1.16-8.91. This results show that similarity laws of $\Delta F \Delta M$ and ΔF_s are not satisfied exactly in the South Han River model. That is because practically available bed materials are not various as shown in Table 2. As a result, the South Han River model experiments are performed under condition that similarity laws of flow and bed formation are not satisfied exactly, which means that the results are analyzed qualitatively. In this study, the applicability of similarity laws of $\Delta F \Delta M$ and ΔF_s was examined by analyzing the behavior of total river-bed change.

The Fig. 3 shows dimensionless shear stress versus particle Reynolds number at each measuring section in river models and the prototype. In this figure mean depths at each section computed by Hec-2 are used in computing dimensionless shear stress and particle Reynolds number. In the Fig. 3, the particle Reynolds numbers of most models are greater than 3.5 with the exception of model IV. Most values of dimensionless shear stress are on or over the threshold line of Shields' diagram, which means the bed formation occurs in all models and prototype.

4.2 River-Bed Change

Because experiment was done for the flood in 1995 when the observed river-bed change data do not exist, the behaviour of total river-bed change in model and prototype cannot be examined by means of comparing experiment data with observed data directly. In addition, as described above, similarity laws were not fully satisfied, which makes this comparison even more difficult. For this experiment, two models were built for the same region with different scales and also different sizes. This makes it possible to examine the applicability of the proposed theory qualitatively because $\Delta F \Delta M$ and ΔF_s are different between these models. Fig. 4 shows the cross-sectional distribution of river-bed change at the measuring section(No. 136) located 60,331km upstream from Paldang dam. Here, the positive value indicates deposited amount, the negative scoured amount. In this study, total river-bed change is defined by the

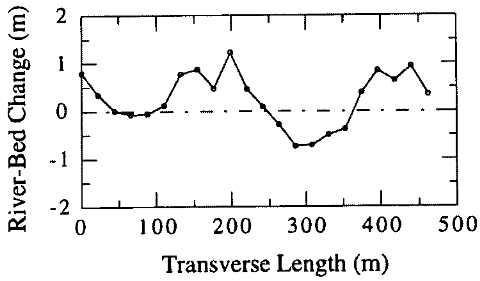

Fig. 4. Cross-Sectional Distribution of River-Bed Change (No. 136)

sum of the deposited amount and the absolute value of scoured amount. The deposited and scoured amounts are obtained per longitudinal unit length at each measuring section. All deposited and scoured amounts were translated to the scale of the prototype.

Because the discharge was controlled in the experiment such that the similarity of Froude number was satisfied, $\varDelta F$ was one. Therefore, if model bed material is selected such that $\varDelta F \varDelta M$ is one, the roughness in the model represents the one which satisfies exactly the relationship required by the similarity of the viscous force. If $\varDelta F \varDelta M$ is smaller than one, roughness in the model can be considered smaller than the value required. And if the relationship between the grain diameter and roughness follows Strickler formula, the grain diameter becomes smaller in the model than in the prototype when $\varDelta F \varDelta M$ is smaller than one. These situation can cause exaggerated total river-bed change in the models. When this value of $\varDelta F_s$, which is the ratio of the dimensionless shear stress at the bottom, is one, the tractive force is equal in model and prototype. When $\varDelta F_s$ is smaller than one, dimensionless tractive force becomes larger in the model than in the prototype, which results in exaggerated total river-bed change in the model. $\varDelta F_s$ is a function of D_r^{-1} and sensitive to particle diameter in model. In contrast, $\varDelta F \varDelta M$ is a function of $D_r^{-1/6}$ shown in Eq. (4), and far less sensitive to particle diameter in the model experiments. This means that a small difference of $\varDelta F \varDelta M$ results in a great difference in the total river-bed change between model and prototype.

As in this study, if the length scales were determined beforehand, the particle diameter of the bed material is important in satisfying similarity laws. Therefore, similarity in $\varDelta F \varDelta M$ and $\varDelta F_s$ become important. To examine the sensitivity of total river-bed change to these values, total river-bed changes along distance are presented in Fig. 5. Fig. 5(a) and (b) are the case in which the difference of $\varDelta F_s$ between models is smaller than that of $\varDelta F \varDelta M$ and show the sensitivity of $\varDelta F \varDelta M$. Fig. 5(c) shows the contrary case in which the difference of $\varDelta F \varDelta M$ is smaller than that of $\varDelta F_s$, and shows the sensitivity of $\varDelta F_s$. Fig. 5(a) and (b) correspond to the case 1 and 3 in table 3 and show that total river-bed change of model I with smaller value of $\varDelta F \varDelta M$ than that of model II and III. Therefore, it can be concluded that the smaller

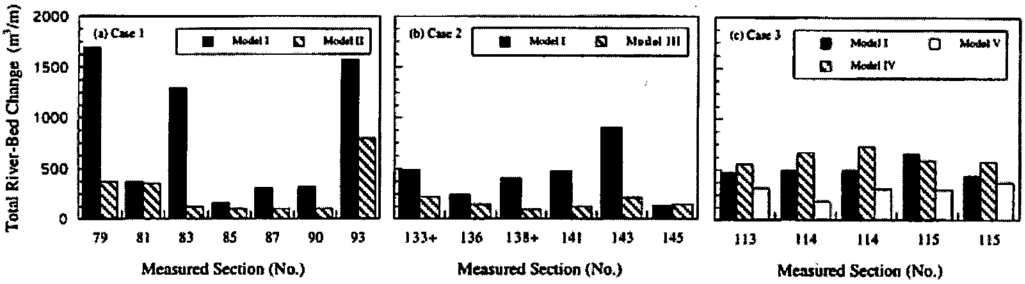

Fig. 5. Comparisons of Total River-Bed Change of Each Models

Table 4. Comparisons of Variation Ratio of River-Bed Change

Case	Equation	Difference of $\Delta F \Delta M$	Difference of ΔF_s	Measuring Section No.	Variation Ratio of River Bed Change
1	$\frac{\text{Model I} - \text{Model II}}{\text{Model II}} \times 100(\%)$	-9.8	1.7	79	360
				81+	4.1
				83	953
				85	61.5
				87	205
				90	227
				93+	96.3
2	$\frac{\text{Model I} - \text{Model III}}{\text{Model III}} \times 100(\%)$	-9.3	1.9	133+	124
				136	67.7
				138+	331
				141	295
				143	326
				145	-9.3
3	$\frac{\text{Model I} - \text{Model IV}}{\text{Model IV}} \times 100(\%)$	-24.4	393	113	-14.5
				114	-26.0
				114+	-31.9
				115	12.5
				115+	-23.0
	$\frac{\text{Model IV} - \text{Model V}}{\text{Model V}} \times 100(\%)$	-19.1	-86.9	113	75.5
				114	259
				114+	134
				115	92.2
				115+	55.4

$\Delta F \Delta M$ is, the larger total river-bed change is. Table 4 shows relative variation as $\Delta F \Delta M$ and ΔF_s change. The result in Case 1 shows that the relative decrease of $\Delta F \Delta M$ is -9.8% to -9.3%, which is mostly similar compared with these of Case 1 and relative increase of total river-bed change is 4.1-953% which is somewhat bigger than the range, -9.3%-331% of Case 2. Fig. 5(c) corresponds to case 3 in table 3 and shows that the smaller ΔF_s is, the larger total river-bed change is. Model IV, where ΔF_s is much smaller than other models shows relatively smaller increase in total river-bed change as shown in Fig. 5(c). It can be reasoned that anthracite, the bed material, has very small diameter and also cohesive, hence sediment transport becomes inactive. Above experimental results show good agreement with theoretical analysis of similarity laws developed in this study, and certifies the applicability qualitatively.

5. Conclusions

In this study, a relaxed similarity theory which can be applied to movable-bed river model is established by modifying existing theory by Einstein and Chien(1954). Experimental data collected from river models with movable beds were used to evaluate the applicability of the proposed theory. Effects of similarity of flow, $\Delta F \Delta M$, and similarity of sediment transport, ΔF_s, were examined by analyzing the behavior of total river-bed change. The results show that the bed material should be selected to satisfy similarities of $\Delta F \Delta M$ and ΔF_s as closely as possible. Especially, great care should be taken in interpreting the value of $\Delta F \Delta M$, because the value of $\Delta F \Delta M$ is not sensitive to model sediment and consequently a small change in $\Delta F \Delta M$ could cause a large change in total river-bed change. The smaller $\Delta F \Delta M$ or ΔF_s is, respectively, the larger total river-bed change is.

The modified similarity theory established in this study would be useful and practical whenever it is impossible or very difficult to satisfy strict theoretical requirements concerning model experiments with movable beds. Because the South Han River model with movable beds is not built to examine effects of similarity theories, it is not possible to examine the effects of the selection of various equations of sediment transport rate. Comparisons of bed formation between model and prototype could not be examined in this study and would be left as a future study.

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