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A Note on the Selected Multicriteria Decision Methods and Their Applications

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Abstract

In this study, we are concerned with the comparative aspects of five selected methods of decision making with respect to their adequacy in helping complex decisions about real-world problems. We examine the methods in terms of outlines, procedures and advantages/disadvantages to identify similarities and differences among them with the aim of showing which method is more likely to gain greater attention both in academia and in practice. To illustrate different courses of deployment of the methods, we offer an application with a case of airport transportation project in this paper. Some discussions on AHP and other methods are presented.

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1. Introduction

Multiple Criteria Decision Making (MCDM) has proven to be important in policy and choice analysis, both by taking account of the wide variety of aspects inherent in any decision problem and by offering an operational framework for a multidisciplinary approach to practical choice problems. Many of the methods in MCDM have been optimization-related. Goal Programming (GP), conceived by Charnes and Cooper [3], was an early contribution. Lately, Data Envelopment Analysis (DEA) has attracted considerable interest. On the other hand, Multiple Attribute Utility/Value Theory (MAU(V)T), first axiomatized by Debreu [6], has long been a major technique in the field of Multiattribute Decision Making (MADM). A French school also has developed several approaches including the ELECTRE methods that employ outranking relations. Additionally, the Analytic Hierarchy Process (AHP) is sometimes classified as a MADM approach.

Including these principal ones, there has been a large body of research on the individual method in MCDM procedures, but little on comparative evaluation of some of those methods. Comparing works are necessary in that multicriteria decision methods do not all yield compatible outcomes some of which may amount to wrong decisions.

The objective of this paper is to improve understanding of multicriteria decision methods with comparative, not expository purposes, by delineating the above-mentioned five methods, identifying their procedures, discussing their pros and cons, and, finally, dealing with a specific case of transportation project.

2. Multicriteria Decision Making Methods

2.1 The Analytic Hierarchy Process (AHP)

Outline

The AHP, developed by Saaty [20] in the early of 1970's, is a general theory of measurement. It is used to derive ratio scales from both discrete and continuous paired comparisons in multilevel hierarchic structures. These comparisons may be taken from actual measurements or from a fundamental scale that reflects the relative strength of preferences feelings. It has found its widest applications in multicriteria decision making, in planning and resource allocation, and in conflict resolution [21].

It is a systematic procedure for representing the elements of any problem. It organizes the basic rationality by breaking down a problem into its smaller constituent parts and then calls for only simple pairwise comparison judgments to develop priorities in each hierarchy. It provides a comprehensive framework to cope with the intuitive, the rational, and the irrational in us all at the same time. It is a method we can use to integrate our perceptions and purposes into an overall synthesis. The AHP does not require that judgments be consistent or even transitive. The degree of inconsistency of the judgments is revealed at the end of the AHP process.

Procedure

Step 1. Define the problem and structure the hierarchy of it from the top through the intermediate levels to the lowest level.

Step 2. Construct a set of pairwise comparison matrices for each level and make all the pairwise comparisons. The consistency is determined using the eigen value.

Step 3. Hierarchical composition is used to weight the eigen vectors by the weights of the criteria and the sum is taken over all weighted eigen vector entries corresponding to those in the next lower level of the hierarchy.

Step 4. The consistency of the entire hierarchy is found by multiplying each consistency index by the priority of the corresponding criterion and adding them together. The result is then divided by the same type of expression using the random consistency index corresponding to the dimensions of each matrix weighted by the priorities as before.

Advantages and Disadvantages

It permits us to collect all the relevant elements of the problem, whether objective or subjective, into one model and then to interactively work out their interdependencies and their perceived consequences. The availability of Expert Choice software with its sophistication and user-friendliness for AHP has made it a popular technique. One of the main strengths the AHP has is the inconsistency measure as a means for measuring the consistency of the decision maker's judgements. Users need to know when they have made inconsistent judgments, especially if they are working as a group. In fact, it is the only approach for solving multicriteria decision problems that has such a capability [2].

However, the AHP users must rely too heavily on their experience and intuitive judgment. The pairwise comparisons give us arbitrary reference points as we select one criterion or one alternative as a starting point for the comparisons. The AHP also has a problem called rank reversal that is the reversal of the preference order of the alternatives when new options are introduced in the problem.

2.2 Goal Programming (GP)

Outline

GP, developed by Charnes and Cooper [3] in 1961 and extended since then, is designed to deal with problems involving multiple conflicting objectives to achieve. The GP approach, as a modification and extension of linear programming, allows a simultaneous solution of a system of complex objectives rather than a single objective. The objective function of GP model may be composed of non-homogeneous units of measure, such as pounds and dollars, rather than one type of unit.

The procedure to formulate a GP model starts with specifying a target or aspiration value for each objective, thus transforming all objectives into goals. The resultant objective function, termed the achievement function in GP, is then the summation of deviations from these goals. Since it is usually impossible to attain goals simultaneously, the problem then becomes minimizing the sum of the deviations from the goals; that is, minimizing the achievement function.

The GP model can be described as follows:

$$\text{Minimize} \qquad Z = \sum_{k=1}^{k} (y_k^+ + y_k^-)$$

Subject to $\sum_{j=1}^{n} c_{jk}x_j-(y_k^+-y_k^-) = g_k$, for $k=1,2,\ldots,K$

$y_k^+ \geq 0,\ y_k^- \geq 0,\ x_j \geq 0\ (j=1,2\ldots,n)$

In the model presented above, the objective function is to minimize the sum of the deviations of each goal from its specified target level. This property eliminates the necessity of converting all objectives into one common unit of measurement [16].

Procedure

The steps described by Lee [15] are as follows.

Step 1. Define variables and constants. The first step in model formulation is the definition of choice variables and the right-hand side.

Step 2. Formulate constraints. Through an analysis of the relationships among choice variables and their relationships to the goals, a set of constraints should be formulated.

Step 3. Develop the objective function. Through the analysis of the decision maker's goal structure, the objective function must be developed.

Step 4. Seek a solution. The last step is to find the solution that minimizes the sum of deviations of these objective functions from their respective goals.

Advantages and disadvantages

GP is a good technique for identifying an acceptable solution when a minimum acceptable achievement level (goal) has been defined for each objective. GP is a suitable approach for deterministic mathematical programming problems, and, as long as the decision maker is capable of articulating target levels for objectives, the method can accommodate decision problems that include a relatively large number of objectives [11][13].

Despite its advantage, GP has a few drawbacks. A major one is that decision makers must specify the goals and their priorities a priori. The subjectivity inherent in the determination of the desired level of attainment for each objective and the penalty weights assigned to its over-attainment or under-attainment may present a problem. Another drawback of GP is the lack of a systematic approach to set priorities and trade-off among objectives. This shortcoming is even more evident when both tangible and intangible factors need to be considered and when many interests are involved and a number of people need to participate in the judgment process. This approach also are the often ad hoc nature of goal selection and the fact that the decision maker is expected to supply information with no knowledge of what the feasible trade-off in the nondominated set are. It may also be difficult for the decision maker to construct the mathematical relationships.

2.3 Outranking Method(ELECTRE)

Outline

The Outranking method, which was developed by Bernard Roy in 1968, based on the MAUT principles with the motivation to improve efficiency without affecting the outcome while considering less information is an outranking approach to multicriteria decision making used when faced with conflicting objectives. It is a procedure that sequentially reduces the number of alternatives the decision maker is faced with in a set of non-dominated alternatives.

The concept of an outranking relation S was introduced as a binary relation defined on the set of alternatives A .Given two alternatives A_i and A_j, A_i outranks A_j, or A_iSA_j ,if given all that it is known about the two alternatives, there are enough arguments to decide that A_i is at least as good as A_j .The goal of outranking methods is to find all alternatives that dominate other alternatives while they cannot be dominated by any other alternative. To find the best alternative the criteria weights are assumed to be measured on some scale, probably a ratio scale. Each criterion $C_j \in C$ is assigned a weight w_j, and every pair of alternatives A_i and A_j is assigned a concordance index $c(A_i, A_j)$ given by:

$$c(A_i, A_j) = \frac{1}{\sum_{k=1}^{n} w_k} \sum_{\{k: g_k(A_i) \geq g_k(A_j)\}} w_k$$

and a discordance index $d(A_i, A_j)$ given by:

$$d(A_i, A_j) = \begin{cases} 0 \ \ \textit{if } g_k(A_i) \geq g_k(A_j) \ \ \textit{for all } k, \\ \frac{1}{\delta}\max\{g_k(A_i) - g_k(A_j)\}, \ \ \textit{otherwise.} \end{cases}$$

where $.\delta = \max\{g_k(A_i) - g_k(A_j)\}$ Obviously, the discordance index is only valid if the operation subtraction is well defined. Once the two indices are defined, an outranking relation S is defined by:

$$A_i S A_j \text{ if and only if } \begin{cases} c(A_i, A_j) \geq \hat{c}, \\ d(A_i, A_j) \geq \hat{d}, \end{cases}$$

where $\hat{c}$ and $\hat{d}$ are thresholds. A problem with this discordance index is that the criteria levels be quantifiable. If that is not the case, then a discordance set D_j is defined for each criterion j with all the ordered pairs (x_j, y_j) such that if $g_j(A) = x_j$ and $g_j(B) = x_j$ then the outranking of B by A is refused. The outranking relation is defined now:

$$A_i S A_j \text{ if and only if } \begin{cases} c(A_i, A_j) \geq \hat{c}, \\ (g_j(A_i), g_j(A_j)) \notin D_j, \forall j. \end{cases}$$

Given the outranking relation it is now possible to find the set of alternatives $\mathrm{N} \subset \mathrm{A}$ for which:

$$\forall B \in \mathrm{A} - \mathrm{N} \exists A \in \mathrm{N} \textit{ such that } ASB$$
$$\forall A, B \in \mathrm{N}, ASB.$$

The outranking relation determines the set of non-dominated alternatives. The alternatives in N form the kernel of the graph defined by the alternatives (vertices) and the outranking relation (edges). Thus, if alternative A_i outranks alternative A_j, then a directed arc exists from A_i to A_j:: A_i. → A_j

Procedure

Step 1. Obtain the values of the attributes for every alternative with respect to the criteria.

Step 2. Construct the outranking relations by the following concordance and discordance, and construct a graph representing these relations.

Step 3. Obtain a minimum dominating subset. If a kernel exists, this is chosen as the minimum dominating subset.

Step 4. If the subset has a single element or is small enough to apply value judgment, select the final decision. Otherwise, step 2 through 4 is repeated until a single element or small subset exists.

Advantages and Disadvantages

It is suitable for multi-objective decision problems under certainty with a relatively small set of alternatives. The method is most appropriate when a subset of actions that enlighten the final choice of a single action must be selected. The method also has the ability to consider both objective and subjective criteria and the least amount of information required from the decision maker [19].

The first and most obvious weak point is the arbitrariness of assigning weights to the criteria as well as assigning values to the attributes. In the literature reviewed, it was always assumed that the decision maker had the capabilities to assign these values. The second shortcoming is that a complete ranking of alternatives may not be achieved. Only a partial prioritization of alternatives is computed. The best the method can do is to reduce the number of alternatives to a subset (i.e. minimal dominating subset) of solutions to the problem. The last weakness of the method is the ordinal way used to combine concordance and discordance that leaves one in doubt about the accuracy of its outcome.

2.4 Multiple Attribute Utility (Value) Theory (MAU(V)T)

Outline

MAUT, developed by Keeney and Raiffa [12], attempts to maximize a decision maker's utility or value (preference) which is represented by a function that maps an object measured on an absolute scale into the decision maker's utility or value relations. It is based on the following fundamental axiom: any decision maker attempts unconsciously to maximize a real valued function $U = U(g_1, g_2, \ldots, g_n)$, aggregating the criteria $g_1, g_2, \ldots g_n$, that is, all the different points of view which are taken into account. The role of the researcher is to try to estimate that function by asking the decision maker some well-chosen questions.

Utility independence is one of central concepts in MAUT and various utility-independence conditions imply specific forms of utility functions. However, only the additive and multiplicative forms are generally used in practice. The additive utility function can be represented as

$$u(x_1, \ldots, x_m) = k_1 u_1(x_1) + \ldots + k_m u_m(x_m)$$

where $u(x_1, \ldots, x_m)$ is on a scale from 0 to 1, the component utility functions $u_i(x_i)$ are on a scale from 0 to 1, and the scaling constants k_i are positive and sum to one. The multiplicative form is given as

$$1 + ku(x_1, \ldots, x_m) = \prod_{i=1}^{m} [1 + kk_i u_i(x_i)]$$

where $u(x_1, \ldots, x_m)$ is on a scale from 0 to 1 and the component utility functions $u_i(x_i)$ are on a scale from 0 to 1. However, the scaling constants k_i may be greater or less than one, and the constant k is chosen to satisfy the following equation.

$$1 + k = \prod_{i=1}^{m} [1 + kk_i]$$

Procedure

Step 1. Identify relevant characteristic (attributes).

Step 2. Assign quantifiable variables to each of the attributes and specify their restrictions.

Step 3. Select and construct utility functions for the individual attributes.

Step 4. Synthesize the individual utility functions into a single additive or multiplicative utility function.

Step 5. Evaluate the alternatives using the function obtained in the fourth step.

Advantage and Disadvantages

The primary advantage of MAUT is that the problem becomes a single objective problem once the utility function has been assessed correctly, thus ensuring achievement of the best-compromise solution.

The disadvantages of MAUT are: (1) because a single-attribute utility function must be defined for each criterion (attribute), the solution process becomes time consuming as the number of criteria increases; (2) the decision maker must articulate his or her preferences among alternatives that may not have any practical reality; and (3) certain assumptions (for example, mutual preferential independence, mutual utility independence) must be satisfied in order for this approach to apply [23].

2.5 Data Envelopment Analysis (DEA)

Outline

DEA was originated by Charnes, Cooper, and Rhodes [4] as a methodology to analyze the relative efficiency of each of several decision making units (DMUs). DEA takes a fundamentally different viewpoint from the other multiple criteria approaches, because it explicitly considers inputs and outputs that are associated with each of the alternatives.

The basic elements of a DEA analysis are the decision making units, inputs (where less is better), and outputs (where more is better). The viewpoint taken is that an increase in an input is expected to yield an increase in an output. However, the viewpoint is also that it is desirable to minimize inputs because the inputs require resources and thus result in a cost. In the context of a multicriteria problem, the DMUs can be considered to be the alternatives and the outputs can be considered to be the criteria. It should also be noted that the inputs could be viewed as attributes that we would like to minimize.

The approach used to develop the envelopment frontier is based on the additive model, and its uses a pair of dual linear programming models as shown below. In particular, the following formulation is used: given m inputs, p outputs, and n DMUs, let xij

correspond to the value of the ith input variable ($i=1,2,\cdots,m$) for the jth DMU ($j=1,2,\cdots,n$), and let ykj correspond to the value of the kth output variable ($k=1,2,\cdots,p$) for the jth DMU.

Primal Problem for the jth *DMU:*

$$\text{Minimize} \quad -\sum_{i=1}^{p} s_i - \sum_{i=1}^{m} \varepsilon_i$$

$$\text{Subject to} \quad \sum_{l=1}^{n} \lambda_l y_{il} - s_i = y_{ij} \qquad i = 1,2,\ldots,p$$

$$-\sum_{l=1}^{n} \lambda_l x_{il} - \varepsilon_i = -x_{ij} \qquad i = 1,2,\ldots,m$$

$$\sum_{l=1}^{n} \lambda_l = 1$$

$$\lambda_i, \varepsilon_i, s_i \geq 0 \quad \textit{for all } i$$

Dual Problem for the jth *DMU:*

$$\text{Maximize} \quad -\sum_{i=1}^{p} \mu_i y_{ij} - \sum_{i=1}^{m} v_i x_{ij} + \omega_j$$

$$\text{Subject to} \quad -\sum_{i=1}^{p} \mu_i y_{ij} - \sum_{i=1}^{m} v_i x_{ij} + \omega_j \leq 0 \qquad k = 1,2,\ldots,n$$

$$-\mu_i \leq -1 \qquad i = 1,2,\ldots,p$$

$$-v_i \leq -1 \qquad i = 1,2,\ldots,m$$

$$\omega_j \quad \textit{free}$$

Note that only one of the preceding problems needs to be solved for the jth DMU.

An underlying concept of DEA is the notion of efficiency. In the dual problem, the variables correspond to prices for the inputs and outputs. Efficiency is defined to be the sum of the value of the outputs divided by the sum of the value of the inputs. It is assumed that no DMU or alternative can be more than 100 percent efficient.

Consequently, we obtain constraints of the form:

$$\frac{\sum_{i=1}^{p} \mu_i y_{ik}}{\sum_{i=1}^{m} v_i x_{ik}} \leq 1.0 \qquad k = 1,2,\ldots,n$$

By multiplying both sides of the inequality by the denominator, we obtain linear expressions that become part of the dual problem.

DEA analysis provides several useful results. It constructs an envelopment surface that is alternatively referred to as a production function or an efficient frontier. This allows us to determine which of the DMUs are efficient and which are inefficient. The envelopment surface shows the maximum amount of outputs that can be achieved by combining various inputs. Alternatively, it shows the minimum amount of inputs required to achieve a given output level.

3. A Case; Pittsburgh Airport Transportation Project

The purpose of this exercise is to report on the application of five different decision methods in selecting the best means of transportation to the Pittsburgh Airport. The data is based on the information provided by transportation planning manager for Southwestern Pennsylvania Regional Planning Commission. However, the problem appears to be very complex and the information obtained is very limited. The problem is very much simplified and some of the quantitative data is assumed for the purpose of this exercise only. A brief summary of facts surrounding airport transportation planning is:

In 1989, the Parkway West Corridor Study was done as a first attempt at comprehensive planning involving the airport and Golden Triangle areas. The result has been a series of projects:

a. Beltway (A): Improvements around Golden Triangle

b. Tollroad (B): A new, possibly multi-modal, corridor to the parkway.

c. Newlanes (C): Airport access improvements

d. Busway (D): Similar to ones already in existence in Pittsburgh.

Over the past three years, the busway has become the major priority. The other projects are coming along, but the toll road idea is proceeding at the slowest pace. Other projects have come into focus, as well. Due to the eventual closing of the Fort Pitt Bridge for repairs, there is a need for improving other arteries to take up the traffic load this main conduit provides. Another situation presenting itself is that the Wheeling and Lake Erie Railroad right-of-way parallels the airport corridor and it may become available as an alternative corridor, possibly for use by car pools and buses. New ideas impacting thinking about transportation projects seem to be, in a sense, "nearly endless." All this change makes transportation decision making a highly dynamic process. So, the problem is summarized as a multicriteria decision problem as follows;

Definition of the problem consists of the alternatives to be evaluated, attributes (criteria) of the problem and their quantification, as well as the consequences of each alternative in terms of the attributes are presented in Table 1 and 2 below:

Table 1 Attributes and their ranges

Attributes		Range
C1	Cost to build	50 to 800 millions $
C2	# of jobs created	0 to 5000 jobs
C3	Travel time	10 to 60 minutes
C4	Travel cost	$2 to $10
C5	Environmental impact	low, medium, high
C6	Housing dislocation	yes, no

Table 2 Estimated values of alternatives for criteria

Attributes	Beltway(A)	Tollroad(B)	Newlanes(C)	Busway(D)
C1	70	190	200	600
C2	0	100	0	500
C3	25	20	30	40
C4	2	2	2	5
C5	low	medium	low	medium
C6	no	yes	no	yes

3.1 The Analytic Hierarchy Process

The decision problem is to decide which of four candidate projects to choose. The matrices of pairwise comparisons of the criteria and alternatives are shown below in Table 3 and 4, along with the resulting vectors of priorities. Of course, these comparisons are performed on the basis of the values in Table 2.

Table 3 Pairwise Comparison Matrix for Criteria

	C1	C2	C3	C4	C5	C6	Priority vector
C1	1	3	2	4	3	9	0.365
C2	1/3	1	1/3	1	1	6	0.113
C3	1/2	3	1	3	3	7	0.267
C4	1/4	1	1/3	1	1	6	0.109
C5	1/3	1	1/3	1	1	8	0.122
C6	1/9	1/6	1/7	1/6	1/8	1	0.025

(Inconsistency Ratio = 0.03)

Table 4 Pairwise Comparison Matrices for Alternatives

C1	A	B	C	D	Priority vector
A	1	3	3	9	0.556
B	1/3	1	2	3	0.224
C	1/3	1/2	1	3	0.158
D	1/9	1/3	1/3	1	0.062

(Inconsistency Ratio = 0.02)

C2	A	B	C	D	Priority vector
A	1	1/3	1	1/9	0.070
B	3	1	2	1/4	0.177
C	1	1/2	1	1/9	0.077
D	1/9	4	9	1	0.675

(Inconsistency Ratio = 0.01)

C3	A	B	C	D	Priority vector
A	1	1	2	3	0.351
B	1	1	2	3	0.351
C	1/2	1/2	1	2	0.189
D	1/3	1/3	1/2	1	0.109

(Inconsistency Ratio = 0.00)

C4	A	B	C	D	Priority vector
A	1	1	1	2	0.280
B	1	1	1	3	0.312
C	1	1	1	2	0.280
D	1/2	1/3	1/2	1	0.127

(Inconsistency Ratio = 0.01)

C5	A	B	C	D	Priority vector
A	1	5	1	5	0.417
B	1/5	1	1/5	1	0.083
C	1	5	1	5	0.417
D	1/5	1	1/5	1	0.083

(Inconsistency Ratio = 0.00)

C6	A	B	C	D	Priority vector
A	1	7	1	7	0.437
B	1/7	1	1/7	1	0.063
C	1	7	1	7	0.437
D	1/7	1	1/7	1	0.063

(Inconsistency Ratio = 0.00)

After applying the principle of composition of priorities, we obtained such final results as A=0.396, B=0.241, C=0.209, and D=0.154 with overall inconsistency ratio 0.03 in distributive mode (A=0.389, B=0.254, C=0.221, and D=0.136 with overall inconsistency ratio 0.03 in ideal mode). Therefore, the best alternative is to improve Beltway.

3.2 Goal Programming

We can define this problem as in the Table 5. The problem includes all three types of goals: a low bound goal (C2), a specific numerical goal (C5, C6), and an upper bound goal (C1, C3, C4).

Table 5 Each projects contribution to six criteria

	A	B	C	D	Goal
C1	70	190	200	600	≤ 800
C2	0	100	0	500	
C3	25	20	30	40	
C4	2	2	2	5	
C5	L(0)	M(0.5)	L(0)	M(0.5)	
C6	N(0)	Y(1)	N(0)	Y(1)	

Let the decision variables be the construction rates of the project A, B, C and D, respectively. These goals can be stated as.

$$\begin{aligned} 70x_1+190x_2+200x_3+600x_4 &\le 800 \\ 100x_2+\qquad 500x_4 &\ge 5000 \\ 25x_1+20x_2+30x_3+40x_4 &\le 60 \\ 2x_1+2x_2+2x_3+5x_4 &\le 10 \\ 0.5x_2+\qquad 0.5x_4 &= 0 \\ x_2+\qquad x_4 &= 0 \end{aligned}$$

We now introduce the new (auxiliary) variables,

$$\begin{aligned} y_1 &= 70x_1+190x_2+200x_3+600x_4-800 \\ y_2 &= 100x_2+500x_4-5000 \\ y_3 &= 25x_1+20x_2+30x_3+40x_4-60 \\ y_4 &= 2x_1+2x_2+2x_3+5x_4-10 \\ y_5 &= 0.5x_2+0.5x_4 \\ y_6 &= x_2+x_4 \end{aligned}$$

As well as their positive and negative components

$$y_1 = y_1^+ - y_1^-, \quad where\ y_1^+ \geq 0,\ y_1^- \geq 0$$
$$y_2 = y_2^+ - y_2^-, \quad where\ y_2^+ \geq 0,\ y_2^- \geq 0$$
$$y_3 = y_3^+ - y_3^-, \quad where\ y_3^+ \geq 0,\ y_3^- \geq 0$$
$$y_4 = y_4^+ - y_4^-, \quad where\ y_4^+ \geq 0,\ y_4^- \geq 0$$
$$y_5 = y_5^+ - y_5^-, \quad where\ y_5^+ \geq 0,\ y_5^- \geq 0$$
$$y_6 = y_6^+ - y_6^-, \quad where\ y_6^+ \geq 0,\ y_6^- \geq 0$$

This leads to the following linear programming formulation if this goal programming problem

Minimize $\quad Z = y_1^+ + y_2^- + y_3^+ + y_4^+ + y_5^+ + y_5^- + y_6^+ + y_6^-$

Subject to

$$70x_1 + 190x_2 + 200x_3 + 600x_4 - (y_1^+ - y_1^-) = 800$$
$$100x_2 + \quad 500x_4 - (y_2^+ - y_2^-) = 5000$$
$$25x_1 + 20x_2 + 30x_3 + 40x_4 - (y_3^+ - y_3^-) = 60$$
$$2x_1 + 2x_2 + 2x_3 + 5x_4 - (y_{34}^+ - y_3^-) = 10$$
$$0.5x_2 + \quad 0.5x_4 - (y_3^+ - y_3^-) = 0$$
$$x_2 + \quad x_4 - (y_3^+ - y_3^-) = 0$$

and $\quad y_k^+ \geq 0,\ y_k^- \geq 0,\ x_j \geq 0\ (j = 1,\ldots,6;\ k = 1,\ldots,6)$

Applying the simplex method to this formulation yields a following optimal solution, $x_1 = 11.43$, and .$x_2 = x_3 = x_4 = 0$ So, the best alternative is improving beltway.

3.3 Outranking Method

The criteria and the value of the attributes for the four alternatives are presented in Table 6 as applied to the method. The six criteria were assigned the following weights:, C1=.294, C2=.142, C3=.237, C4=.126, C5=.138, and C6=.063.

Table 6 Alternatives and Criteria

	A	B	C	D
C1	.566	.189	.179	.066
C2	.000	.167	.000	.833
C3	.300	.350	.200	.150
C4	.294	.294	.294	.118
C5	.325	.175	.325	.175
C6	.333	.333	.333	.001

Using these values of the weights for the criteria and the values in Table 6, the complete set of concordance and discordance indexes is represented in Table 7 and 8.

Table 7 the Concordance Matrix

	A	B	C	D
A	-	.527	.766	.858
B	.474	-	.768	.789
C	.235	.233	-	.858
D	.142	.211	.142	-

Table 8 the Discordance Matrix

	A	B	C	D
A	-	.167	.000	.833
B	.377	-	.150	.666
C	.387	.167	-	.833
D	.332	.332	.332	-

Now suppose that the decision maker has specified a minimum concordance of 0.50 and a maximum discordance of 0.50, that is, $c(A_i, A_j) > 0.50$ and $d(A_i, A_j) < 0.50$. With this specification the graph can now be constructed. The directed paths which appear in the graph are determined by the set of indices that simultaneously satisfy the requirement that $p > 0.50$ and $q < 0.50$. These indices are: (A,B) (A,C) (B,C).

The resulting graph is depicted below in Figure 1. Using the graph, the decision maker can determine the optimal choice by elimination of the nodes. The direction of the arrow determines which alternative outranks which. As shown in this figure, alternative

A is outranked alternative B and alternative C. Alternative B outranks or is more preferred than alternative C. So, the best alternative is to improve Beltway.

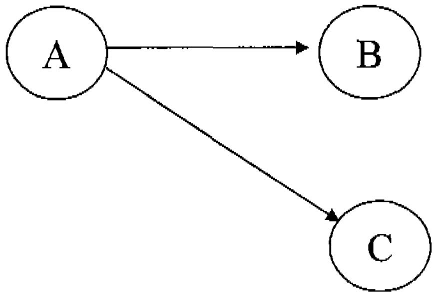

Figure 1 Resulting Graph

3.4 Multiple Attribute Utility Theory

It is assumed that the utility functions are monotonic and that the decision makers are risk averse. The utility function may be monotonically increasing (that is, if x_k is greater than x_j, x_k is always preferred to x_j) or monotonically decreasing (that is, if is less than x_k, x_k is always preferred to x_j). Assuming risk aversion decision makers mean that I can use either $u(x) = a + b\log(x + c)$ for monotonically decreasing utility functions or either $u(x) = a + b\log(c - x)$ for monotonically increasing utility functions.

Evaluation procedure of utility function for the six attributes enables us to obtain the utility functions below:

(1) C1: $u_1(x_1) = 29.87 - 3.70\log(x_1 + 2400)$

(2) C2: $u_2(x_2) = 11.23 - 1.23\log(900 - x_2)$

(3) C3: $u_3(x_3) = 1.43 - 0.36\log(x_3 - 6.67)$

(4) C4: $u_4(x_4) = 2.46 - 0.97\log(x_4 + 2.5)$

(5) C5: $u_5(high) = 0,\ u_5(med) = 0.5,\ u_5(low) = 1$

(6) C6: $u_6(yes) = 0\ and\ u_6(no) = 1$

For the given consequences of each alternative as shown in Table 2, we obtain such utilities as in Table 9.

Table 9 Utilities of alternatives

Attributes	A	B	C	D
C1	0.97	0.79	0.78	0.25
C2	0	0.04	0	0.10
C3	0.38	0.50	0.30	0.17
C4	1	1	1	0.51
C5	1	0.5	1	0.5
C6	1	0	1	0

The scaling constants are $k_1 = 0.25, k_3 = 0.25, k_4 = 0.14, k_5 = 0.14, k_2 = 0.14,$ and $k_6 = 0.05$. Since $k_1 + \cdots + k_6 = 0.97 \approx 1$, we should use an additive utility function.

Utility of the alternatives : $u(Beltway) = 0.6675$, , $u(Toll\ road) = 0.5381$, $u(New lanes) = 0.6000$ and $u(Busway) = 0.2604$. Improving beltway has the highest utility, which means this is the best alternative.

If the scaling constants are $k_3 = 0.50$, $k_4 = 0.28$, $k_5 = 0.28$, $k_2 = 0.28$, and $k_6 = 0.1$, since $k_1 + \cdots + k_6 = 1.94 \neq 1$, we should use a multiplicative utility function.

We obtain $k = -0.8762$ and the utility function is:

$$u(x_1,\dots,x_6) = 1.1413 + [0.5u_1(x_1) - 1.1413][1 - 0.2453u_2(x_2)][1 - 0.4381u_3(x_3)]$$
$$[1 - 0.2453u_4(x_4)][1 - 0.2453u_5(x_5)][1 - 0.0876u_6(x_6)]$$

Utility of the alternatives : $u(Beltway) = 0.857$, $u(Toll\ road) = 0.759$, $u(New lanes)$ =0.802, and $u(Busway) = 0.706$. Here the best alternative is improving Beltway.

3.5 Data Envelopment Analysis

To understand how DEA works, consider four projects (DMUs). Suppose that each project uses three inputs to produce one output. Namely, the inputs are cost of construction, travel time, and travel cost; the output is number of job created. The other two criteria named the environmental impact and dislocation of housing is excluded due to their intangible attributes. DEA deals with only the measurable inputs and outputs. The inputs and outputs for four projects are summarized in Table 10.

Table 10 Input and output data for four alternative projects

	Inputs			Output
Projects	1	2	3	1
A	70	25	2	0
B	190	20	2	100
C	200	30	2	0
D	600	40	5	500

Using the preceding LP formulation for the dual problem, the following LPs are solved:

Dual Problem for Project A:

$$\text{Maximize} \quad -70\nu_1 - 25\nu_2 - 2\nu_3 + \omega_1$$

$$\text{Subject to} \quad -70\nu_1 - 25\nu_2 - 2\nu_3 + \omega_1 \le 0$$

$$100\mu_1 - 190\nu_1 - 20\nu_2 - 2\nu_3 + \omega_1 \le 0$$

$$-200\nu_1 - 30\nu_2 - 2\nu_3 + \omega_1 \le 0$$

$$500\mu_1 - 600\nu_1 - 40\nu_2 - 5\nu_3 + \omega_1 \le 0$$

$$\mu_i \ge 1 \quad i = 1, \quad \nu_i \ge 1, \ i = 1,2,3$$

Dual Problem for Project B

$$\text{Maximize} \quad 100\mu_1 - 190\nu_1 - 20\nu_2 - 2\nu_3 + \omega_2$$

$$\text{Subject to} \quad -70\nu_1 - 25\nu_2 - 2\nu_3 + \omega_2 \le 0$$

$$100\mu_1 - 190\nu_1 - 20\nu_2 - 2\nu_3 + \omega_2 \le 0$$

$$-200\nu_1 - 30\nu_2 - 2\nu_3 + \omega_2 \le 0$$

$$500\mu_1 - 600\nu_1 - 40\nu_2 - 5\nu_3 + \omega_2 \le 0$$

$$\mu_i \ge 1 \quad i = 1, \quad \nu_i \ge 1, \ i = 1,2,3$$

Dual Problem for Project C:

$$\text{Maximize} \quad -200\nu_1 - 30\nu_2 - 2\nu_3 + \omega_3$$

$$\text{Subject to} \quad -70\nu_1 - 25\nu_2 - 2\nu_3 + \omega_3 \le 0$$

$$100\mu_1 - 190\nu_1 - 20\nu_2 - 2\nu_3 + \omega_3 \le 0$$

$$-200\nu_1 - 30\nu_2 - 2\nu_3 + \omega_3 \le 0$$

$$500\mu_1 - 600\nu_1 - 40\nu_2 - 5\nu_3 + \omega_3 \le 0$$

$$\mu_i \ge 1 \quad i=1, \quad \nu_i \ge 1, \; i=1,2,3$$

Dual Problem for Project D:

$$\text{Maximize} \quad 500\mu_1 - 600\nu_1 - 40\nu_2 - 5\nu_3 + \omega_4$$

$$\text{Subject to} \quad -70\nu_1 - 25\nu_2 - 2\nu_3 + \omega_4 \le 0$$

$$100\mu_1 - 190\nu_1 - 20\nu_2 - 2\nu_3 + \omega_4 \le 0$$

$$-200\nu_1 - 30\nu_2 - 2\nu_3 + \omega_4 \le 0$$

$$500\mu_1 - 600\nu_1 - 40\nu_2 - 5\nu_3 + \omega_4 \le 0$$

$$\mu_i \ge 1 \quad i=1, \quad \nu_i \ge 1, \; i=1,2,3$$

The 'problem Solver' in Microsoft EXCEL was used to solve the preceding problems. For illustration purpose, selected output is shown below. Specifically, the objective function values for all projects are shown in Table 11.

Table 11 Objective function values for the four projects

	Original value of the objective	Final value of the objective
Project A	0	0
Project B	0	0
Project C	0	-135
Project D	0	0

As shown in Table 11, the values of the objective function at the optimal solution for the four projects A,B,C, and D are 0,0,-135, and 0, respectively. Thus we see that project A,B, and D are efficient (lie on the efficient frontier) and project C is inefficient (that is, project C is dominated by project A,B, and D). The value of -135 is a measure of the distance of project C from the efficient frontier.

As shown in the above example, DEA is used not to have a weight value of each alternative but only to analyze the efficiency of sets of alternatives.

4. Concluding Remarks

We have briefly described several different methods for solving multicriteria decision problems and illustrated each method with a specific case of Pittsburgh transportation project. From the descriptions, each of these methods has its own advantages and disadvantages. Some ideas on how each method works and types of applications where each may be used were provided.

Even though all the techniques examined in this study have their limitations, they are useful for certain decision making scenarios, as long as the assumptions underlying each technique are carefully considered before a choice is made. It is certainly possible to imagine problem situations where more than one technique can be employed for effective transportation planning.

The AHP is free of paradoxes because so far it has solved the two issues against it: the need to preserve rank in relative measurement when new alternatives are added, and the need to diminish the number of judgments provided. It solved both the seeming paradox of rank reversal and the criticism that too many judgments are required. So far as real life applications are concerned, the AHP has reached its theoretical and technical maturity. Decision science, the AHP in particular, is cross-discipline area of research. Substantial development in the field can only be obtained if researchers working in other disciplines, such as the social sciences, organizational development, and strategic management embrace the AHP more widely.

Vargas [25] commented the major differences between the AHP and other methods in the conclusion of his conference paper. As we may understand from the above case study, the AHP produces ratio scales, while MAUT yields interval scales and ELECTRE derives ordinal scales. In the process of constructing the scales, however, these three methods need quantification of criteria. In MAUT unlike in the AHP, the criteria weights that appear in the multiattribute utility function do not represent how important the criteria weights are but speak of scaling constants. This is also a problem with ELECTRE. The major philosophical difference between the AHP and the other methods is that rank reversal is acceptable and that the criteria are just as measurable as alternatives are.

Moreover, the final results of ELECTRE do not provide a priority vector, but preference relations only. This implies that the ELECTRE method is more adequate for clustering plentiful alternatives rather than prioritizing smaller alternatives. The optimization-related techniques considered in this paper also do not come up with priorities, but simply efficiency of the alternatives.

Despite of the above some differences between the AHP and the other methods, we have same results. After applying five methods to the transportation problem as an example, 'Beltway' was decided as the best alternative by all five methods. The results are seemed to depend on not only the initial value of each alternative for each criterion but also the weight values of criteria. In this case, if we emphasize the second criterion (number of job created), alternative D might be the best one.

References

[1] Booker, J.M., Bryson, M.C., Decision Analysis in Project Management: An Overview, *IEEE Transactions on Engineering Management*, Vol.32, pp.3-9, 1985.

[2] Carlsson, C., Walden, P., AHP in Political Group Decisions: A study in the Art of Possibilities, *Interfaces*, Vol.25, pp.14-29, 1995.

[3] Charnes, A., Cooper, W.W., *Management Model and Industrial Application of Linear Programming*, John Wiley, New York, 1961.

[4] Charnes, A., Cooper, W.W., Rhodes, E., Measuring the Efficiency of Decision Making Units, *European Journal of Operational Research,* Vol.2, pp.429-444, 1978.

[5] Charnes, A., Cooper, W.W., Ferguson, R.O., Optimal Estimation of Executive Compensation by Linear Programming, *Management Science*, Vol.1, pp.138-151,1955.

[6] Debreu, G., Topological Methods in Cardinal Theory, In Arrow, K.J., et al. (eds), *Mathematical Methods in the Social Science 1959,* Stanford University Press, Stanford, CA, PP.16-26, 1960

[7] Dyer, J.S., *et al.*, Multiple Criteria Decision Making, Multiattribute Utility Theory: the Next Ten Years, *Management Science*, Vol.38, pp.645-654, 1992.

[8] Forgionne, G.A., *Quantitative Decision Making,* Wadsworth Publishing Company, 1986.

[9] Goicoechen, A., *et al., Multiobjective Decision Analysis with Engineering and Business Applications*, John Wiley and Sons, 1982.

[10] Hillier, F.S., Lieberman, G.J., *Introduction to Operations Research*, 3rd ed., Holden-Day, Inc., 1980.

[11] Ignizio, J.P., *Goal Programming and Extensions*, Lexington Books, 1976.

[12] Keeney, R.L., Raiffa, H., *Decisions with Multiple Objectives: Preference and Value Tradeoffs*, Wiley, New York, 1976.

[13] Khorramshahgol, R., *et al.*, An Integrated Approach to Project Evaluation and Selection, *IEEE Transactions on Engineering Management*, Vol.35, pp.265-270, 1988.

[14] Khorramshahgol, R., Gousty, Y., Delphic Goal Programming (DGP): A multi-objective Cost/Benefit Approach to R&D Portfolio Analysis, *IEEE Transactions on Engineering Management*, Vol.33, pp.172-175, 1986.

[15] Lee, S.M., *Goal Programming for Decision Analysis*, Auerbach Publishers Inc., 1972.

[16] Mollaghasemi, M., Edwards, J.P., *Technical Briefing: Making Multiple Objective Decisions*, IEEE Computer Society Press, 1997.

[17] NijKamp, P., Van Delft, A., *Multi-criteria Analysis and regional decision-making,* Martinus Nijhoff, Leiden, 1977.

[18] Romero, C., *Handbook of Critical Issues in Goal Programming*, Pergamon Press,

1991.

[19] Roy, B., Vincke, P., Multicriteria Analysis: Survey and New Directions, *Euroupean Journal of Operational Research*, Vol.8, pp.207-218, 1981.

[20] Saaty, T.L., *The Analytic Hierarchy Process*, McGraw-Hill, 1980.

[21] Saaty, T.L., Priority Setting in Complex problems, *IEEE Transactions on Engineering Management*, Vol.30, pp.140-155, 1983.

[22] Schniederjans, M.J., *Goal Programming: Methodology and Applications*, Kluwer Academic Publishers, 1995.

[23] Schoemaker, P.J.H., Waid, C.C., A Comparison of Several Methods for Constructing Additive Representations of Multi-Attribute Preferences, *Wharton applied Research Center*, University of Philadelphia, August, 1978.

[24] Vargas, L.G., An Overview of the Analytic Hierarchy Process and Its Applications, *Euroupean Journal of Operational Research*, Vol.48, pp.2-8, 1990.

[25] Vargas, L.G., Comparison of Three Multicriteria Decision Making Theories: The Analytic Hierarchy Process, Multiattribute Utility Theory and Outranking Methods, *The International Society of Analytic Hierarchy Process Conference paper*, 1993.

[26] Vincke, P., *et al.*, *Multicriteria Decision-Aid*, John Wiley and Sons, 1992.

[27] Zahedi, F., The Analytic Hierarchy Process: A Survey of the Method and Its Application, *Interfaces*, Vol.16, pp.96-108, 1986.

[28] Zeleny, M., *Multiple Criteria Decision Making*, MaGraw Hill, 1982.

Table 2 Estimated values of alternatives for criteria

[98년 11월 2일 접수, 99년 5월 10일 심사완료]