

# Suppression of Output Distortion in a Gyroscope using Fiber Amplifier/Source by Tracking of Optimum Modulation Amplitude

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We propose and demonstrate a new scheme for suppression of output distortion in an open-loop gyroscope employing an erbium-doped fiber amplifier/source (FAS). In addition to the main modulation for the rotation rate measurement, a small auxiliary modulation at a different frequency is used to extract an error signal, which is necessary for keeping the quasi-dc component of the feedback signal power at a constant level for varying rotation rate. By active tracking of the optimum modulation condition using this two-frequency modulation scheme, we obtain stable gyro output with suppressed distortion as well as stable FAS characteristics. We also calculate the distortion in the gyro response due to the feedback effect, from which we estimate the FAS gyro output distortion due to the residual ac feedback effect when the dc feedback effect is removed by the proposed scheme. The measured residual deviation agrees reasonably with the estimation.

## I. INTRODUCTION

Broadband erbium-doped fiber sources have been promising sources of choice for navigation-grade fiber-optic gyroscope, owing to their good spectral stability and high output power [1-4]. Various source configurations with doped fibers have been studied [5-10]. Among them, fiber amplifier/source (FAS) configuration [6,10,12] uses the doped fiber not only as a source but also as an optical amplifier for the returning gyro signal. This configuration offers great advantages in high detected power as well as simplicity, which can relax requirements in electronic detection circuitry. Despite these advantage, however, this configuration has suffered a feedback problem owing to its bidirectional nature. Since the feedback from the gyro coil to the source is the returning gyro signal that is rotation dependent, the FAS characteristics such as power and spectrum become also dependent on the rotation rate, which leads to distortion in gyro output [11,12].

One possible approach to this problem is to set the amplitude of the phase difference modulation at the proper value (2.405 rad) where the time average of the feedback signal power is kept constant for varying rotation rate [12]. In fact, however, some difficulties remain in employing this approach effectively. First of all, it is not so easy to set the modulation amplitude at the exact value of the proper modulation. Secondly, even if

the voltage amplitude of the driving signal (applied to the modulator) is initially set at the exact value and is kept constant, the actual amplitude of the modulated phase difference may drift due to environmental changes, especially changes in temperature. Thus it is necessary for stabilization of an FAS gyro to automatically set and track the optimum amplitude value of the phase modulation.

In this paper, we propose and demonstrate a scheme to automatically set and track the optimum phase modulation amplitude in an open-loop gyroscope employing an erbium-doped FAS (EDFAS). The operating principle is based on the assumption that the characteristics of EDFAS are mostly affected by the quasi-dc component of the feedback signal owing to the slow gain dynamics of the erbium ion [13]. We found a new way of extracting an error signal from the gyro signal output to keep the quasi-dc component of the feedback power at a constant level. In addition to the main modulation for the rotation rate measurement, a small auxiliary modulation at a different frequency was used to extract an error signal. By tracking the optimum modulation amplitude using the two-frequency modulation scheme, we obtained stable gyro output with suppressed distortion as well as stable FAS. We also calculated the deviation from the ideal gyro response due to the feedback effect. From the derived expression for the feedback signal distortion, we estimated the FAS gyro output distortion due to the residual ac feedback effect when the dc feedback effect was

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removed by the proposed scheme.

## II. PRINCIPLE OF OPERATION

Suppose that in an open-loop fiber-optic gyroscopes, incorporating a Sagnac loop the phase difference between counter-propagating waves is modulated such that

$$\phi(t) = \phi_1 \sin \omega_1 t + \phi_2 \sin \omega_2 t. \quad (1)$$

Here,  $\phi_1 \sin \omega_1 t$  is the main modulation for the rotation rate measurement, and  $\phi_2 \sin \omega_2 t$  with small  $\phi_2$  is the auxiliary one for extracting an error signal. Then the total phase difference between counter-propagating waves,  $\Delta\phi$  is expressed as  $\Delta\phi = \Delta\phi_R + \phi(t)$  where  $\Delta\phi_R$  is the Sagnac phase shift. When an optical wave from the source with power  $P_s(t)$  is sent into the gyroscope, the feedback power returning to the source  $P_f(t)$  is

$$P_f(t) = \frac{1}{2} \gamma P_s(t - \tau) (1 + \cos \Delta\phi) \quad (2)$$

where  $\gamma$  is the optical loss factor through the gyroscope and  $\tau$  is the transit time of the gyroscope.

As a first approximation, we ignore the ac feedback response, the residual ac feedback effect will be treated later in the Appendix. While ignoring the ac feedback response, we can set  $P_s(t) = P_s(t - \tau) = P_s$  that is independent of time. Then the feedback signal power from the Sagnac loop can be expanded in a series as a function of time with harmonics of frequencies of  $\omega_1$  and  $\omega_2$ . Among the terms in the series expansion, components of the quasi-dc, frequencies of  $\omega_1$ ,  $\omega_2$  and  $2\omega_2$  are expressed as

$$P_f^{dc} = \frac{1}{2} \gamma P_s [1 + J_0(\phi_1) J_0(\phi_2) \cos \Delta\phi_R], \quad (3)$$

$$P_f^{\omega_1} = -\gamma P_s J_1(\phi_1) J_0(\phi_2) \sin \Delta\phi_R \sin \omega_1 t, \quad (4)$$

$$P_f^{\omega_2} = -\gamma P_s J_1(\phi_2) J_0(\phi_1) \sin \Delta\phi_R \sin \omega_2 t, \quad (5)$$

$$\text{and } P_f^{2\omega_2} = \gamma P_s J_2(\phi_2) J_0(\phi_1) \cos \Delta\phi_R \cos 2\omega_2 t, \quad (6)$$

respectively. Here  $J_n$  is the first kind of Bessel function of order  $n$ .

The  $\omega_1$  frequency component is still proportional to  $\sin \Delta\phi_R$  and is used for the rotation rate measurement. The quasi-dc component,  $P_f^{dc}$ , generally depends on the rotation rate. However, it can be made rotation-independent if  $\phi_1$  is kept at  $J_0(\phi_1) = 0$ . The second harmonic component of frequency  $\omega_2$ ,  $P_f^{2\omega_2}$ , is used as an error signal to keep  $\phi_1$  at  $J_0(\phi_1) = 0$ , since  $P_f^{2\omega_2}$  is proportional to  $J_0(\phi_1)$ . The voltage amplitude of the driving signal (frequency  $\omega_1$ ) applied to the phase modulator can be controlled in such a way that the error signal vanishes. Automatic setting and tracking of the optimum driving voltage amplitude can simply be im-

plemented through an appropriate electrical feedback circuit.

In the Appendix, we calculated the feedback signal distortion due to the residual ac feedback effect. We extended the previous treatment [12] to the case where the ac feedback response is not ignorable. An expression for the modulation frequency component of the feedback signal was derived as a function of the Sagnac phase shift. From the derived expression, we can estimate the magnitude of the residual gyro output distortion due to ac feedback when the phase difference modulation amplitude is kept exactly at  $J_0(\phi_1) = 0$  by the proposed tracking scheme.

## III. EXPERIMENTAL

Fig. 1 shows the experimental configuration. A depolarized gyroscope was used as a testbed gyroscope. It consisted of a 3.4 km-long single-mode fiber loop and a 7 m-long erbium-doped fiber (EDF) with 800 ppm concentration to form a FAS configuration. A 1.48  $\mu\text{m}$  wavelength laser diode was used as a pump source and the backward amplified spontaneous emission (ASE) from the EDF was the light source of the gyroscope. The Sagnac loop included a PZT phase modulator and a depolarizer composed of two pieces of highly birefringent fibers that were spliced in such a way that the angle between their birefringence axes was  $45^\circ$ . Two harmonic signal outputs of frequency 30.6 kHz ( $\omega_1/2\pi$ ) and 103 kHz ( $\omega_2/2\pi$ ) from the separate function generators 1 and 2, respectively, were combined and were used to drive the PZT phase modulator. The main modulation frequency of 30.6 kHz was the proper frequency of the gyroscope. The amplitude of the auxiliary modulation at 103 kHz was chosen as small as possible. The returning gyro signal was amplified while transmitting through the EDF and was detected using an InGaAs detector. The detected signal at the main modulation frequency  $\omega_1$  was measured with the lock-in amplifier 1 that gave rotation rate information. The frequency  $2\omega_2$  component of the detected signal, which is the error signal, was

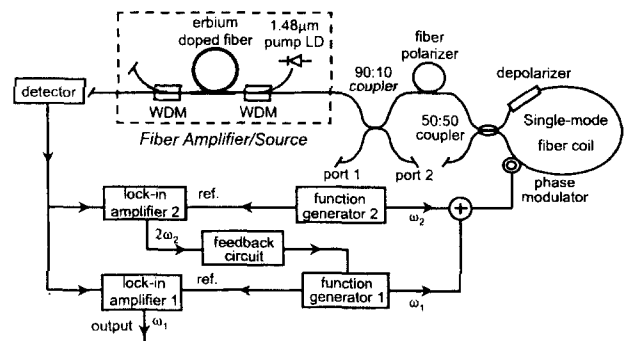


FIG. 1. Experimental Setup

measured with the lock-in amplifier 2 and was sent to a proportional-and-integral feedback circuit to control the voltage amplitude of the driving signal from the function generator 1. A 10 % tapping coupler, which was not required in this configuration, was placed between the FAS and the gyroscope to measure the power and the spectrum of the source output (backward ASE) from port 1, and those of the feedback signal from port 2. All the unused fiber ends were angle-polished to eliminate an additional feedback due to reflection.

The pump power of 15 mW was coupled into the EDF throughout the experiment. The feedback level (from source to source) was  $-10$  dB ( $\gamma=0.1$ ) in the absence of modulation. At the modulation amplitude of 2.4 rad, the average feedback level was  $-13$  dB. The average optical power as high as 2.6 mW was detected when the tracking system was operating. In the detected power, the background ASE not carrying the rotation information was included, but it was below 7 % of the peak power.

#### IV. RESULTS AND DISCUSSIONS

The source output power variation was measured while varying the rotation rate rapidly and continuously within the range of  $\pm 7$  deg/sec, and the result is shown in Fig. 2. Initially  $\phi_1$  was set at 1.8 rad before the tracking started, then the source power varied by up to 6.4 % depending on the rotation rate. After the tracking started, however, the source power variation was much reduced to 0.4 % for the rotation rate variation. In addition, the source output spectrum remained almost unchanged for varying rotation rate.

Fig. 3 shows the variation of the error signal output (upper trace) with the ambient temperature variation (lower trace). Initially the driving amplitude voltage from function generator 1 was manually set so that it yielded zero error signal. As time passed, the error

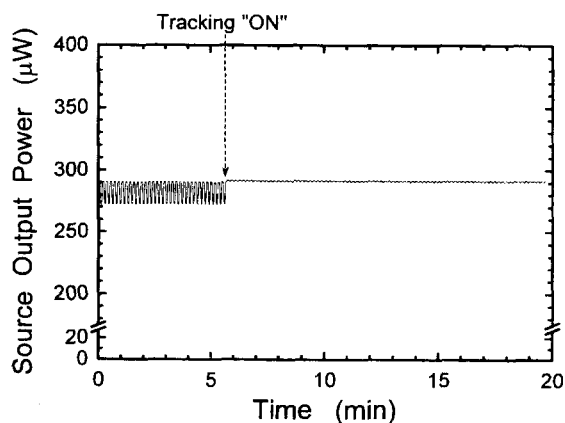


FIG. 2. Source power variation for continuously varying rotation rate within  $\pm 7$  deg/sec.

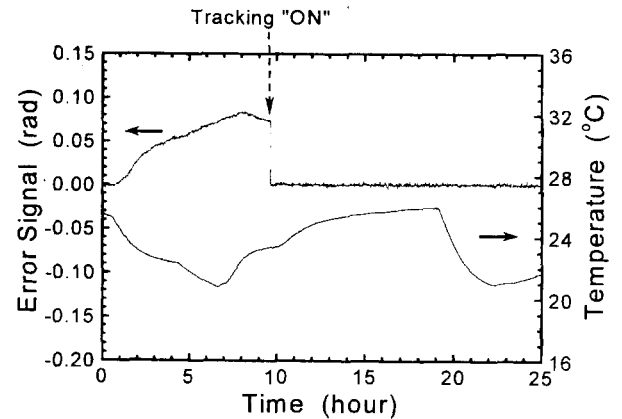


FIG. 3. Variation of the error signal output with temperature variation. (upper trace: error signal, lower trace: temperature)

signal drifted and deviated from zero, even though the driving voltage amplitude was kept constant. We can see from Fig. 3 that the error signal variation resembles the temperature variation, which means that the main cause of the drift was the change in temperature. This was also confirmed in an independent experiment where the amplitude of the modulated phase difference was measured as a function of the temperature (figure is not shown). Note that the error signal around zero value is approximately proportional to the deviation from the optimum amplitude of 2.405 rad. The scale of the error signal output in Fig. 3 was so calibrated that it could also be read approximately as the deviation from the optimum in rad. Hence, we can see that the phase difference modulation drifted about 0.077 rad for the temperature variation of  $5^{\circ}\text{C}$ , before the tracking started. When the tracking was started by enabling the feedback circuit, the error signal locked to zero for varying temperature. Then the amplitude of the phase difference modulation was maintained at the optimum of  $\phi_1 = 2.405$  rad, regardless of the temperature variation.

The detected signal of the amplified output at the main modulation frequency, which is the FAS gyro output, was measured as a function of the rotation rate. The deviation of its normalized results from an ideal sine curve is plotted in Fig. 4. In Fig. 4, two cases for the modulation amplitudes of 1.8 and 3.2 rad without tracking are also plotted together with the theoretical prediction. When the gyroscope was operated with automatic tracking, the measured maximum deviation from the ideal sine curve was about 0.1 %. ( More accurate deviation value was not available, since it was limited by our measurement resolution. ) By contrast, considerable deviations up to  $\pm 10.3$  % resulted showing  $\sin 2\Delta\phi_R$  dependence in the case of 1.8 rad without tracking.

Using the result derived in the Appendix, we es-

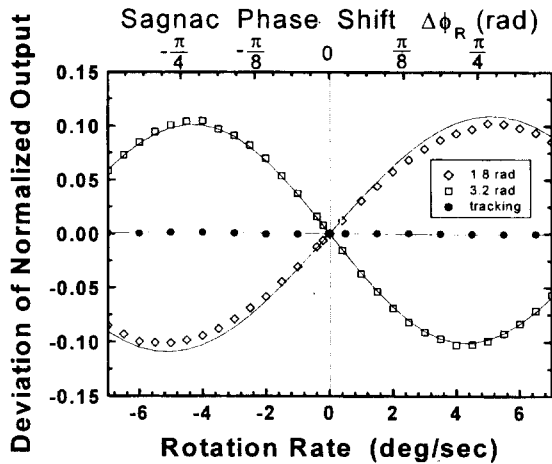


FIG. 4. Deviation of the normalized FAS gyro output from an ideal sine curve with the tracking. For comparison, deviations for the modulation amplitudes of 1.8 rad and 3.2 rad (without tracking) are plotted together with the theoretical prediction (solid line).

estimated the residual gyro output distortion when the modulation amplitude is maintained exactly at  $J_0(\phi_1) = 0$  by the automatic tracking. First, we obtained  $\alpha_0 = -6.5$  in an independent experiment by measuring the dependence of the source output power on the dc feedback power ( $\alpha_i$ 's are defined in the Appendix). From the known frequency dependence of ac feedback response that is normalized to dc response [14], the magnitude of  $\alpha_1$  was estimated about 20 dB below  $\alpha_0$  at the present modulation frequency (30.6 kHz). As derived in the Appendix, the residual distortion of the normalized feedback signal is  $-\frac{1}{2J_1(\phi_1)}\gamma \sum_{n=1}^{\infty} \alpha_n J_n(\phi_1) J'_n(\phi_1) \sin 2\Delta\phi_R$ , where  $J'_n$  is the derivative of  $J_n$ . Since the frequency dependence of ac feedback response falls off like a typical single-pole system (10 dB per decade) with a pole at around 300 Hz [14],  $\alpha$ 's can be approximated as  $\alpha_n \approx \alpha_1/n$  at the harmonics of the modulation frequency that is much higher than the pole frequency. The maximum residual deviation estimated in this way was  $3.8 \times 10^{-4}$  with  $\gamma = 0.1$ . In the mean time, during the optical amplification of the feedback signal through the EDF, the gyro signal will experience a rotation-rate dependent gain variation through a gain saturation, in almost the same way as the source power variation. Rigorous treatment of the distortion of the amplified output (FAS gyro output) becomes very complicated when the ac feedback effects are taken into consideration. However, by the simplified treatment including only the dominant dc feedback effect, it can be shown that the distortion in the FAS gyro output approximately doubles that in the feedback signal. This has experimentally been confirmed in the previous publication [12]. Hence the net maximum deviation of the FAS gyro output was estimated to be about  $8 \times 10^{-4}$ ,

which is in reasonable agreement with the experiment.

We compared the feedback signal distortion between the proposed tracking FAS scheme and the conventional gyroscope operating at 1.8 rad. The calculated distortion in the tracking FAS scheme was found to be same as in the conventional gyroscope that uses an isolator with the isolation of 21 dB and 31 dB, for modulation at 30 kHz and 300 kHz, respectively. Considering that the typical single-stage isolator provides an isolation around 30~35 dB over the wide wavelength range of interest, the proposed tracking FAS scheme may become comparable with the conventional configuration, provided that the operating frequency is properly chosen. In any case, owing to the definite advantage of high detected power, the proposed FAS scheme may find applications where a very accurate scale factor is not required.

The proposed scheme is directly applicable within the range where  $\cos \Delta\phi_R$  is sizable, since the error signal is proportional to  $J_0(\phi_1) \cos \Delta\phi_R$ . When the rotation rate exceeds the range such that  $|\cos \Delta\phi_R|$  becomes too small, the measurement range could easily be extended by instantly switching the error signal from the second harmonic ( $2\omega_2$ ) component to the first harmonic ( $\omega_2$ ) component, which is proportional to  $J_0(\phi_1) \sin \Delta\phi_R$  as seen in Eq. (5).

## V. CONCLUSION

We have proposed and demonstrated a scheme for suppression of output distortion in an open-loop gyroscope employing an EDFAS. By active tracking of optimum phase difference modulation amplitude using the two-frequency modulation scheme, we have obtained stable gyro output with suppressed distortion. The dc feedback effect on the gyro output distortion has been eliminated with this scheme. The residual distortion could be attributed to the residual ac feedback effect. The measured maximum deviation agrees reasonably with the one estimated from the theory. The residual distortion could be reduced if the higher modulation frequency can be used, since the residual distortion due to ac feedback is expected to vary approximately with the inverse of the modulation frequency. It is believed that the FAS gyroscope with the proposed scheme may be practical, provided that the operating parameters are optimized further.

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## APPENDIX A: CALCULATION OF FEEDBACK SIGNAL DISTORTION DUE TO FEEDBACK EFFECT

For simplicity, we suppose only the case of single frequency modulation such that  $\phi(t) = \phi_m \sin \omega_m t$ , ignoring the effect of small auxiliary modulation. We start with the constant source power. Then the dc (or quasi-dc) component of the feedback power can be expressed as

$$P_f^{dc} = P_{fC}^{dc} + P_{fR}^{dc} \quad (A1)$$

where  $P_{fC}^{dc} = \frac{1}{2} \gamma P_s$  is the constant feedback term and  $P_{fR}^{dc} = \frac{1}{2} \gamma P_s J_0(\phi_m) \cos \Delta \phi_R$  is the rotation dependent portion of the dc feedback. Henceforth, we take  $P_s$  as the source power under the constant feedback  $P_{fC}^{dc}$ .

As the next step, we include the effect of the rotation dependent portion of the dc feedback along with the ac feedback which is also dependent on the rotation rate. Then the source power becomes not only dependent on the rotation rate, but also modulated owing to the ac feedback. In this case, the approximation  $P_s(t - \tau) \approx P_s(t) \approx P_s$  is no longer valid and the original form of Eq. (2) must be used for the calculation of the feedback signal. Inclusion of these additional feedback effects results in a new source power  $P'_s(t)$  as

$$P'_s(t) = P_s + \Delta P_s(t) \quad (A2)$$

where  $\Delta P_s(t)$  is the change in source power due to the ac feedback as well as the rotation dependent portion of the dc feedback. We assume further that the effects of feedback on the source power are additive in the small range of source power variation. Since the feedback signal is composed of harmonics of the modulation frequency, we can approximate  $\Delta P_s(t)$  as

$$\Delta P_s(t) = \alpha_0 P_{fR}^{dc} + \sum_{n=1}^{\infty} \alpha_n P_f^{n\omega_m}(t) \quad (A3)$$

where  $\alpha_0$  and  $\alpha_n$ 's are proportionality constants, and  $P_f^{n\omega_m}(t)$  is the n-th harmonic frequency component of the feedback signal given by

$$P_f^{n\omega_m}(t) = \begin{cases} -\gamma P_s J_n(\phi_m) \sin \Delta \phi_R \sin n\omega_m t, & \text{for odd } n \\ \gamma P_s J_n(\phi_m) \cos \Delta \phi_R \cos n\omega_m t, & \text{for even } n \end{cases} \quad (A4)$$

The new feedback signal power  $P'_f(t)$ , influenced by the change in source power, becomes

$$P'_f(t) = \frac{1}{2} \gamma P'_s(t - \tau) [1 + \cos(\Delta \phi_R + \phi_m \sin \omega_m t)]. \quad (A5)$$

The feedback signal power and the source power under the feedback can be obtained by the iteration of these steps. By substituting Eqs. (A2) and (A3) into Eq. (A5), the feedback signal power is calculated as the first approximation. After some calculation and by collecting  $\omega_m$  frequency terms, we can obtain the modulation frequency component of the feedback signal, which is proportional to the gyro output in case of the conventional (non-FAS) gyro configuration, as

$$P_f^{\omega_m}(t) = -\gamma P_s J_1 \left[ \left\{ \sin \omega_m t + \frac{1}{2} \alpha_1 \gamma \sin \omega_m(t - \tau) \right\} \sin \Delta \phi_R + \frac{1}{4} \gamma \alpha_0 J_0 \sin \omega_m t \sin 2\Delta \phi_R \right] \\ + \frac{1}{4} \gamma^2 P_s \sum_{n=1}^{\infty} (-1)^n \alpha_n J_n [J_{n-1} \sin \omega_m(t - n\tau) - J_{n+1} \sin \omega_m(t + n\tau)] \sin 2\Delta \phi_R, \quad (\text{A6})$$

where all the arguments of Bessel functions are  $\phi_m$ .

If we take the example of proper frequency modulation ( $\omega_m = \pi/\tau$ ) as in the present experiment,  $P_f^{\omega_m}$  becomes somewhat simplified as follows:

$$P_f^{\omega_m} = -\gamma P_s \sin \omega_m t \left[ \left( 1 - \frac{\gamma \alpha_1}{2} \right) J_1 \sin \Delta \phi_R + \frac{1}{4} \gamma \left\{ \alpha_0 J_0 J_1 + \sum_{n=1}^{\infty} \alpha_n J_n (J_{n-1} - J_{n+1}) \right\} \sin 2\Delta \phi_R \right]. \quad (\text{A7})$$

The first term in Eq. (A7) corresponds to the undistorted gyro response. The second term, which is proportional to  $\sin 2\Delta \phi_R$ , represents the deviation from an ideal gyro response. As a special case, when  $\alpha_n$ 's are all equal to  $\alpha_0$ , the deviation term in Eq. (A7) becomes zero. In this case, the source response to feedback is so fast and instantaneous that the change in source power can be approximated to be proportional to the entire feedback signal power. The result is that the feedback signal distortion can be removed by proper frequency modulation, which is consistent with the literature [15]. In the case of erbium-doped fiber sources, however, gain response to feedback is very slow and  $\alpha_n$ 's fall off as  $n$  increases.

Eq. (A7) may also be expressed in the following form that shows the effects of individual harmonic terms in Eq. (A3) by using the Bessel function relation  $J_{n-1} - J_{n+1} = 2J'_n$  where  $J'_n$  is the derivative of  $J_n$ .

$$P_f^{\omega_m} = -\gamma P_s \sin \omega_m t \left[ \left( 1 - \frac{\gamma \alpha_1}{2} \right) J_1 \sin \Delta \phi_R - \frac{1}{4} \gamma \left\{ \alpha_0 J_0 J'_0 - 2 \sum_{n=1}^{\infty} \alpha_n J_n J'_n \right\} \sin 2\Delta \phi_R \right]. \quad (\text{A8})$$

If  $J_0(\phi_m)$  is kept at zero, distortion due to dc feedback is completely suppressed. In this case, the normalized residual distortion due to ac feedback effect is approximately  $-\frac{\gamma}{2J_1} \sum_{n=1}^{\infty} \alpha_n J_n J'_n \sin 2\Delta \phi_R$ , since  $1 - \frac{1}{2} \gamma \alpha_1 \approx 1$  in most cases.