Recursive Optimal State and Input Observer for Discrete Time-Variant Systems

Youngjin Park and J. L. Stein

Abstract : One of the important challenges facing control engineers in developing automated machinery is to be able to monitor the machines using remote sensors. Observers are often used to reconstruct the machine variables of interest. However, conventional observers are unable to observe the machine variables when the machine models, upon which the observers are based, have inputs that cannot be measured. Since this is often the case, the authors previously developed a steady-state optimal state and input observer for time-invariant systems [1], this paper extends that work to time-variant systems. A recursive observer, similar to a Kalman-Bucy filter, is developed. This optimal observer minimizes the trace of the error variance for discrete, linear, time-variant, stochastic systems with unknown inputs.

keywards : state and input observer, Kalman-Bucy filter, discrete time varying system

I. Introduction

Stein and Park [2] observed that, in many model-based machine monitoring situations, insufficient knowledge of the machine or machine process exists and that models of these systems have unknown inputs or disturbances. Therefore, a classical state observer cannot be used to estimate the states and inputs when the unknown inputs cannot be measured. They proposed a simultaneous state and input observer (SSIO) based on minimizing the number of measurements (both state and input) required to observe all the states and inputs.

In a subsequent paper [3], they showed that by relaxing the requirement of using only a minimum of measurements, a closed-loop state and input observer (CSIO) could be developed. This solution allows a designer to tune the observer dynamics to meet some performance criterion. They also developed a steady-state optimal, state and input observer (OSIO) [1] for discrete, time-invariant systems that minimizes the effect of system and measurement noise on the estimates. This is particularly important for an input and state observer because the observer is acausal and measurement noise can strongly influence the input estimates, which are, in part, based on derivatives of the state measurements [2].

While the class of state and input observers developed by Stein and Park have many applications in machine monitoring and other areas where determining inputs or disturbances to a system is important, they only apply to systems that can be represented by linear, time-invariant models. The purpose of this paper is to develop a state and input observer for discrete, time-variant, stochastic systems with unknown inputs.

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Youngjin Park : Department of Mechanical Engineering at Korea Advanced Institute of Science and Technology

J. L. Stein : Department of Mechanical Engineering at University of Michigan

This observer is recursive in nature, as is a Kalman-Bucy filter [4] for time-variant systems. The observer gain is chosen to minimize the effect of system and measurement noise on the estimates of the state and input. This is achieved by minimizing the trace of the estimation error variance matrix at each time step. The observer structure is identical to the observer developed in OSIO [1] except that it is in recursive form.

II. Notation

Singular value decomposition and the generalized inverse (Moor-Penrose inverse) are briefly explained and the notation is given as follows. This notation is used without further explanation throughout the paper.

1. Singular value decomposition

Any $\mu\times\nu$ matrix $(\cdot)$, whose rank is α, can be decomposed as follows:

$$(\cdot)=U_{(\cdot)}\Sigma_{(\cdot)}V_{(\cdot)}t={}_1U_{(\cdot)}\sigma(\cdot)\,{}_1V^t_{(\cdot)}$$

where

$$V_{(\cdot)}=[{}_1V_{(\cdot)}\ {}_2U(\cdot)]$$
$$V_{(\cdot)}=[{}_1V_{(\cdot)}\ {}_2V_{(\cdot)}]$$
$$\Sigma_{(\cdot)}=\begin{pmatrix}\sigma_{(\cdot)} & 0\\ 0 & 0\end{pmatrix}$$

$U_{(\cdot)}$: left singular matrix of matrix $(\cdot)$, $U_{(\cdot)}\in R^{\mu\times\mu}$

$V_{(\cdot)}$: right singular matrix of matrix $(\cdot)$ $V_{(\cdot)}\in R^{\nu\times\nu}$

$\Sigma_{(\cdot)}$: singular matrix of matrix $(\cdot)$, $\Sigma_{(\cdot)}\in R^{\mu\times\nu}$

${}_1U_{(\cdot)}$: singular matrix of matrix $(\cdot)$, ${}_1U_{(\cdot)}\in R^{\mu\times\alpha}$

${}_2U_{(\cdot)}$: null space matrix of matrix $(\cdot)^t$, ${}_2U_{(\cdot)}\in R^{\mu\times(\mu-\alpha)}$

${}_1V_{(\cdot)}$: range space matrix of matrix $(\cdot)^t$, ${}_1V_{(\cdot)}\in R^{\nu\times\alpha}$

${}_2V_{(\cdot)}$: null space matrix of matrix $(\cdot)$, ${}_2V_{(\cdot)}\in R^{\nu\times(\nu-\alpha)}$

$\sigma_{(\cdot)}$: positive definite diagonal matrix, $\sigma_{(\cdot)}\in R^{\alpha\times\alpha}$ (1)

The left and right singular matrices are orthogonal matrices and are not necessarily unique. The nonzero diagonal elements of a singular matrix are called singular values.

2. Generalized inverse

A generalized inverse (Moore- Penrose inverse; [5]) can be defined using the singular value decomposition technique as follows:

$$(\cdot)^{+}=V_{(\cdot)}\Sigma_{(\cdot)}{}^{+}U_{(\cdot)}{}^{t}={}_{1}V_{(\cdot)}\sigma_{(\cdot)}{}^{-1}{}_{1}U_{(\cdot)}{}^{t} \quad (2)$$

where $\Sigma_{(\cdot)}{}^{+}=\begin{pmatrix}\sigma_{(\cdot)}{}^{-1} & 0\\ 0 & 0\end{pmatrix} \in R^{\nu\times\mu}$

$(\cdot)^{+}$: moore-Penrose inverse of matrix

$(\cdot), (\cdot)^{+} \in R^{\nu\times\mu}$

III. Analysis

1. System model

A class of linear time variant discrete systems can be represented by the following equations:

$$x_{k+1}=A_k x_k+B_k U_k+w_{0k} \quad (3)$$

$$y_{1k}=C_k x_k+w_{1k} \quad (4)$$

$$y_{2k}=D_k u_k+w_{2k} \quad (5)$$

where

x_k : system state at time step k, $x_k \in R^n$

u_k : system input at time step k, $u_k \in R^m$

y_{1k} : state measurements at time step k, $y_{1k} \in R^P$

y_{2k} : input measurements at time step k, $y_{2k} \in R^q$

w_{0k}: system noise at time step k, $w_{0k} \in R^n$

w_{1k}: state measurement noise at time step k, $w_{1k} \in R^p$

w_{2k}: input measurement noise at time step k, $w_{2k} \in R^q$

A_k : system matrix at time step k

B_k : system matrix at time step k

C_k : system matrix at time step k

D_k : system matrix at time step k

$w_k=[\,w_{0k}^t \ \ w_{1k}^t \ \ w_{2k}^t\,]^t$

The following assumptions have been made. First, the noise vector w_k is assumed to be a zero mean, white, uncorrelated random sequence whose variance is represented by R_w, as follows:

$$R_w=E[\,w_k\ w^t{}_k]=\begin{bmatrix} R_{w_0} & 0 & 0\\ 0 & R_{w_1} & 0\\ 0 & 0 & R_{w_2}\end{bmatrix} \quad (6)$$

where

$R_{w0}=E[\,w_{0k}w_{0k}^t]$: positive semi-definite matrix

$R_{w1}=E[\,w_{1k}w_{1k}^t]$: positive definite matrix

$R_{w2}=E[\,w_{2k}w_{2k}^t]$: positive semi-definite matrix

Second, the A_k, B_k, C_k, D_k, and R_w matrices are assumed to be known. The B_k matrix is of full column rank and the C_k and D_k matrices are of full row rank for all time step k. If these matrices are rank deficient, then the order of input or output must be reduced to prevent rank deficiency. The null space of the D_k matrix is fixed for all time step k.

2. Existence condition

One of the sufficient and necessary conditions for the existence of the discrete observer is as follows:

$$N_k L=[\,C_k B_{k-1\,2}V_D]^{+}C_k B_{k-1\ 2}V_D=I_{m-q} \quad (7)$$

where I_{m-q} : $(m-q)\times(m-q)$ unit matrix

This condition is the same as the one of the necessary and sufficient conditions of CSIO [3] and OSIO [1] and can be derived in a similar manner. This condition dictates the necessary measurements to estimate states and inputs of the systems. This can be satisfied if and only if the matrix $C_k B_{k-1\,2}V_D$ is of full rank for all time step k. Note that the stability condition (detectability condition in CSIO and OSIO) is not presented. However, the sufficient condition for the asymptotic stability similar to that of the Kalman-Bucy filter [7~9] can be derived. This condition is not further investigated in the paper because of its little practical importance. The stability of the stochastic observers such as the Kalman filter is usually examined during the recursive calculation by monitoring the error covariance.

3. System equation decoupling

The system equation (3) can be decoupled into two equations one of which does not depend on the input u_k. This can be done by premultiplying (3) by an invertible matrix E_k :

$$E_k\,x_{k+1}=E_k\,A_k\,x_k+B'u_k+E_k\,w_{0k} \quad (8)$$

where $E_k=\begin{bmatrix} B_k^{+}\\ U\end{bmatrix}$

and by applying the following invertible transformations

$$E_{k-1}x_k=[\,{}_2V_{Nk}\ L]\begin{pmatrix}\zeta_k\\ y_k^*\end{pmatrix}\Leftrightarrow\begin{pmatrix}\zeta_k\\ y_k^*\end{pmatrix}=\begin{bmatrix}{}_2V_{Nk}^t(I-LN_k)\\ N_k\end{bmatrix}E_{k-1}x_k \quad (9)$$

$$u_k=[\,D_k^{+}\ \ {}_2V_D]\begin{pmatrix}y_{2k}-w_{2k}\\ u_k^*\end{pmatrix}\Leftrightarrow\begin{pmatrix}y_{2k}-w_{2k}\\ u_k^*\end{pmatrix}=\begin{bmatrix}D_k\\ V_D^t\end{bmatrix}u_k \quad (10)$$

system equation can be represented by (11) and (12)

$$y_{k+1}^*=N_{k+1}E_kA_kE_{k-1}^{-1}({}_2V_{Nk}\zeta_k+Ly_k^*)+N_{k+1}B'u_k+N_{k+1}E_kW_{0k} \quad (11)$$

$$\begin{aligned}\zeta_{k+1}=\ & {}_2V_{Nk+1}^1[I-LN_{k+1}]E_kA_kE_{k-1}^{-1}{}_2V_{Nk}\zeta_k\\ & +{}_2V_{Nk+1}^t[I-LN_{k+1}]E_KA_kE_{k-1}^{-1}LM_ky_{1k}\\ & +{}_2V_{Nk+1}^t[I-LN_{k+1}]B'D_k^{+}y_{2k}\\ & +{}_2V_{Nk+1}[I-LN_{k+1}]\\ & \quad [E_k-E_kA_kE_{k-1}^{-1}LM_k\ \ -B'\ \ D_k^{+}]w_k\end{aligned} \quad (12)$$

Note that (12) does not include the input term u_k. A detailed derivation can be found in Appendix I. (11) and (12) are an alternate representation of (3) and are required for the observer derivation.

4. Observer derivation

It can be easily shown that state and input of the system can be represented by sub-state vector ζ_k and measurement vectors and noise vector as follows (detailed in Appendix II):

$$\begin{pmatrix} x_k \\ u_k \end{pmatrix} = G_{1k}y_{1k} + G_{2k}y_{2k} + G_{3k}y_{3k} + P_k\zeta_k + Q_k w_k + R_k w_{k+1} \quad (13)$$

where

$$G_{1k} = \begin{bmatrix} E_{k-1}^{-1}LM_k \\ -{}_2V_D N_{k+1} E_k A_k E_{k-1}^{-1} LM_k \end{bmatrix}$$

$$G_{2k} = \begin{bmatrix} 0 \\ (I - {}_2V_D N_{k+1} B')D_k^+ \end{bmatrix}$$

$$G_{3k} = \begin{bmatrix} 0 \\ {}_2V_D M_{k+1} \end{bmatrix}$$

$$P_k = \begin{bmatrix} E_{k-1}^{-1}\,{}_2V_{Nk} \\ -{}_2V_D N_{k+1} E_k A_k E_{k-1}^{-1}\,{}_2V_{Nk} \end{bmatrix}$$

$$Q_k = \begin{bmatrix} 0 & -E_{k-1}^{-1}LM_k & 0 \\ -{}_2V_D N_{k+1} E_k & {}_2V_D N_{k+1} E_k A_k E_{k-1}^{-1} LM_k & -(I - {}_2V_D N_{k+1} B')D_k^+ \end{bmatrix}$$

$$R_k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -{}_2V_D M_{k+1} & 0 \end{bmatrix}$$

By defining a one-step-delayed (*a posteriori*) estimation of state and input at time step k, as follows

$$\begin{pmatrix} \hat{x}_k \\ \hat{u}_k \end{pmatrix} = G_{1k}y_{1k} + G_{2k}y_{2k} + G_{3k}y_{1k+1} + P_k\hat{\zeta}_k \quad (14)$$

an estimation error ε_k and its covariance Π_k can be represented as

$$\varepsilon_k = \begin{pmatrix} x_k \\ u_k \end{pmatrix} - \begin{pmatrix} \hat{x}_k \\ \hat{u}_k \end{pmatrix} = P_k(\zeta_k - \hat{\zeta}_k) + Q_k w_k + R_k w_{k+1} \quad (15)$$

$$\Pi_k = E(\varepsilon_k \varepsilon_k^t)$$

where sub-state (projected state) estimation error e_k and its covariance Ψ_k are defined as

$$e_k = \zeta_k - \hat{\zeta}_k \quad (16)$$

$$\Psi_k = E(e_k e_k^t).$$

The authors [1] have shown that state and input estimation error covariance, Π_k, is minimum when sub-state estimation error covariance, Ψ_k is minimum for non-recursive (time invariant) observer. However, that is not the case in this paper, and a recursive observer will be designed to minimize the state and input estimation error covariance at any given time step k.

Assume that *a priori* sub-state estimate, $\hat{\zeta}_k^-$ and the associated error covariance matrix, Ψ_k^- are known, where Ψ_k^- is defined as

$$\Psi_k^- = E(e_k^- e_k^{-t}) \quad (17)$$

where $e_k^- = \zeta_k - \hat{\zeta}_k^-$

Define *a posteriori* sub-state estimate $\hat{\zeta}_k$ and *a priori* state estimate $\hat{x}_k^-$ from (9) as follows:

$$\hat{\zeta}_k = \hat{\zeta}_k^- + K_k\,{}_2V_{Mk}^t(y_{1k} - C_k\hat{x}_k^-) \quad (18)$$

$$\hat{x}_k = E_{k-1}^{-1}({}_2V_{Nk}\hat{\zeta}_k^- + LM_k y_{1k}) \quad (19)$$

These two equations can be simplified as:

$$\hat{\zeta}_k = (I - K_k C'_k)\hat{\zeta}_k^- + K_k\,{}_2V_{Mk}^t y_{1k} \quad (20)$$

where $C'_k = {}_2V_{Mk}^t C_k E_{k-1}^{-1}\,{}_2V_{Nk}$.

The sub-state estimation error e_k can then be represented by the following equation (see Appendix III for detailed derivation)

$$e_k = [I - K_k C'_k]e_k^- - K_{k_2} V_{Mk}^t w_{1k}$$

$$= [I - K_k C'_k]e_k^- - K_k F_k w_k \quad (21)$$

where $F_k = [0 \;\; {}_2V_{Mk}^t \;\; 0]$

and the estimation error covariance, Ψ_k is given by

$$\Psi_k = E(e_k e_k^t) = [I - K_k C'_k]\Psi_k^-[I - K_k C'_k]^t + K_k F_k R_w F_k^t K_k^t$$

$$= \Psi_k^- - K_k C'_k \Psi_k^- - \Psi_k^- C'^t_k K_k^t + K_k[C'_k \Psi_k^- C'^t_k + F_k R_w F_k^t]K_k^t \quad (22)$$

Note that state measurement noise w_k and *a priori* sub-state estimation error e_k^- are assumed to be uncorrelated. This assumption will be validated later.

From (15), (16) and (22), the state and input estimation error covariance can be represented as follows (see Appendix IV for detailed derivation:)

$$\Pi_k = P_k \Psi_k^{-1} P_k^t + Q_k R_w Q_k^t + R_k R_w R_k^t$$

$$+ [P_k K_k S_k - \Sigma_k][P_k K_k S_k - \Sigma_k]^t - \Sigma_k \Sigma_k^t \quad (23)$$

where

$$S_k S_k^t = C'_k \Psi_k^- C'^t_k + F_k R_w F_k^t = C'_k \Psi_k^- C'^t_k + {}_2V_{Mk}^t R_{w1\,2} V_{Mk}$$

$$\Sigma_k = [P_k \Psi_k^- C'^t_k + Q_k R_w F_k^t][S_k^T]^{-1}$$

Note that $S_k S_k^t$ term is always positive definite because the matrix ${}_2V_{Mk}$ is of full column rank and state measurement noise covariance R_{w1} is positive definite by definition. Hence, the given factored form is possible. Also note that all the terms in (23) are positive semi definite and the first three and the last terms do not involve gain K_k. To minimize the trace of matrix Π_k, optimal gain matrix is chosen to make the forth term of (23) minimum as follows:

$$K_k = P_k^+ \Sigma_k [S_k^t]^{-1} = P_k^+ \Sigma_k [S_k^t]^{-1}$$

$$= P_k^+ [P_k \Psi_k^- C'^t_k - Q_k R_w F_k^t][S_k S_k^t]^{-1} \quad (24)$$

$$= P_k^+ [P_k \Psi_k^- C'^t_k + Q_k R_w F_k^t][C'_k \Psi_k^- C'^t_k + F_k R_w F_k^t]^{-1}$$

By noting the fact that P_k is of full column rank, the optimal gain can be simplified as follows:

$$K_k(\Psi_k^- C'^t_k + T_k Q_k R_w F_k^t)[C'_k \Psi_k^- C'^t_k + F_k R_w F_k^t]^{-1} \quad (25)$$

where $T_k = [P_k^t P_k]^{-1} P_k^t$

This gain minimizes the mean square error of the state and input estimation. Note that the matrix S_k does not need to be actually calculated to obtain optimal gain.

The error covariance matrix associated with the optimal gain can be obtained by substituting (25) into (22):

$$\Psi_k = \Psi_k^- - (\Psi_k^- C'^t_k + T_k Q_k R_w F_k^t)[C'_k \Psi_k^- C'^t_k + F_k R_w F_k^t]^{-1} C'_k \Psi_k^-$$
$$- \Psi_k^- C'^t_k [C'_k \Psi_k^- C'^t_k + F_k R_w F_k^t]^{-1} (\Psi_k^- C'^t_k + T_k Q_k R_w F_k^t)^t$$
$$+ (\Psi_k^- C'^t_k + T_k Q_k R_w F_k^t)[C'_k \Psi_k^- C'^t_k + F_k R_w F_k^t]^{-1}$$
$$(\Psi_k^- C'^t_k + T_k Q_k R_w F_k^t)^t = [I - K_k C'_k] \Psi_k^- + T_k Q_k R_w F_k^t K_k^t \quad (26)$$

The one-step ahead (*a priori*) sub-state estimate $\hat{\zeta}_{k+1}^-$ is provided based on the *a posteriori* sub-state estimate $\hat{\zeta}_k$, the decoupled system equation (12) and by ignoring the contribution of the noise term.

$$\hat{\zeta}_{k+1}^- = {}_2V_{Nk+1}^t [I - LN_{k+1}] E_k A_k E_{k-1}^{-1} {}_2V_{Nk} \hat{\zeta}_k$$
$$+ {}_2V_{Nk+1}^t [I - LN_{K+1}] E_k A_k E_{k-1}^{-1} L M_k y_{1k}$$
$$+ {}_2V_{Nk+1}^t [I - LN_{k+1}] B' D_k^+ y_{2k} \quad (27)$$

The error covariance matrix associated with $\hat{\zeta}_{k+1}^-$ is obtained by first forming the expression for the *a priori* error:

$$e_{k+1}^- = \zeta_{k+1} - \hat{\zeta}_{k+1}^- = {}_2V_{Nk+1}^t [I - LN_{k+1}] E_k A_k E_{k-1}^{-1} {}_2V_{Nk} (\zeta_k - \hat{\zeta}_k)$$
$$+ {}_2V_{Nk+1}^t [I - LN_{k+1}][E_k - E_k A_k E_{k-1}^{-1} L M_k - B' D_k^+] w_k$$
$$= A'_k e_k + H_k w_k \quad (28)$$

where $A'_k = {}_2V_{Nk+1}^t [I - LN_{k+1}] E_k A_k E_{k-1}^{-1} {}_2V_{Nk}$

$H_k = {}_2V_{Nk+1}^t [I - LN_{k+1}][E_k - E_k A_k E_{k-1}^{-1} L M_k - B' D_k^+]$

Ψ_{k+1}^- can then be expressed as follows by using (21) and the assumption that w_k and e_k^- are uncorrelated:

$$\Psi_{k+1}^- = E[e_{k+1}^- \; e_{k+1}^{-t}]$$
$$= A'_k \Psi_k A'^t_k + H_k R_w H_k^t + A'_k E(e_k w_k^t) H_k^t + H_k E(w_k e_k^t) A'^t_k \quad (29)$$
$$= A'_k \Psi_k A'^t_k + H_k R_w H_j^t - A'_k K_k F_k R_w H_k^t - H_k R_w F_k^t K_k^t A'^t_k$$

From (21) and (28), it is obvious that state measurement noise w_{k+1} and *a priori* sub-state estimation error e_{k+1}^- are uncorrelated. Hence, the assumption that w_k and e_k^- are uncorrelated is valid if the initial estimation error e_0^- is chosen to be uncorrelated to the initial noise w_0.

By using (25), (20), (26), (27), and (29) recursively and in sequence, sub-state estimate $\hat{\zeta}_k$ can be obtained with initial guess of sub-state estimate and the associated error covariance. The estimates of the states and inputs can be obtained from (14) after the sub-state estimates are given.

IV. Discussion

The recursive state and input observer (RSIO) developed in this paper is a minimum error covariance linear estimator whose feedback structure is a weighted sum of the state measurements. Thus the optimal gain does not account for noise amplified by differentiation. This is similar to the time-invariant case [1]. This can lead to input estimates with an insufficient signal-to-noise (S/N) ratio. This situation can be improved by using a low-pass filter or a band-limited differentiator, as demonstrated by Stein and Park [2].

Note that the state estimates for time step k can be determined just after the measurements of time step k are obtained but the input estimates for time step k can be calculated only after a one step time delay. This phenomenon occurs because RSIO is acausal when derived from a causal system.

The flexibility that RSIO offers over OSIO is significant in that a much larger class of monitoring problems can be addressed. For example, many manufacturing machines have parameters which vary in a known way with time. One example of this is a spindle system on a CNC lathe. The workpiece inertia can be significant compared to the rest of the spindle systems inertia and would, therefore, be an important parameter to include in a spindle model [9]. Since, over the entire cutting process a significant amount of material may be removed, a significant change in the system inertia would occur. Since the cutting paths are well known, this change in the workpiece inertia can be calculated in advance. Thus RSIO could be used to monitor the spindle drive system using remote sensing. For a specific example of using input and state estimation on a spindle system see Stein and Park [9].

One of the assumptions used in deriving the recursive observer is that the input measurement matrix D_k is fixed (i.e., its null space is fixed with time). This is a reasonable assumption since the input measurements would typically be just a subset of the model inputs (i.e. the D_k matrix would have 1's or 0's on the diagonal). If the D_k matrix is assigned arbitrarily (i.e., its null space is not fixed with time) the following additional condition is necessary for the existence of an unbiased observer:

$${}_2V_{Nk+1}^t [I - L_{k+1} N_{k+1}] L_k = 0 \quad \text{for} \quad k = 1, 2, 3, \cdots \quad (30)$$

where $L_k = B' {}_2V_{Dk}$

$N_k = (C_k B_{k-1} {}_2V_{Dk})^+ C_k E_{k-1}^{-1}$

Note the definitions of the observer parameters need to be redefined to accommodate the time-varying characteristics of ${}_2V_{Dk}$. This general time varying case is not considered in this paper because it is not a very realistic case. (30) would have to be checked for every time step k to determine if the estimates may be biased. There would be no other obvious indication that this is true. However, when the D_k matrix is fixed, the existence condition (7) need not be evaluated for every time step, because if it is not satisfied, an ill condition is created, and the recursive algorithm will blow-up.

As a final note about computation time, the recursive algorithm requires (25), (20), (26), (27), (29), and (14) to be processed in sequence. Only the evaluation of (25) poses any computation complexity (a matrix inversion).

Thus the computational load for this state and input observer is similar to the conventional Kalman-Bucy observer.

V. Conclusions

An optimal state and input observer has been developed to observe the states and inputs of discrete, linear, time-variant systems with unknown inputs. This technique guarantees minimum error variance among linear observers for a system subject to system and measurement noise. The observer increases the flexibility of applying input and state observers to model-based machine monitoring problems.

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Appendix I.

Derivation of (11) and (12)

To obtain (11), pre-multiply both side of (8) by N_{k+1} and combine with (9):

$$N_{k+1}E_k x_{k+1} = y^*_{k+1}$$
$$= N_{k+1}E_k A_k E^{-1}_{k-1}({}_2V_{Nk}\zeta_k + Ly^*_k) + N_{k+1}B' u_k + N_{k+1}E_k w_{0k} \quad (11)$$

To obtain (12), pre-multiply both side of (8) by ${}_2V^t_{Nk+1}[I-LN_{k+1}]$ then combine with (9), (10), and (4) :

$$\begin{aligned}
\zeta_{k+1} &= {}_2V^t_{Nk+1}[I-LN_{k+1}]E_k x_{k+1} \\
&= {}_2V_{Nk+1}[I-LN_{k+1}]E_k A_k E^{-1}_{k-1}({}_2V_{nk}\zeta_k + Ly^*_k) \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}B'\{D^+_k(y_{2k}-w_{2k}) + {}_2V_D u^*_k\} \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]E_k w_{0k} \\
&= {}_2V^t_{Nk+1}[I-LN_{k+1}]E_k A_k E^{-1}_{k-1}\,{}_2V_{Nk}\zeta_k \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]E_K A_k E^{-1}_{k-1}LM_k(y_{1k}-w_{1k}) \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]B'D^+_k(y_{2k}-w_{2k}) \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]B'\,{}_2V_D u^*_k \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]E_k w_{0k} \\
&= {}_2V^t_{Nk+1}[I-LN_{k+1}]E_k A_k E^{-1}_{k-1}\,{}_2V_{Nk}\zeta_k \qquad (12) \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]E_k A_k E^{-1}_{k-1}LM_k y_{1k} \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]B'D^+_k y_{2k} \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]Lu^*_k \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]E_k w_{0k} - {}_2V^t_{Nk+1}[I-LN_{k+1}] \\
&\qquad E_k A_k E^{-1}_{k-1}LM_k w_{1k} - {}_2V^t_{Nk+1}[I-LN_{k+1}]B'D^+_k w_{2k} \\
&= {}_2V^t_{Nk+1}[I-LN_{k+1}]E_k A_k E^{-1}_{k-1}\,{}_2V_{Nk}\zeta_k \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]E_k A_k E^{-1}_{k-1}LM_k y_{1k} \\
&\quad + {}_2V^t_{Nk+1}[I-LN_{k+1}]B'D^+_k y_{2k} + {}_2V^t_{Nk+1}[I-LN_{k+1}] \\
&\qquad [E_k - E_k A_k E^{-1}_{k-1}LM_k \ - B'D^+_k]w_k
\end{aligned}$$

Appendix II.

Derivation of (13)

From (4), (9) and the definition of N_k, the following equation is derived:

$$\begin{aligned}
&x_k - E^{-1}_{k-1}LM_k y_{1k} \\
&\quad = x_k - E^{-1}_{k-1}LM_k(C_k x_k + w_{1k}) \\
&\quad = E^{-1}_{k-1}[I - LM_k C_k E^{-1}_{k-1}]E_{k-1}x_k - E^{-1}_{k-1}LM_k w_{1k} \qquad (a2.1) \\
&\quad = E^{-1}_{k-1}[I-LN_k]({}_2V_{Nk}\zeta_k + Ly^*_k) - E^{-1}_{k-1}LM_k w_{1k} \\
&\quad = E^{-1}_{k-1}[I-LN_k]\,{}_2V_{Nk}\zeta_k - E^{-1}_{k-1}LM_k w_{1k}
\end{aligned}$$

From (4), (9), (5), (11), and (7), the following equation is derived:

$$\begin{aligned}
&u_k + {}_2V_D N_{k+1}E_k A_k E^{-1}_{k-1}\,{}_2V_{Nk}y_{1k} - [I - {}_2V_D N_{k+1}B']D^+_k y_{2k} \\
&\quad - {}_2V_D M_{k+1}y_{1k+1} = u_k + {}_2V_D N_{k+1}E_k A_k E^{-1}_{k-1}LM_k y_{1k} \\
&\quad - [I - {}_2V_D N_{k+1}B']D^+_k y_{2k} - {}_2V_D(y^*_{k+1} + M_{k+1}w_{1k+1}) \\
&= u_k + {}_2V_D N_{k+1}E_k A_k E^{-1}_{k-1}LM_k(C_k x_k + w_{1k}) \\
&\quad - [I - {}_2V_D N_{k+1}B']D^+_k(D_k u_k + w_{2k}) \\
&\quad - {}_2V_D\{N_{k+1}E_k A_k E^{-1}_{k-1}({}_2V_{Nk}\zeta_k + Ly^*_k) + N_{k+1}B'u_k \\
&\quad + N_{k+1}E_k w_{0k}\} - {}_2V_D M_{k+1}w_{1k+1} \\
&= [I - D^+_k D_k - {}_2V_D N_{k+1}B'[I - D^+_k D_k]]u_k \\
&\quad + {}_2V_D N_{k+1}E_k A_k E^{-1}_{k-1}L(M_k C_k x_k + M_k w_{1k} - y^*_k) \\
&\quad - [I - {}_2V_D N_{k+1}B']D^+_k w_{2k} - {}_2V_D N_{k+1}E_k A_k E^{-1}_{k-1}\,{}_2V_{Nk}\zeta_k \\
&\quad - {}_2V_D N_{k+1}E_k w_{0k} - {}_2V_D M_{k+1}w_{1k+1} \\
&= [{}_2V_{D_2}V_D t - {}_2V_D N_{k+1}B'\,{}_2V_{D2}V^t_D]u_k
\end{aligned}$$

$$
\begin{aligned}
&+{}_2V_DN_{k+1}E_kA_kE_{k-1}^{-1}LM_kw_{1k}\\
&-[I-{}_2V_DN_{k+1}B']D_k^{+}w_{2k}-{}_2V_DN_{k+1}E_kA_kE_{k-1}^{-1}{}_2V_{Nk}\zeta_k\\
&-{}_2V_DN_{k+1}E_kw_{0k}-{}_2V_DM_{k+1}w_{1k+1}\\
=&{}_2V_D[I-N_{k+1}L]{}_2V_{Dt}u_k+{}_2V_DN_{k+1}E_kA_kE_{k-1}^{-1}LM_kw_{1k}\\
&-[I-{}_2V_DN_{k+1}B']D_k^{+}w_{2k}-{}_2V_DN_{k+1}E_kA_kE_{k-1}^{-1}{}_2V_{Nk}\zeta_k\\
&-{}_2V_DN_{k+1}E_kw_{0k}-{}_2V_DM_{k+1}w_{1k+1}\\
=&-{}_2V_DN_{k+1}E_kA_kE_{k-1}^{-1}{}_2V_{Nk}\zeta_k-{}_2V_DNk_{+1}E_kw_{0k}\\
&+{}_2V_DN_{k+1}E_kA_kE_{k-1}^{-1}LM_kw_{1k}\\
&-[I-{}_2V_DN_{k+1}B']D_k^{+}w_{2k}-{}_2V_DM_{k+1}w_{1k+1} \qquad \text{(a2.2)}
\end{aligned}
$$

(13) is obtained by combining (a2.1) and (a2.2):

$$
\begin{pmatrix} x_k \\ u_k \end{pmatrix} = G_{1k}y_{1k}+G_{2k}y_{2k}+G_{3k}y_{1k+1}+P_k\zeta_k+Q_kw_k+R_kw_{k+1} \qquad (13)
$$

where

$$
G_{1k}=\begin{bmatrix} E_{k-1}^{-1}L\ M_k \\ -2V_D\,N_{k+1}E_kA_kE_{k-1}^{-1}L\ M_k \end{bmatrix}
$$

$$
G_{2k}=\begin{bmatrix} 0 \\ (I-{}_2V_DN_{k+1}B')D_k^{+} \end{bmatrix}
$$

$$
G_{3k}=\begin{bmatrix} 0 \\ {}_2V_D\ M_{k+1} \end{bmatrix}
$$

$$
P_k=\begin{bmatrix} E_{k-1}^{-1}{}_2V_{Nk} \\ -{}_2V_DN_{k+1}E_kA_kE_{k-1}^{-1}{}_2V_{Nk} \end{bmatrix}
$$

$$
Q_k=\begin{bmatrix} 0 & -E_{k-1}^{-1}LM_k & 0 \\ -{}_2V_DN_{k+1}E_k & {}_2V_DN_{k+1}E_kA_kE_{k-1}^{-1}LM_k & -(I-{}_2V_DN_{k+1}B')D_k^{+} \end{bmatrix}
$$

$$
R_k=\begin{bmatrix} 0 & 0 & 0 \\ 0 & -{}_2V_DM_{k+1} & 0 \end{bmatrix}
$$

Appendix III.

Derivation of (20)

From the definitions of error estimate e_k, *a priori* estimate e_k^- and (20), (4), and (9), the following equation for e_k can be derived:

$$
\begin{aligned}
e_k=&\zeta_k-\hat{\zeta}_k=\zeta_k-\hat{\zeta}_k^-+K_k[C'_k\hat{\zeta}_k^- -{}_2V_{Mk}^ty_{1k}]\\
=&\zeta_k-\hat{\zeta}_k^-+K_{k\,2}V_{Mk}^t[C_kE_{k-1}^{-1}{}_2V_{Nk}\hat{\zeta}_k^- -C_kx_k-w_{1k}]\\
=&\zeta_k\hat{\zeta}_k^-+K_{k\,2}V_{Mk}^t\{C_kE_{k-1}^{-1}({}_2V_{Nk}\hat{\zeta}_k^- -E_{k-1}x_k)-w_{1k}\}\\
=&\ \zeta_k-\hat{\zeta}_k^-+K_{k\,2}V_{Mk}^t\\
&\{C_kE_{k-1}^{-1}{}_2V_{Nk}(\hat{\zeta}_k^- -\zeta_k)-C_kE_{k-1}^{-1}Ly_k^*-w_{1k}\}\\
=&[I-K_{k\,2}V_{Mk}^tC_kE_{k-1}^{-1}{}_2V_{Nk}]e_k^- -K_{k\,2}V_{Mk}^tM_k^+y_k^*-K_{k_2}V_{Mk}^tw_{1k} \qquad \text{(a3.1)}
\end{aligned}
$$

The middle term of (a3.1) can be eliminated from the definition of the generalized inverse and orthogonal characteristics of a right singular matrix. Then, by using the definition of matrix C'_k, (20) is obtained:

$$
e_k=[I-K_kC'_k]e_k^- -K_{k_2}V_{Mk}^tw_{1k}. \qquad (20)
$$

Appendix IV.

Derivation of (23)

From (15) and (16), state and input estimation error covariance can be represented as:

$$
\begin{aligned}
\Pi_k=&\ E(\varepsilon_k\varepsilon_k^t)=E[\{P_ke_k+Q_kw_k+R_kw_{k+1}\}\\
&\{P_ke_k+Q_kw_k+R_kw_{k+1}\}^t]\\
=&P_kE(e_ke_k^t)P_k^t+P_kE(e_kw_k^t)Q_k^t+P_kE(e_kw_{k+1}^t)R_k^t\\
&+Q_kE(w_ke_k^t)P_k^t+Q_kE(w_kw_k^t)Q_k^t+Q_kE(w_kw_{k+1}^t)R_k^t\\
&+R_kE(w_{k+1}e_k^t)P_k^t+R_kE(w_{k+1}w_k^t)Q_k^t\\
&+R_kE(w_{k+1}w_{k+1}^t)R_k^t \qquad \text{(a4.1)}
\end{aligned}
$$

From (21) and (27), sub-state estimation error, e_k can be obtained as:

$$
\begin{aligned}
e_k=&[I-K_kC'_k]\{A'_{k-1}e_{k-1}+H_{k-1}w_{k-1}\}-K_kF_kw_k\\
=&\ \prod_{i=1}^{k}[I-K_{k-i+1}C_{k-i+1}]A'_{k-i}e_0\\
&+\sum_{i=0}^{k-2}\prod_{j=0}^{i}[I-K_{k-j}C'_{k-j}]A'_{k-j-1}H_{k-i-2}w_{k-i-2}\\
&+[I-K_kC'_k]H_{k-1}w_{k-1}-\sum_{i=0}^{k-2}\prod_{j=0}^{i}[I-K_{k-j}C_{k-j}]\\
&A'_{k-j-1}K_{k-i-1}F_{k-i-1}w_{k-i-1}-K_kF_kw_k \qquad \text{(a4.2)}
\end{aligned}
$$

Based on (a4.2) and (6), the following relations can be derived:

$$
\begin{aligned}
&E(e_kw_k^t)=-K_kF_kR_w\\
&E(w_ke_k^t)=-R_wF_k^tK_k^t\\
&E(w_{k+1}w_k^t)=E(w_kw_{k+1}^t)=0\\
&E(e_kw_{k+1}^t)=E(w_{k+1}e_k^t)=0 \qquad \text{(a4.3)}
\end{aligned}
$$

and (a4.1) can be simplified by substituting (a4.3), (6) and (16):

$$
\begin{aligned}
\Pi_k=&\ P_k\Psi_kP_k^t-P_kK_kF_kR_wQ_k^t-Q_kR_wF_k^tK_k^tP_k^t\\
&+Q_kR_wQ_k^t+R_kR_WR_k^t \qquad \text{(a4.4)}
\end{aligned}
$$

(a.4.4) can be represented as follows by substituting (22):

$$
\begin{aligned}
\Pi_k=&\ P_k\Psi_k^-P_k^t-P_kK_k(C'_k\Psi_k^-P_k^t+F_kR_wQ_k^t)\\
&-(P_k\Psi_k^-C'^t_k+Q_kR_wF_k^t)K_k^tP_k^t+P_kK_k[C'_k\Psi_k^-C'^t_k\\
&+F_kR_wF_k^t]K_k^tP_k^t+Q_kR_wQ_k^t+R_kR_WR_k^t
\end{aligned}
$$

Let's assume that Π_k can be represented as follows:

$$
\begin{aligned}
\Pi_k=&\ P_k\Psi_k^-P_k^t+Q_kR_WQ_k^t+R_kR_WR_k^t+[P_kK_kS_k-\Sigma_k]\\
&[P_kK_kS_k-\Sigma_k]^t-\Sigma_k\Sigma_k^t \qquad \text{(a4.5)}
\end{aligned}
$$

then, the following equation can be derived:

$$
\begin{aligned}
\Pi_k=&\ P_k\Psi_k^-P_k^t+Q_kR_WQ_k^t+R_kR_WR_k^t-P_kK_kS_k\Sigma_k^t\\
&-\Sigma_kS_k^tK_k^tP_k^t+P_kK_kS_kS_k^tK_k^tP_K^t \qquad \text{(a4.6)}
\end{aligned}
$$

The following relations are obtained by comparing (a4.4) and (a4.6):

$$
\begin{aligned}
&S_kS_k^t=C'_k\Psi_k^-C'^t_k+F_kR_wF_k^t\\
&\Sigma_kS_k^t=[P_k\Psi_k^-C'^t_k+Q_kR_wF_k^t] \qquad \text{(a4.7)}
\end{aligned}
$$

From (a4.5) and (a4.7), (23) can be obtained assuming that S_k^t is invertible.

Nomenclature

A_k : discrete system matrix at time step k

$A'_k={}_2V_{k+1}^t[I-LN_{k+1}]E_kA_k\,E_{k-1}^{-1}{}_2V_{Nk}$

B_k = discrete input matrix at time step k

$B' = \begin{bmatrix} I \\ o \end{bmatrix}$

C_k = state measurement matrix at time step k

$C'_k = {}_2V_{Mk}^t C_k E_{k-1}^{-1} {}_2V_{Nk}$

D_k : input measurement matrix at time step k, ${}_2V_{Dk} = {}_2V_D$

$E_k = \begin{bmatrix} B_k^+ \\ {}_2U_{Bk}^t \end{bmatrix}$

$e_k = \zeta_k - \hat{\zeta}_k$

$e_k^- = \zeta_k - \hat{\zeta}_k^{-1}$

$F_k = [0 \ {}_2V_{Mk}^t \ 0]$

$G_{1k} = \begin{bmatrix} E_{k-1}^{-1} L M_k \\ -{}_2V_D N_{k+1} E_k A_k E_{k-1}^{-1} L M_k \end{bmatrix}$

$G_{2k} = \begin{bmatrix} 0 \\ (I - {}_2V_D N_{k+1} B') D_k^+ \end{bmatrix}$

$G_{3k} = \begin{bmatrix} 0 \\ {}_2V_D M_{k+1} \end{bmatrix}$

$H_k = {}_2V_{Nk+1}^t [I_L N_{k+1}][E_k - E_k A_k E_{k-1}^{-1} L M_k \ \ -B' \ D_k^t]$

K_k : optiomal gain matrix at time step k

$L = B' {}_2V_D = \begin{bmatrix} {}_2V_D \\ 0 \end{bmatrix}$

$M_k = (C_k E_{k-1}^{-1} L)^+ = (C_k B_{k-1} {}_2V_D)^+$

$N_k = M_k C_k E_{k-1}^{-1}$

$P_k = \begin{bmatrix} E_{k-1}^{-1} {}_2V_{Nk} \\ -{}_2V_D N_{k+1} E_k A_k E_{k-1}^{-1} {}_2V_{Nk} \end{bmatrix}$

$Q_k = \begin{bmatrix} 0 & -E_{k-1}^{-1} L M_k & 0 \\ -{}_2V_D N_{k+1} E_k & {}_2V_D N_{k+1} E_k A_k E_{k-1}^{-1} L M_k - (I - {}_2V_D N_{k+1} B') D_k^+ \end{bmatrix}$

$R_k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -{}_2V_D M_{k+1} & 0 \end{bmatrix}$

$R_w = E[w_k w_k^t] = \text{Diag}(R_{w0}, R_{w1}, R_{w2})$

$R_{w0} = E[w_{0k} w_{0k}^t]$:positive define matrix

$R_{w1} = E[w_{1k} w_{1k}^t]$: positive definite matrix

$R_{w2} = E[w_{2k} w_{2k}^t]$: positive semi-definite matrix

$U_{(\cdot)}$: $[{}_1U_{(\cdot)} {}_2U_{(\cdot)}]$ left singular matrix of matrix $(\cdot)$,
$U_{(\cdot)} \in R^{\mu\times\mu}$, where $(\cdot) \in R^{\mu\times\nu}$

${}_1U_{(\cdot)}$: singular matrix of matrix $(\cdot)$, ${}_1U_{(\cdot)} \in R^{\mu\times\alpha}$,
where $(\cdot) \in R^{\mu\times\nu}$ rank $(\cdot) = \alpha$

${}_2U_{(.)}$: null space matrix of matrix $(.)^t$, ${}_2U_{(.)} \in R^{\mu\times(\mu-\alpha)}$,
where $(\cdot) \in R^{\mu\times\nu}$, rank $(\cdot) = \alpha$

u_k : system input at time step k, $u_k \in R^m$

$\hat{u}_k$: estimate of input variables at time step k

$u_k^* = {}_2V_D^t u_k$

$V_{(\cdot)}$: $[{}_1V_{(\cdot)} {}_2V_{(\cdot)}]$ right singular matrix fo matrix $(\cdot)$
$V_{(\cdot)} \in R^{\nu\times\nu}$, where $(\cdot) \in R^{\mu\times\nu}$

${}_1V_{(\cdot)}$: range space matrix of matrix $(\cdot)^t$, ${}_1V_{(\cdot)} \in R^{\nu\times\alpha}$,
where $(\cdot) \in R^{\mu\times\nu}$, rank$(\cdot) = \alpha$

${}_2V_{(\cdot)}$: null space matrix of matrix $(\cdot)$, ${}_2V_{(\cdot)} \in R^{\nu\times(\nu-\alpha)}$,
where $(\cdot) \in R^{\mu\times\nu}$, rank$(\cdot) = \alpha$

$w_k = [w_{0k}^t \ w_{1k}^t \ w_{2k}^t]^t$

w_{0k} : system noise at time step k, $w_{0k} \in R^n$

w_{1k} : state measurement noise at time step k,
$w_{1k} \in R^p$

w_{2k} : input measurement noise at time step k,
$w_{2k} \in R^q$

x_k : system state at time step k, $x_k \in R^n$

$\hat{x}_k$: estimate of state variables at time step k

y_{1k} : state measurements at time step k, $y_{1k} \in R^P$

y_{2k} : input measurements at time step k, $y_{2k} \in R^q$

$y_k^* = N_k E_{k-1} x_k = M_k C_k x_k$

$\Sigma_{(\cdot)} = \begin{bmatrix} \alpha_{(\cdot)} & 0 \\ 0 & 0 \end{bmatrix}$: singular matrix of matrix (.),
$\Sigma_{(\cdot)} \in R^{\mu\times\nu}$, where$(\cdot) \in R^{\mu\times\nu}$

$\sigma_{(\cdot)}$: positive definite diagonal matrix, $\sigma_{(\cdot)} \in R^{\alpha\times\alpha}$,
where $(\cdot) \in R^{\mu\times\nu}$, rank$(\cdot) = \alpha$

$\varepsilon_k = \begin{pmatrix} x_k \\ u_k \end{pmatrix} - \begin{pmatrix} \hat{x}k \\ \hat{u}k \end{pmatrix}$: estimate error

$\zeta_k = {}_2V_{Nk}^t (I - LN_k) E_{k-1} x_k$: sub-state variables at time step k, $\zeta_k \in R^{n-m}$

$\hat{\zeta}_k$: estimate of sub-state variables at time step k

$\hat{\zeta}_k^-$: *a prior* estimate of sub-state variables at time step k

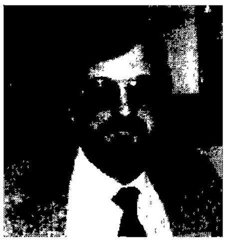
$\Pi_k = E[\varepsilon_k \varepsilon_k^t]$: error covariance of state and input estimation at time step k

$\Psi_k = E[\varepsilon_k \epsilon_k^t]$: error covariance of sub-state estimation at time step k

$(\cdot)^+ = V_{(\cdot)} \Sigma_{(\cdot)}^+ U_{(\cdot)}^t = {}_1V_{(\cdot)} \alpha_{(\cdot)}^{-1} {}_1U_{(\cdot)}^t$

Youngjin Park

For photography and biography, see the Transaction in Control, Automation and systems Engineering, Vol. 1, No. 1. 1999.



J. L. Stein

He received his Ph. D. in mechanical Engineering from MIT in 1983. He has been on faculty at the University of Michigan since 1983, becoming a Professor in 1996. He is interested in Computer-aided design, particularly from the perspective of automating the development of dynamic mathematical models to be used as part of the design process. He is a former Associate Editor of the ASME, Journal of Dynamic Systems, Measurement and Control and a member of ASME, SME, ASEE, Sigma Xi and Phi Beta Kappa. He has received several professional honors and awards including the 1987 National Science Foundation Young Investigator Awards. He has authored or co-authored over 60 articles in journals and conference proceedings.