Fuzzy H^{∞} Filtering for Nonlinear Systems with Time-Varying Delayed States

Kap Rai Lee, Jang Sik Lee, Do Chang Oh, and Hong Bae Park

Abstract : This paper presents a fuzzy H^{∞} filtering problem for a class of uncertain nonlinear systems with time-varying delayed states and unknown initial state on the basis of Takagi-Sugeno(T-S) fuzzy model. The nonlinear systems are represented by T-S fuzzy models, and the fuzzy control systems utilize the concept of the so-called parallel distributed compensation. Using a single quadratic Lyapunov function, the stability and L_2 gain performance from the noise signals to the estimation error are discussed. Sufficient conditions for the existence of fuzzy H^{∞} filters are given in terms of linear matrix inequalities(LMIs). The filtering gains can also be directly obtained from the solutions of LMIs.

Keywords : fuzzy filter, H^{∞} filtering, nonlinear systems, fuzzy model, time-varying delays

I. Introduction

Recently, stability analysis and systematic design are among the most important issues for fuzzy control systems. There have been significant research efforts on these issues. With the development of fuzzy systems, it is known that the qualitative knowledge of a system can also be represented in nonlinear functional form. On the basis of this idea, some fuzzy models based fuzzy control have appeared in the fuzzy control field [1]-[6]. These methods are conceptually simple and straightforward. The nonlinear system is represented by a Takagi-Sugeno(T-S) fuzzy model. And then, the control design is carried out on the basis of the fuzzy model via the so-called parallel distributed compensation(PDC) scheme. Since uncertainties are frequently a source of instability, Tanaka *et al.* [5],[6] presented stability analysis for a class of uncertain nonlinear systems and method for designing robust fuzzy controllers to stabilize the uncertain nonlinear systems. However, their design method has to predetermine the state feedback gains before checking the stability condition of the closed-loop system. It is well known that observer or filter design is also a very important problem in control systems. Ma *et al.* [8] presented the analysis and design of the fuzzy controller and fuzzy observer on the basis of T-S fuzzy model using separation property Tanaka *et al.* [9] also presented systematic design method of the fuzzy regulator and fuzzy observer on the basis of T-S fuzzy model.

The H^{∞} filtering approach is concerned with the design of filter which ensures that L_2 induced gain from the noise signals to the estimation error signals will be less than a prescribed level. In H^{∞} filtering, the noise sources are arbitrary signals with bounded energy, or bounded average power, which renders this approach insensitive to the noise statistics, and thus very appropriate to applications where the statistics of the noise signals are not exactly known [10]-[14]. Since time delay is frequently a source of instability and encountered in various engineering systems, the H^{∞} control problem for delayed systems has received considerable attention over the last few decades [15]-[18]. However for a fuzzy control system, there are few publications on H^{∞} filter design for delayed systems.

In this paper, we design a fuzzy H^{∞} filter for nonlinear system with time-varying delayed states. To design a fuzzy H^{∞} filter, the nonlinear systems are represented by T-S fuzzy models and the PDC is employed. A sufficient condition is derived such that the closed-loop system is exponentially stable and L_2 gain of the input-output map is bounded. Based on the derivation, we obtain a sufficient condition, which is given in terms of LMIs, for the existence of fuzzy H^{∞} filters for T-S fuzzy model. And the filter gains can be directly obtained from the solutions of LMIs.

II. Problem formulation

The continuous fuzzy dynamic model, proposed by Takagi and Sugeno, is described by fuzzy IF-THEN rules which represented local linear input-output relations of nonlinear system. Consider a nonlinear system with time-varying delayed states that can be described by the following T-S fuzzy model with time-varying delayed states:

Plant Rule 1 :

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Kap Rai Lee, Jang Sik Lee : Dept. of Computer Applied Control, Doowon Technical College.
Do Chang Oh : Dept. of Control and Instrumentation, Kon-yang University.
Hong Bae Park : School of Electronic and Electrical Engineering, Kyungpook National University.

$$\begin{aligned}&\text{IF } z_1(t) \text{ is } M_{i1} \text{ and ... and } z_g(t) \text{ is } M_{ig}\\&\text{THEN } \dot{x}(t) = A_i x(t) + A_{d_i} x(t-d_1(t)) + B_{1_i} w(t)\\&\quad y(t) = C_{y_i} x(t) + C_{yd_i} x(t-d_2(t)) + D_{y_i} w(t)\\&\quad e(t) = C_i x(t), \quad x(0) = x_0, \quad i=1,2,\dots,r\\&\quad x(t) = \phi(t), \quad t \in [-\max(d_1(t), d_2(t)), 0]\end{aligned} \tag{1}$$

where M_{ij} is the fuzzy set, $x(t) \in R^n$ is the state vector, $\phi(t) \in R^n$ is the continuous initial value function, $w(t) \in R^p \in L_2(0, T)$ is the square-integrable noise signal, $y(t) \in R^m$ is the measurement, $e(t) \in R^q$ is the signal to be estimated, r is the number of IF-THEN rules, $z_1 \sim z_g$ are some measurable system variables, i.e., the premise variables, and all matrices are constant matrices with appropriate dimensions, $d_i(t), i=1,2,$ are the time- varying delays with following assumptions:

$$0 \le d_i(t) < \infty, \quad \dot{d}_i(t) \le \beta_i < 1, \quad i=1,2. \tag{2}$$

The final output of the fuzzy system is inferred as follows:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + A_{d_i} x(t-d_1(t)) + B_{1_i} w(t)\}\\ y(t) &= \sum_{i=1}^{r} h_i(z(t))\{C_{y_i} x(t) + C_{yd_i} x(t-d_2(t)) + D_{y_i} w(t)\}\\ e(t) &= \sum_{i=1}^{r} h_i(z(t)) C_i x(t)\\ x(0) &= x_0, \quad x(t) = \phi(t), \quad t \in [-\max(d_1(t), d_2(t)), 0]\end{aligned} \tag{3}$$

where

$$\begin{aligned} w_i(z(t)) &= \prod_{j=1}^{n} M_{ij}(z_j(t))\\ h_i(z(t)) &= w_i(z(t)) / \sum_{i=1}^{r} w_i(z(t))\\ z(t) &= [z_{1(t)}\ z_{2(t)}\ \dots\ z_{n(t)}]^T\end{aligned}$$

where $M_{ij}(z_{j(t)})$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed that

$$\begin{aligned} w_i(z(t)) &\ge 0, \quad i=1,2,\dots,r\\ \sum_{i=1}^{r} w_{i(z(t))} &> 0\end{aligned} \tag{4}$$

for all t. Then we can obtain the following conditions:

$$\begin{aligned} h_i(z(t)) &\ge 0, \quad i=1,2,\dots,r\\ \sum_{i=1}^{r} h_{i(z(t))} &= 1\end{aligned} \tag{5}$$

for all t. As a fuzzy H^∞ filter of the fuzzy system (1), we consider the following structure:

Filtering Rule I :

$$\begin{aligned}&\text{IF } z_1(t) \text{ is } M_{i1} \text{ and ... and } z_n(t) \text{ is } M_{in}\\&\text{THEN } \dot{\hat{x}}(t) = F_i \hat{x}(t) + G_i y(t); \quad \hat{x}(0)=0\\&\quad \hat{e}(t) = C_i \hat{x}(t), \quad i=1,2,\dots,r\end{aligned} \tag{6}$$

where the matrix F_i and G_i are to be determined. The final output of this fuzzy filter is

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(z(t))\{F_i \hat{x}(t) + G_i y(t)\}; \quad \hat{x}(0)=0,\\ \hat{e}(t) &= \sum_{i=1}^{r} h_i(z(t)) C_i \hat{x}(t).\end{aligned} \tag{7}$$

From (3) and (7), we obtain the following estimation error system

$$\begin{aligned}\dot{\zeta}(t) &= \hat{A}(z)\zeta(t) + \hat{A}_1(z)x(t-d_1(t))\\ &\quad + \hat{A}_2(z)x(t-d_2(t)) + \hat{B}(z)w(t)\\ \zeta(0) &= [x_0^T\ x_0^T]^T, \quad \zeta(t) = [\phi(t)^T\ \phi(t)^T]^T,\\ t &\in [-\max(d_1(t), d_2(t)), 0]\\ \tilde{e}(t) &= \hat{C}(z)\zeta(t)\end{aligned} \tag{8}$$

with the following notations

$$\begin{aligned}\hat{A}(z) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\ \hat{A}_{ij},\\ \hat{C}(z) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\ \hat{C}_{ij},\\ \hat{A}_1(z) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\ \hat{A}_{1_{ij}},\\ \hat{A}_2(z) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\ \hat{A}_{2_{ij}},\\ \hat{B}(z) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\ \hat{B}_{ij},\end{aligned} \tag{9}$$

where $\zeta(t) = [(x(t) - \hat{x}(t))^T \ \ x^T(t)]^T$, $\tilde{e}(t) = e(t) - \hat{e}(t)$,

$$\begin{aligned}&\hat{A}_{ij} = \begin{bmatrix} F_i & A_i - G_i C_{y_j} - F_i \\ 0 & A_i \end{bmatrix} \quad \hat{A}_{2_{ij}} = \begin{bmatrix} -G_i G_{yd_j} \\ 0 \end{bmatrix},\\ &\hat{A}_{1_{ij}} = [A_{d_i}^T\ \ A_{d_i}^T]^T,\\ &\hat{B}_{ij} = [(B_{1_i} - G_i D_{y_j})^T\ \ B_{1_i}^T]^T \quad \hat{C}_{ij} = [C_i\ \ 0].\end{aligned} \tag{10}$$

For given γ, we define L_2 gain γ-performance of the system (10) as the quantity

$$\begin{aligned}\int_0^T \|\tilde{e}(t)\|^2 dt \le \gamma^2 [\int_0^T \|w(t)\|^2 dt\\ + x_0^T R_0 x_0 + \sum_{i=1}^{2} \int_{-\beta_i}^{0} x^T(\tau) R_i x(\tau) d\tau]\end{aligned} \tag{11}$$

for all $T > 0$ and all $w \in L_2[0\ \ T]$, where $\|\cdot\|$ denotes the Euclidean norm. The weighting matrix $R_i, i=0,1,2,$ in (11) are measure of the initial state uncertainty at $t \le 0$ relative to the uncertainty in $w(t)$. A large value of R_i indicates that the state at $t \le 0$ is very close to zero. In the case of filtering problems for the systems with no time delays, the performance measure of (11) is replaced by

$$\int_0^T \|\tilde{e}(t)\|^2 dt \le \gamma^2 [\int_0^T \|w(t)\|^2 dt + x_0^T R_0 x_0] \tag{12}$$

This paper addresses designing fuzzy H^∞ filter (7) for the system (3) such that the closed-loop system is globally exponentially stable and achieves L_2 gain γ -performance (Globally ES- γ).

III. Robust fuzzy H^∞ filter design

Our first result deals with the problem of analysing if a given fuzzy H^∞ filter as in (7) provides a globally exponentially stable error dynamics and satisfies the performance constraint of (11).

Lemma 1 : Consider the unforced system of (8). If there exist matrices $P>0, S_{11}>0$ and $S_{22}>0$, and positive scalar α satisfying the following inequalities

$$\Omega^s_{ii} < 0, \quad i=1,2,\dots,r, \tag{13}$$

$$\Omega^s_{ij}+\Omega^s_{ji} < 0, \quad i<j<r, \tag{14}$$

where

$$\Omega^s_{ij} = \begin{bmatrix} \hat{A}_{ij}{}^T P+P\hat{A}_{ij}+\tilde{S}+\tilde{S}_1 & P\hat{A}_{1_{ij}} & P\hat{A}_{2_{ij}} \\ \hat{A}_{1_{ij}}{}^T P & -\tilde{S}_{11} & 0 \\ \hat{A}_{2_{ij}}{}^T P & 0 & -\tilde{S}_{22} \end{bmatrix}, \tag{15}$$

$i,j = 1,\text{cdots},r$

$$\tilde{S} = \begin{bmatrix} 0 & 0 \\ 0 & S_{11}+S_{22} \end{bmatrix}, \quad \tilde{S}_1 = \begin{bmatrix} 0 & 0 \\ 0 & \alpha I \end{bmatrix}$$

$$\tilde{S}_{ii} = (1-\beta_i)S_{ii}, \quad i=1,2,$$

then the equilibrium of the unforced system of (8) is globally exponentially stable.

Proof : Define a Lyapunov functional

$$V(\zeta,t)=\zeta^T(t)P\zeta(t)+\sum_{i=1}^{2}\int_{t-d_i(t)}^{t} x(\tau)^T S_{ii}x(\tau)d\tau, \tag{16}$$

where $P>0$ and $S_{ii}>0$. Then there exist positive scalars δ_1 and δ_2 such that $\delta_1\|\zeta\|^2 \le V(\zeta,t) \le \delta_2\|\zeta\|^2$. If there exists scalar $\alpha>0$ such that $\dot{V}(\zeta,t) \le -\alpha\|x\|$, then the unforced system of (8) is globally exponentially stable[19]. using

$$\dot{V}(\zeta,t)\le \dot{\zeta}^T(t)P\zeta(t)+\zeta^T(t)P\dot{\zeta}(t)$$

$$+\sum_{i=1}^{2}\{x(t)^T S_{ii}x(t)-x(t-d_i(t))^T\tilde{S}_{ii}x(t-d_i(t))\} \tag{17}$$

$$:= \dot{V}_a(\zeta,t),$$

and assuming the zero input, the condition $\dot{V}_a(\zeta,t) \le -\alpha\|x\|$ is equivalent to

$$\begin{aligned} &\zeta^{T(t)}\{\hat{A}^T(z)P+P\hat{A}(z)+\tilde{S}+\tilde{S}_1\}\zeta(t) \\ &+x^T_{d_1}(t)\hat{A}_1{}^T(z)P\zeta(t)+x^T_{d_2}(t)\hat{A}_2{}^T(z)P\zeta(t) \\ &+\zeta^T(t)P\hat{A}_1(z)x_{d_1}(t)+\zeta^T(t)P\hat{A}_2(z)x_{d_2}(t) \\ &-x^T_{d_1}(t)\tilde{S}_{11}x_{d_1}(t)-x^T_{d_2}(t)\tilde{S}_{22}x_{d_2}(t)\le 0, \end{aligned} \tag{18}$$

where $x_{d_i}(t)=x(t-d_i(t)), i=1,2.$

This is equivalent to the existence of $P>0$ satisfying

$$\begin{bmatrix} \overline{A}_{11}(z) & P\hat{A}_1(z) & P\hat{A}_2(z) \\ \hat{A}_1{}^T(z)P & -\tilde{S}_{11} & 0 \\ \hat{A}_2{}^T(z)P & 0 & -\tilde{S}_{22} \end{bmatrix} \le 0. \tag{19}$$

where $\overline{A}_{11}(z)=\hat{A}^T(z)P+P\hat{A}(z)+\tilde{S}+\tilde{S}_1$

By using the Schur complement [20], it can be easily shown that the inequality (19) is equivalent to

$$\begin{aligned} &\sum_{i=1}^{r} h_i(z(t))h_i(z(t))\Omega^s_{ii} \\ &+\sum_{i<j}^{r} h_i(z(t))h_j(z(t))\{\Omega^s_{ij}+\Omega^s_{ji}\} \le 0. \end{aligned} \tag{20}$$

Thus, if (13) and (14) are satisfied, then the unforced system of (8) is globally exponentially stable. ■

Lemma 2 : Consider the system (3) with assumption (2), and let $R_i>0, i=1,2,$ be a given initial state weighting matrix and $\gamma>0$ a given scalar. Given a fuzzy H^∞ controller of the form of (7), then the corresponding closed-loop system is Globally ES-γ for all $w\in L_2[0,T]$, if there exist matrices $P>0, S_{11}>0$ and $S_{22}>0$, and positive scalar α satisfying the following inequalities

$$\Omega^{L_2}_{ii}<0, \quad i=1,2,\dots,r \tag{21}$$

$$\Omega^{L_2}_{ij}+\Omega^{L_2}_{ji}<0, \quad i<j<r \tag{22}$$

$$[I_n \;\; I_n]P[I_n \;\; I_n]^T-\gamma^2 R_0\le 0 \tag{23}$$

$$S_{ii}-\gamma^2 R_i\le 0, \;\; i=1,2. \tag{24}$$

In here

$$\Omega^{L_2}_{ij}= \begin{bmatrix} \hat{A}^T_{ij}P+P\hat{A}_{ij}+\tilde{S}+\tilde{S}_1 & P\hat{A}_{1_{ij}} & P\hat{A}_{2_{ij}} & P\hat{B}_{ij} & \hat{C}^T_{ij} \\ \hat{A}^T_{1_{ij}}P & -\tilde{S}_{11} & 0 & 0 & 0 \\ \hat{A}^T_{2_{ij}}P & 0 & -\tilde{S}_{22} & 0 & 0 \\ \hat{B}_{ij}{}^T P & 0 & 0 & -\gamma^2 I & 0 \\ \hat{C}_{ij} & 0 & 0 & 0 & -I \end{bmatrix},$$

$$i,j=1,2,\dots,r \tag{25}$$

where $\tilde{S}$, $\tilde{S}_1$ and $\tilde{S}_{ii}, i=1,2$ are given by (15).

Proof : The positive definite matrices P, S_{11} and S_{22}, and positive scalar α satisfying (21) and (22) also satisfy the inequalities (13) and (14). Using a Lyapunov functionals (16), (17) and the following condition

$$J_a(t):=\dot{V}_a(\zeta,t)+\tilde{e}^T(t)\tilde{e}(t)-\gamma^2 w^{T(t)}w(t)\le 0, \tag{26}$$

(21) and (22) are obtained. From (17) and (26)

$$\begin{aligned} &\int_0^T\|\tilde{e}(t)\|^2dt-\gamma^2\int_0^T\|w(t)\|^2dt \le \\ &\zeta^T(0)P\zeta(0)+\sum_{i=1}^{2}\int_{-d_i(t)}^{0}x^T(\tau)S_{ii}x(\tau)d\tau \end{aligned} \tag{27}$$

because $V(\zeta,T)>0$. It follows from initial condition (8) that

$$\begin{aligned} &\int_0^T\|\tilde{e}(t)\|^2dt-\gamma^2[\int_0^T\|w(t)\|^2dt+x_0^TR_0x_0 \\ &+\sum_{i=1}^{2}\int_{-\beta_i}^{0}x^T(\tau)R_ix(\tau)d\tau]\le\zeta^T(0)P\zeta(0)-\gamma^2x_0^TR_0x_0+ \\ &\sum_{i=1}^{2}\int_{-d_i(t)}^{0}x^T(\tau)S_{ii}x(\tau)d\tau-\gamma^2\sum_{i=1}^{2}\int_{-\beta_i}^{0}x^T(\tau)R_ix(\tau)d\tau \\ &\le x_0^T\{[I_n \;\; I_n]P[I_n^T \;\; I_n^T]^T-\gamma^2R_0\}x_0 \\ &+\sum_{i=1}^{2}\int_{-\beta_i}^{0}x^T(\tau)[S_{ii}-\gamma^2R_i]x(\tau)d\tau. \end{aligned} \tag{28}$$

Thus if (21)-(24) are satisfied, the closed-loop system is Globally ES-γ. ■

The next Theorem 1 presents a solution to the fuzzy

H^∞ filtering problem for the uncertain T-S fuzzy model with time-varying delayed states in terms of LMIs.

Theorem 1 : Consider the system (3), and let $R_i > 0, i=0,1,2,$ be given initial state weighting matrices and $\gamma > 0$ a given scalar. Then there exists a fuzzy H^∞ filter (7) such that the estimation error system (8) is Globally ES- γ. if there exist common matrix $X > 0$, $Z > 0$, $S_{11} > 0$ and $S_{22} > 0$, and matrices W_i, Y_i, $i=1,2,\ldots,r$, and positive scalar α satisfying the following LMIs:

$$\Phi_{ii} < 0, \quad i=1,2,\ldots,r, \tag{29}$$

$$\Phi_{ij}+\Phi_{ji} < 0, \quad i<j<r, \tag{30}$$

$$X+Z-\gamma^2 R_0 \le 0, \tag{31}$$

$$S_{ii}-\gamma^2 R_i \le 0, \ \ i=1,2. \tag{32}$$

In here

$$\Phi_{ij}= \begin{bmatrix} \tilde{\Psi}_i & \tilde{\Gamma}_{ij} & XA_{d_i} & -Y_i C_{yd_i} & \tilde{U}_{ij} & C^{T_i} \\ \tilde{\Gamma}_{ij^T} & \tilde{\Lambda}_i & ZA_{d_i} & 0 & ZB_{1_i} & 0 \\ A^T_{d_i}X & A^T_{d_i}Z & -\tilde{S}_{11} & 0 & 0 & 0 \\ -C^T_{yd_i}Y_i & 0 & 0 & -\tilde{S}_{22} & 0 & 0 \\ \tilde{U}_{ij^T} & B^{TZ}_{1_i} & 0 & 0 & -\gamma^2 I & 0 \\ C_i & 0 & 0 & 0 & 0 & -I \end{bmatrix}, \tag{33}$$

$i,j=1,2,\ldots,r$

where

$$\tilde{\Psi}_i= W_i+W_i^T, \quad \tilde{\Gamma}_{ij}=XA_i-Y_iC_{y_i}-W_i,$$

$$\tilde{\Lambda}_i=ZA_i+A_i^TZ+S_{11}+S_{22}+\alpha I, \tag{34}$$

$$\tilde{U}_{ij}=XB_{1_i}-Y_iD_{y_i}, \quad \tilde{S}_{ii}=(1-\beta_i)S_{ii}, \ \ i=1,2.$$

Furthermore, filter gains are given by

$$F_i=X^{-1}W_i, \quad G_i=X^{-1}Y_i. \tag{35}$$

Proof : Let $P=\mathrm{diag}\{X,Z\}$, where X and Z are symmetric positive definite matrices to be found. Denoting $W_i=XF_i$ and $Y_i=XG_i$, $i=1,2,\ldots,r$, and considering (10), it can be easily shown that (21)-(24) are equivalent to (29)-(32), respectively. ■

It has been seen that the filter design problem of the fuzzy system (5) can be transformed into a linear algebra problem. This set of LMIs constitutes a finite-dimensional convex feasibility problem. There are several efficient algorithms to solve the above convex LMIs problem [22]-[23].

Remark 1 : Consider the system (3) with no time delays, i.e., $x(t-d_1(t))=0$, $x(t-d_2(t))=0$, and let $R_0 > 0$ be a given initial state weighting matrix and a given scalar. Then there exists a fuzzy H^∞ filter $\gamma > 0$(7) such that the estimation error system is Globally ES- γ, if there exist common matrices $X > 0$, $Z > 0$, and matrices W_i, Y_i, $i=1,2,\ldots,r$, and positive scalar α satisfying the following LMIs:

$$\Phi^d_{ii} < 0, \quad i=1,2,\ldots,r, \tag{36}$$

$$\Phi^d_{ij}+\Phi^d_{ji} < 0, \quad i<j<r, \tag{37}$$

$$X+Z-\gamma^2 R_0 \le 0. \tag{38}$$

In here

$$\Phi^d_{ij}= \begin{bmatrix} \Psi_i & \tilde{\Gamma}_{ij} & \tilde{U}_{ij} & C_i^T \\ \tilde{\Gamma}^T{}_{ij} & \tilde{\Lambda}_i & ZB_{1_i} & 0 \\ \tilde{U}^T{}_{ij} & B^T_{1_i}Z & -\gamma^2 I & 0 \\ C_i & 0 & 0 & -I \end{bmatrix}, \quad i,j=1,2,\ldots,r \tag{39}$$

$$\Psi_i= W_i+W_i^T, \quad \tilde{\Gamma}_{ij}=XA_i-Y_iC_{y_i}-W_i,$$

$$\tilde{\Lambda}_i=ZA_i+A_i^TZ+\alpha I, \quad \tilde{U}_{ij}=XB_{1_i}-Y_iD_{y_i},$$

Furthermore, observer gains are given by (40).

IV. Design example

We will design a fuzzy H^∞ filter for the following nonlinear system;

$$\begin{aligned} \dot{x}_1(t) &= -5.125x_1(t)-0.5x_1(t-d(t))-2x_2(t) \\ &\quad -6.7x_2^3(t)-0.2x_2(t-d(t)) \\ &\quad -0.67x_2^3(t-d(t))+w(t) \\ \dot{x}_2(t) &= x_1(t) \\ y(t) &= x_2(t)-0.01x_2(t-d(t))+0.1w(t) \\ e(t) &= 2x_1(t) \end{aligned}$$

where time-varying delay is

$$d(t)=1+0.5\cos(0.1t).$$

$x_1(t)$ is estimated using a fuzzy H^∞ filter and assume that $x_2(t)$ is observable. It is also assumed that

$$x_1(t)\in[-1.5 \ \ 1.5], \qquad x_2(t)\in[-1.5 \ \ 1.5].$$

Using the same procedure as in [6], the nonlinear term can be represented as

$$-6.7x_2^3(t)=M_{11}\cdot 0\cdot x_{2(t)}-(1-M_{11})\cdot 15.075\,x_2(t).$$

By solving the equation, M_{11} is obtained as follows:

$$\begin{aligned} M_{11}(x_2(t)) &= 1-\frac{x_2(t)^2}{2.25} \\ M_{12}(x_2(t)) &= \frac{x_2(t)^2}{2.25}. \end{aligned}$$

M_{11} and M_{12} can be interpreted as membership functions of fuzzy set. By using these fuzzy sets, the nonlinear system can be presented by the following uncertain T-S fuzzy model

Plant Rule 1 : IF $x_2(t)$ is M_{11} THEN}

$$\begin{aligned} \dot{x}(t) &= A_1x(t)+A_{d_1}x(t-d(t))+B_{1_1}w(t) \\ y(t) &= C_{y_1}x(t)+C_{yd_1}x(t-d(t))+D_{y_1}w(t) \\ e(t) &= C_1x(t), \end{aligned}$$

Plant Rule 2 : IF $x_2(t)$ is M_{12}THEN}

$$\begin{aligned} \dot{x}(t) &= A_2x(t)+A_{d_2}x(t-d(t))+B_{1_2}w(t) \\ y(t) &= C_{y_2}x(t)+C_{yd_2}x(t-d(t))+D_{y_2}w(t) \end{aligned}$$

$$e(t) = C_2 x(t)$$

where

$$x(t) = [x_{1(t)} \;\; x_{2(t)}]^T,$$

$$A_1 = \begin{bmatrix} -5.125 & -2 \\ 1 & 0 \end{bmatrix}, \; A_{d_1} = \begin{bmatrix} -0.5 & -0.2 \\ 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -5.125 & -17.075 \\ 1 & 0 \end{bmatrix}, \; A_{d_2} = \begin{bmatrix} -0.5 & -1.71 \\ 0 & 0 \end{bmatrix}$$

$$B_{1_1} = B_{1_2} = [1 \;\; 0]^T, \; C_1 = C_2 = [2 \;\; 0],$$

$$C_{y_1} = C_{y_2} = [0 \;\; 1], \quad C_{yd_1} = C_{yd_2} = [0 \;\; -0.01].$$

$$D_{y_1} = D_{y_2} = [0 \;\; 0.1]^T.$$

Let $\gamma = 10$, $\beta_1 = \beta_2 = 0.8$, and $R_0 = R_1 = R_2 = \text{diag}(0.5, 0.5)$ then filter gains obtained from Therrem 1 are

$$F_1 = \begin{bmatrix} -13.163 & -104.031 \\ -7.128 & -164.618 \end{bmatrix}, \; F_2 = \begin{bmatrix} -14.092 & -107.002 \\ -11.598 & -163.492 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} -22.460 & 124.823 \\ 15.600 & 148.726 \end{bmatrix}, \; G_2 = \begin{bmatrix} -45.325 & 115.887 \\ -6.180 & 144.510 \end{bmatrix}$$

Also, consider the system with no time delays. The filter gains obtained from Remark 1 under the same conditions are

$$F_1 = \begin{bmatrix} -5.441 & -3.597 \\ -1.464 & -44.925 \end{bmatrix}, \; F_2 = \begin{bmatrix} -5.658 & -3.631 \\ -2.209 & -44.428 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 4.348 & 6.045 \\ 42.054 & 2.947 \end{bmatrix}, \; G_2 = \begin{bmatrix} -14.720 & 0.605 \\ -98.870 & 139.773 \end{bmatrix}$$

The simulation results of nonlinear systems with time-varying delays and with no time-varying delays are shown in Fig. 1 and Fig. 2, respectively.

Here the solid lines are results of the behavior of the designed filters which are not considered initial uncertainty, i.e., $R_i = \text{diag}(100, 100)$, $i = 0, 1, 2$. For these simulation, the noise signal $w(t)$ is

$$w(t) = 0.1 \cdot \cos(\pi t)$$

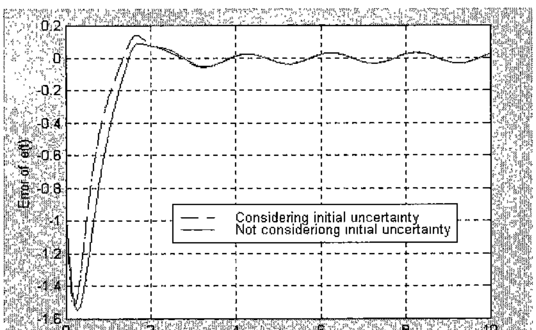

Fig. 1. The simulation result of nonlinear system with time delays.

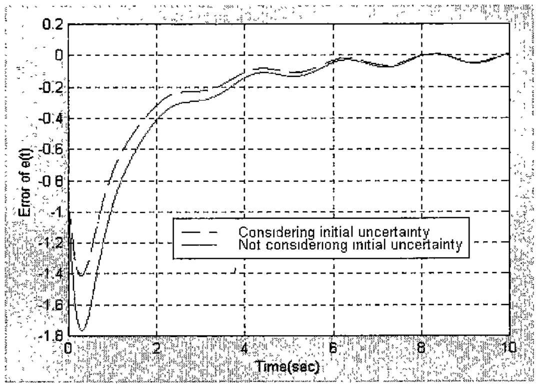

Fig. 2. The simulation result of nonlinear system with no time delays.

and the initial value of the state is assumed by

$$[x_1^T(t) \;\; x_2^T(t)]^T = [-1 \;\; -1]^T, \quad t \le 0.$$

The designed fuzzy H^∞ filter estimates the states of the nonlinear system without the steady state errors.

V. Conclusion

In this paper, we have developed fuzzy H^∞ filter design method for nonlinear systems with time-varying delayed states and unknown initial states described by Takagi and Sugeno fuzzy model. We have obtained sufficient conditions for the existence of fuzzy H^∞ filters such that the estimation error system is globally exponentially stable and achieves L_2 gain γ-performance. The filter design has utilized the concept of parallel distributed compensation and the filter gains can also be directly obtained from the LMI solutions.

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Kap Rai Lee

He received the B. S., M. S. and Ph. D. degree in electronic engineering from Kyungpook National University, Korea, in 1987, 1990 and 1999, respectively. From 1990 to 1995, he has worked in Agency for Defence Development. Since 1997, he is an associate professor of the Department of Computer Applied Control, Doowon Technical College, Korea. His research interests are robust control, H^{∞} control of fuzzy systems and nonlinear systems.

Do Chang Oh

He received the B. S., M.S. and Ph. D. degree in electrical engineering from Kyungpook National University, Korea, in 1991, 1993 and 1997, respectively. Since 1997, he is an associate professor of the Dep. of Control and Instrumentation, Konyang University, Korea. His research interests are robust control, H^{∞} control, model and controller reduction.

Jang Sik Lee

He received the B. S. and M. S. degree in electronic engineering from Kyungpook National University, Korea, in 1984 and 1986, respectively. From 1986 to 1997, he has worked in the Agency for Defence Development. Since 1997, he is an associate professor of the Department of Computer Applied Control, Doowon Technical College, Korea. His research interests are acoustic signal processing and target tracking.

Hong Bae Park

He was born in Korea, on March 6, 1951. He received the B. S. and M. S. degrees in electronic engineering from Kyungpook National University, Taegu, Korea, in 1977 and 1979, respectively, and the Ph. D. degree in electrical and computer engineering from the University of New Mexico, Albuquerque, New Mexico, in 1988. He is currently a Professor in School of Electronic and Electrical Engineering, Kyungpook National University. He received 'Hae-Dong Paper Award' from IEEK in 1998. His current research interests include robust control, optimal control to industrial applications, and guidance control. He is a member of IEEE, IEEK, and ICASE.