Computation of Gradient of Manipulability for Kinematically Redundant Manipulators Including Dual Manipulators System

Jonghoon Park, Wankyun Chung, and Youngil Youm

Abstract : One of the main reason advocating redundant manipulators' superiority in application is that they can afford to optimize a dexterity measure, for example the manipulability measure. However, to obtain the gradient of the manipulability is not an easy task in case of general manipulator with high degrees of redundancy. This article proposes a method to compute the gradient of the manipulability, based on recursive algorithm to compute the Jacobian and its derivative using Denavit-Hartenberg parameters only. To characterize the null motion of redundant manipulators, the null space matrix using square minors of the Jacobian is also proposed. With these capabilities, the inverse kinematics of a redundant manipulator system can be done automatically. The result is easily extended to dual manipulator system using the relative kinematics.

Keywords : kinematically redundant manipulator, manipulability measure, recursive computation

I. Introduction

Imposing kinematic redundancy in robot manipulator system seems to be one trend in robot application, since kinematically redundant manipulators can provide an easier way to overcome complex limitations from which nonredundant manipulators suffer. The limitations are manifested by the kinematic singularity, collision, joint limit, and so on. Nowadays, as the manufacturing cost becomes lower and lower, the more redundancy can be implemented by adding additional degrees of freedom or by coordinating multiple manipulator.

The main advantage of redundant manipulator lies in optimizing a certain performance measure while executing a given task trajectory, using the null motion. The main algorithm for optimizing inverse kinematics is given by the well-known resolved motion rate control with the gradient projection method [1]

$$\dot{q}=J^+(q)\,\dot{p}+\kappa(I-J^+(q)J(q))\nabla m(q) \qquad (1)$$

or the extended Jacobian method

$$\dot{q}=\begin{bmatrix} J(q) \\ \dfrac{\partial Z(q)\nabla m(q)}{\partial q}\end{bmatrix}^{-1}\begin{pmatrix}\dot{p}\\ 0\end{pmatrix} \qquad (2)$$

for the joint velocity $\dot{q}\in R^n$ and the task velocity $\dot{p}\in R^m$, where $J\in R^{m\times n}$ is the Jacobian matrix, such that $J(q)\,\dot{q}=\dot{p}$, $J^+(q)\in R^{n\times m}$ is the Moore-Penrose pseudoinverse of $J(q)$, $Z(q)\in R^{r\times n}$ ($r=n-m$) is a full-row rank null space matrix such that $J(q)Z^T(q)=0$.

In either case, as the redundancy becomes larger, the terms appearing in (1) and (2) such as J, Z and ∇m are difficult to get. As a matter of fact it is easy to compute the Jacobian J automatically using the Denavit-Hartenberg (DH) parameters of the manipulator.

Similarly, the null space matrix Z can be computed by

$$Z=[\,J_2^T \operatorname{adj}\{J_1^T\}\; -\det\{J_1\}I\,] \qquad (3)$$

where J is partitioned to $J=[\,J_1\;\; J_2\,]$ with nonsingular $J_1\in R^{m\times m}$ [4].

In the meantime, to compute a performance measure is much more difficult, except a few simple measure. Consider the manipulability measure [5] one of the performance measure to avoid the kinematic singularity, defined by

$$m(q)=\sqrt{\det\{J(q)J^T(q)\}} \qquad (4)$$

In simple redundant manipulator system, it can be symbolically expressed and differentiated to derive the gradient using numerous available symbolic expression manipulation packages. However our experience tells that in the case of general manipulators with large redundancy the symbolic derivation is frustrating, since the result contains too many terms due to inefficient simplification, and it has no flexibility which means that, for example, addition of additional joint leads to total reevaluation of the gradient.

In this article, we provide an algorithm to automatically solve the inverse kinematics of a general redundant manipulator, including dual manipulator system, using the DH parameters only. More specifically, the recursive formulations of the Jacobian and the gradient

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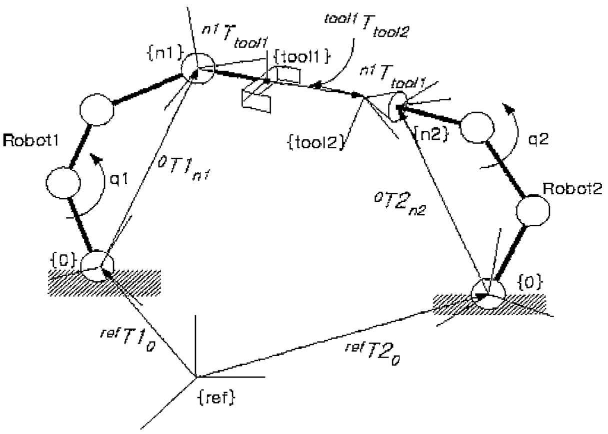


Fig. 1. Schematic diagram of general dual manipulators system.

of the manipulability measure are provided, and effcient method to compute the null space bases matrix using the square minor of the Jacobian is developed. Based on the method, a simulation of dual nine degrees of freedom manipulators will be provided.

II. Kinematic computation using DH parameters for single manipulator

One of the most widely accepted method in modeling manipulator kinematics is the one by the Denavit Hartenberg (DH) notation [6]. The DH method begins by constructing the link transform, with a set of DH parameters $(a_i, d_i, \alpha_i, \theta_i)$, by

$$
{}^{i-1}A_i(q_i) = \begin{bmatrix} c(\theta_i) & -s(\theta_i)c(\alpha_i) & s(\theta_i)s(\alpha_i) & a_i c(\theta_i) \\ s(\theta_i) & c(\theta_i)c(\alpha_i) & -c(\theta_i)s(\alpha_i) & a_i s(\theta_i) \\ 0 & s(\alpha_i) & c(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}
$$

where q_i is the joint variable, i.e. $q_i = d_i$ in case of prismatic joint, and $q_i = \theta_i$ in case of revolute joint, and $s(\cdot)$ and $c(\cdot)$ denotes $\sin(\cdot)$ and $\cos(\cdot)$, respectively.

Then each manipulator consists of a number of links which is driven by the joint of the manipulator. Hence by attaching one frame at each joint, numbered from 0 (the base), 1, through n (the end-effector), where n is the degrees of freedom of the manipulator, we can construct a homogeneous transform of the end-effector, i.e. n - th coordinate frame denoted by $\{n\}$ in Fig. 1 from the base frame denoted by {0}. In general cases, the manipulator's base coordinate frame {0} is not the same as the reference coordinate frame {ref}, with respect to which the manipulator task is specified. Also, some tool, represented as {tool}, to perform a task is attached at the end-effector frame $\{n\}$. To take into account these, one can introduce additional two coordinate transformations, which are ${}^{ref}T_0$ and ${}^{n}T_{tool}$. If the base is fixed and the tool does not have no degrees of freedom, they are constant homogeneous transformation. Therefore, the kinematic modeling of a manipulator can be written as

$$
{}^{ref}T_{tool}(q) = {}^{ref}T_0 \cdot {}^{0}T_n(q) \cdot {}^{n}T_{tool}, \quad q \in R^n. \tag{6}
$$

Then the inverse kinematics aims at solving q for a given ${}^{ref}T_{tool,d}(t)$ such that

$$
{}^{ref}T_{tool,d}(t) = {}^{ref}T_0 \cdot {}^{0}T_n(q) \cdot {}^{n}T_{tool}. \tag{7}
$$

Directly solving (7) involves a case-by-case analysis for each manipulator, and is not useful in redundancy utilization. Hence its linearized version is used to solve $\dot{q}(t)$ such that

$$
\begin{pmatrix} {}^{ref}\dot{x}_{tool,d}(t) \\ {}^{ref}w_{tool,d}(t) \end{pmatrix} = {}^{ref}J_{tool}(q)\,\dot{q}, \quad {}^{ref}J_{tool}(q) \in R^{6\times n} \tag{8}
$$

where ${}^{ref}\dot{x}_{tool} \in R^3$ and ${}^{ref}w_{tool} \in R^3$ is the linear and angular velocity of the tool coordinate frame {tool} along and around the coordinate axis of {ref} expressed with respect to the base frame {ref}. The 6-by-n matrix ${}^{ref}J_{tool}(q)$ is called the manipulator Jacobian and denoted by $J_{MAN}(q)$. The m rows of the manipulator Jacobian $J_{MAN}(q)$ sufficient for specifying a current task can be selected and stacked to yield the Jacobian, denoted also by $J_{MAN}(q)$,

$$
\dot{p}_d(t) = J_{MAN}(q)\,\dot{q}, \quad J_{MAN}(q) \in R^{m\times n}, \quad J_{MAN}(q) \in R^{m\times n} \tag{9}
$$

1. Jacobian computation

A standard technique to compute the manipulator Jacobian matrix $J_{MAN}(q)$ for a general spatial manipulator turns out to be nonrecursive (e.g. See [6]), which is not efficient.

To obtain a recursive formulation, the following matrices U_i representing the transformation from the tool frame of the manipulator to each joint's frame, are recursively calculated by [7][8]

$$
U_{n-1} = {}^{n}A_{tool}^{-1} \tag{10a}
$$

$$
U_i = U_{i-1} \cdot {}^{i-1}A_i^{-1}(q_i) \tag{10b}
$$

for $i = n$ to $i = 1$, where

$$
{}^{i-1}A_i^{-1}(q_i) = \begin{bmatrix} c(\theta_i) & s(\theta_i) & 0 & -a_i \\ -s(\theta_i)c(\alpha_i) & c(\theta_i)c(\alpha_i) & s(\alpha_i) & -d_i s(\alpha_i) \\ s(\theta_i)s(\alpha_i) & -c(\theta_i)s(\alpha_i) & c(\alpha_i) & -d_i c(\alpha_i) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}
$$

Note that

$$
U_1 \cdot {}^{ref}A_0^{-1} = {}^{ref}T_{tool}^{-1} \tag{12}
$$

As a matter of fact, the matrices recursively computed above is related to the hand Jacobian denoted by $J_{\mathrm{HAND}}(q) \in R^{6\times n}$, not the manipulator Jacobian itself. The i-th column of $J_{\mathrm{HAND}}(q)$, denoted by $J^{(i)}_{\mathrm{HAND}}$ is constructed by

$$J^{(i)}_{\mathrm{HAND}} = \begin{bmatrix} a_i \\ 0 \end{bmatrix} \quad \text{if } q_i = d_i$$
$$J^{(i)}_{\mathrm{HAND}} = \begin{bmatrix} p_i \times a_i \\ a_i \end{bmatrix} \quad \text{if } q_i = \theta_i \tag{13}$$

where $U_i = \begin{bmatrix} n_i & o_i & a_i & p_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$ for $i = 1 \cdots n$. The manipulator Jacobian matrix $J_{\mathrm{MAN}}(q)$ is obtained by transforming the reference frame of J_{HAND} as

$$J_{\mathrm{MAN}}(q) = \begin{bmatrix} {}^{\mathrm{ref}}R_{\mathrm{tool}}(q) & 0 \\ 0 & {}^{\mathrm{ref}}R_{\mathrm{tool}}(q) \end{bmatrix} J_{\mathrm{HAND}}(q) \tag{14}$$

Remark 2.1 : In case of a two degrees of freedom planar manipulator, the manipulator Jacobian is

$$J_{\mathrm{MAN}}(q) = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}$$

whereas the hand Jacobian is

$$J_{\mathrm{HAND}}(q) = \begin{bmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{bmatrix}.$$

Note that while computing the Jacobian matrix recursively, the homogeneous coordinate transform from the reference frame to the tool frame is also computed. This method of recursive computation of the Jacobian matrix has already been developed, e.g. [8]. However, the computation of the manipulability seems to still relies on some kind of symbolic calculation, which is extremely difficult for general redundant manipulators with many degrees of redundancy. Hence the manipulability, one of the very popular kinematic measures, is not easy to use for such manipulators. Instead, a simple measure was devised as an alternative or no measure was used. It is well known that proper optimization of a suitable measure can guarantee a dextrous motion in kinematically redundant manipulators. As a matter of fact, many dexterity measures are defined using the manipulator Jacobian [9]. Hence, the gradient of such measures should involve the derivative of the manipulator Jacobian anyway. Then the proposed recursive algorithm can help to achieve the gradient of them. In this article, we propose the manipulability of the Jacobian as an example. This will be presented in the next section.

2. Manipulability computation

First we provide the following lemmas on the gradient of the manipulability.

Lemma 2.1 : The gradient of the manipulability of $J_{\mathrm{MAN}}(q)$ and $J_{\mathrm{HAND}}(q)$ are equal, i.e.

$$\frac{\partial \det\{J_{\mathrm{MAN}} J^T_{\mathrm{MAN}}\}}{\partial q_k} = \frac{\partial \det\{J_{\mathrm{HAND}} J^T_{\mathrm{HAND}}\}}{\partial q_k}. \tag{15}$$

Lemma 2.2 : The gradient of the manipulability given in (4) is given by

$$\frac{\partial m}{\partial q_k} = m(q)\,\mathrm{trace}\left\{\frac{\partial J_{\mathrm{HAND}}}{\partial q_k} J^+_{\mathrm{HAND}}\right\} \tag{16}$$

when $J_{\mathrm{HAND}}(q)$ has full row rank at q.

Hence, the gradient of the manipulability can be computed when $\frac{\partial J_{\mathrm{HAND}}}{\partial q_k}$ is computed. Using the recursive formulation of the Jacobian, it is easy to derive the following equations for the derivative of Jacobian. Similarly, its computation is based on the recursive computation of $\frac{\partial U_i}{\partial q_k}$ expressed as

$$\frac{\partial U_{n+1}}{\partial q_k} = 0 \tag{17a}$$

$$\begin{aligned} \frac{\partial U_i}{\partial q_k} &= 0 && \text{if } k < i \\ \frac{\partial U_i}{\partial q_k} &= U_{i+1} \cdot \frac{\partial\, {}^{i-1}A_i^{-1}(q_i)}{\partial q_k} && \text{if } k = i \\ \frac{\partial U_i}{\partial q_k} &= \frac{\partial U_{i+1}}{\partial q_k} \cdot {}^{i-1}A_i^{-1}(q_i) && \text{if } k > i \end{aligned} \tag{17b}$$

where

$$\frac{\partial\, {}^{i-1}A_i^{-1}(q_i)}{\partial \theta_i} = \begin{bmatrix} -s(\theta_i) & c(\theta_i) & 0 & 0 \\ -c(\theta_i)c(\alpha_i) & -s(\theta_i)c(\alpha_i) & 0 & 0 \\ c(\theta_i)s(\alpha_i) & s(\theta_i)s(\alpha_i) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial\, {}^{i-1}A_i^{-1}(q_i)}{\partial d_i} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s(\alpha_i) \\ 0 & 0 & 0 & -c(\alpha_i) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

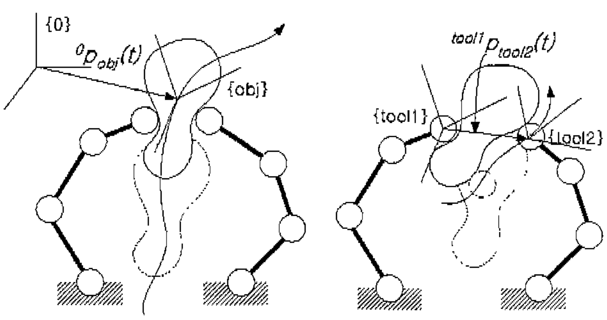
Then i-th column of $\frac{\partial J_{\mathrm{HAND}}}{\partial q_k}$ is constructed

$$\begin{aligned} \frac{\partial J^{(i)}_{\mathrm{HAND}}}{\partial q_k} &= \begin{bmatrix} \frac{\partial a_i}{\partial q_k} \\ 0 \end{bmatrix} && \text{if } q_i = d_i \\ \frac{\partial J^{(i)}_{\mathrm{HAND}}}{\partial q_k} &= \begin{bmatrix} \frac{\partial p_i}{\partial q_k} \times a_i + p_i \times \frac{\partial a_i}{\partial q_k} \\ \frac{\partial a_i}{\partial q_k} \end{bmatrix} && \text{if } q_i = \theta_i \end{aligned} \tag{18}$$

where $\frac{\partial U_i}{\partial q_k} = \begin{bmatrix} \frac{\partial n_i}{\partial q_k} & \frac{\partial o_i}{\partial q_k} & \frac{\partial a_i}{\partial q_k} & \frac{\partial p_i}{\partial q_k} \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

III. Relative kinematic computation for dual manipulator

There are many ways to coordinate a dual manipulators system. One most popular ways is to



(a) Coordination of the object motion (b) Coordination of the relative motion

Fig. 2. Method of coordination with object by dual manipulators.

coordinate them to realize a motion of an object carried by them. We call this way of coordination as the coordination of the object motion (See Fig. 2(a)). For coordination of the object motion, the absolute trajectory of the object is specified with respect to the reference frame. The different way of coordination can be realized by coordinating the dual manipulators so as to execute a task on an object. We call this coordination method as the coordination of relative motion (See Fig. 2(b)). For this coordination method, the relative trajectory of the one manipulator with respect to the other is specified. This method has unique differences from the object motion coordination in many aspects. Above all, the most important thing is that in this case the dual manipulators realize a single redundant manipulator system, since the absolute posture of the object is not of concern, but the relative motion is important.

If the potential redundancy arising in a dual manipulators system is to be exploited, the notion of the relative kinematics is useful [8]. The velocity relation of the relative kinematics is expressed as

$$\dot{r} = {}^{tool1}J_{tool2}(q1, q2)\begin{pmatrix} \dot{q}1 \\ \dot{q}2 \end{pmatrix} \tag{19}$$

where $\dot{r} \in R^m$ is the tool velocity of the second manipulator with respect to the tool frame of the first manipulator, and ${}^{tool1}J_{tool2}(q1, q2) \in R^{m\times(n1+n2)}$, denoted also by J_{REL}, is the relative Jacobian with n_1 the degrees of freedom of the first manipulator, n_2 that of the second one, and m the dimension of the task for the second one. In some expressions below, the single vector $q = (q1^T, q2^T)^T \in R^{n1+n2}$ is used.

1. Relative Jacobian computation

Similarly, the relative Jacobian J_{REL} is computed based on recursive computation of $U1_i$'s and $U2_i$'s given by

$$U2_{n2+1} = {}^{n2}A2^{-1}_{tool} \tag{20a}$$

$$U2_i = U2_{i+1} \cdot {}^{i-1}A2_i^{-1}(q2_i) \tag{20b}$$

for $i = n2$ to $i = 1$ downward, and

$$U1_1 = U2_1 \cdot {}^0A2^{-1}_{base} \cdot {}^0A1_{base} \tag{21a}$$

$$U1_{i+1} = U1_i \cdot {}^{i-1}A1_i(q1_i) \tag{21b}$$

for $i = 0$ to $i = n1 - 1$ upward. Then i-th column of ${}^{tool2}J_{tool1} = [\,{}^{tool2}J1_{tool1} \;\; {}^{tool2}J2_{tool1}\,]$ is constructed by

$$\begin{aligned} {}^{tool2}J2^{(i)}_{tool1} &= \begin{bmatrix} a2_i \\ 0 \end{bmatrix} \quad \text{if } q2_i = d2_i \\ {}^{tool2}J2^{(i)}_{tool1} &= \begin{bmatrix} p2_i \times a2_i \\ a2_i \end{bmatrix} \quad \text{if } q2_i = \theta 2_i \end{aligned} \tag{22}$$

$$\begin{aligned} {}^{tool2}J1^{(i)}_{tool1} &= \begin{bmatrix} -a1_i \\ 0 \end{bmatrix} \quad \text{if } q1_i = d1_i \\ {}^{tool2}J1^{(i)}_{tool1} &= \begin{bmatrix} a1_i \times p1_i \\ -a1_i \end{bmatrix} \quad \text{if } q1_i = \theta 1_i \end{aligned} \tag{23}$$

where $U1_i = \begin{bmatrix} n1_i & o1_i & a1_i & p1_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and similarly for $U2_i$.

Noting that

$$U1_{n1} \cdot {}^{n1-1}A1_{n1}(q1_{n1}) \cdot {}^{n1}A1_{tool} = {}^{tool1}T^{-1}_{tool2} \tag{24}$$

the original relative Jacobian ${}^{tool1}J_{tool2} = J_{REL}$ with respect to the tool frame of the first manipulator is obtained by

$$J_{REL}(q) = \begin{bmatrix} {}^{tool1}R_{tool2}(q) & 0 \\ 0 & {}^{tool1}R_{tool2}(q) \end{bmatrix} {}^{tool2}J_{tool1}(q). \tag{25}$$

3. Relative manipulability computation

As seen in (19), the singularity of the relative Jacobian $J_{REL}(q)$ impose many limitations on validity and smoothness of the inverse kinematic solutions, just as the manipulability does for a single manipulator. Hence, one of the natural measures for dual manipulators systems in the relative kinematics configuration is the relative manipulability defined by

$$m(q1, q2) = \sqrt{\det\{J_{REL}(q1, q2) J^T_{REL}(q1, q2)\}}. \tag{26}$$

To extremize the relative manipulability during the motion of the dual manipulators system, the gradient should be computed. It can be shown that it is computed as

$$\nabla m = \begin{pmatrix} \dfrac{\partial m}{\partial q1} \\ \dfrac{\partial m}{\partial q2} \end{pmatrix}$$

and its element is given by

$$\frac{\partial m}{\partial q1_k} = m(q1, q2)\, \mathrm{trace}\left\{ \frac{\partial\, {}^{tool2}J_{tool1}}{\partial q1_k}\, {}^{tool2}J^{+}_{tool1} \right\} \tag{27}$$

and similarly for $q2_k$.

The proof is similar in the gradient of the single manipulator case. Therefore the computation of the derivative of the relative Jacobian ${}^{tool2}J_{tool1}$, counterpart of the hand Jacobian in single manipulator system, is necessary and its recursive computation is formulated as follows:

$$\frac{\partial U2_i}{\partial q1_k}=0 \tag{28a}$$

$$\begin{aligned} &\frac{\partial U1_{i+1}}{\partial q1_k}=0 && \text{if } k>i \\ &\frac{\partial U1_{i+1}}{\partial q1_k}=U1_i\cdot\frac{\partial\, {}^{i-1}A1_i(q1_i)}{\partial q1_k} && \text{if } k=i \\ &\frac{\partial U1_{i+1}}{\partial q1_k}=\frac{\partial U1_i}{\partial q1_k}\cdot {}^{i-1}A1_i && \text{if } k<i \end{aligned} \tag{28b}$$

where

$$\frac{\partial U2_{n2+1}}{\partial q1_k}=0,\quad \frac{\partial U1_1}{\partial q1_k}=0 \tag{29}$$

and

$$\begin{aligned} &\frac{\partial U2_i}{\partial q2_k}=0 && \text{if } k<i \\ &\frac{\partial U2_i}{\partial q2_k}=U2_{i+1}\cdot\frac{\partial\, {}^{i-1}A2_i^{-1}(q2_i)}{\partial q2_k} && \text{if } k=i \\ &\frac{\partial U2_i}{\partial q2_k}=\frac{\partial U2_{i+1}}{\partial q2_k}\cdot {}^{i-1}A2_i^{-1} && \text{if } k>i \end{aligned} \tag{30a}$$

$$\frac{\partial U1_{i+1}}{\partial q2_k}=\frac{\partial U1_i}{\partial q2_k}\cdot {}^{i-1}A1_i \tag{30b}$$

where

$$\begin{aligned} &\frac{\partial U2_{n2+1}}{\partial q2_k}=0 \\ &\frac{\partial U1_1}{\partial q2_k}=\frac{\partial U2_1}{\partial q2_k}\cdot {}^0A2_{base}^{-1}\cdot {}^0A1_{base}. \end{aligned} \tag{31}$$

Then i-th column of $\frac{\partial\, {}^{tool2}J_{tool1}}{\partial q1_k}$ and $\frac{\partial\, {}^{tool2}J_{tool1}}{\partial q2_k}$ can be similarly defined using (18) in the single manipulator system.

So far, the method to analytically compute the gradient of the manipulability measure for general redundant manipulator systems including dual manipulators system was discussed. To incorporate the gradient into the inverse kinematic solution, the following section will briefly explain one method of efficient inverse kinematics algorithms.

VI. Inverse kinematics of redundant manipulator system

The inverse kinematics based on (1) can be equivalently expressed as

$$\dot{q}=[R(q)(J(q)R(q))^{-1}\ N(q)(Z(q)N(q))^{-1}]\begin{pmatrix}\dot{p}\\ \dot{n}\end{pmatrix}$$

$$=\begin{bmatrix}J(q)\\ Z(q)\end{bmatrix}^{-1}\begin{pmatrix}\dot{p}\\ \dot{n}\end{pmatrix} \tag{32}$$

based on the kinematically decoupled joint space decomposition [10], where $\dot{n}\in R^r(r=n-m)$ is the desired null velocity given by

$$\dot{n}=\kappa Z(q)\nabla m \tag{33}$$

The matrices $R(q)\in R^{n\times m}$ and $N(q)\in R^{n\times r}$ are obtained by the singular value decomposition of $J(q)$

$$J=U[\Sigma\ 0]\,[R\ N]^T \tag{34}$$

where $\Sigma=\text{diag}\{\sigma_1,\cdots,\sigma_m\}$ is the diagonal matrix of the singular values of J. It is easy to see that the manipulability (4) is equal to [5]

$$m(q)=\sigma_1\sigma_2\cdots\sigma_m. \tag{35}$$

The null space matrix $Z(q)\in R^{r\times n}$ given in (3) can be computed using only square minors of the Jacobian by the following lemma.

Lemma 4.1 : Let J be rearranged so that the first square minor is nonsingular. Then the element of the null space matrix Z is derived by

$$\begin{aligned} Z_{i,j} &= \det[J^{(1)}\cdots J^{(j-1)}\ J^{(m+i)}\ J^{(j+1)}\cdots J^{(m)}] \\ Z_{i,m+i} &= -\det[J^{(1)}\cdots J^{(j-1)}\ J^{(j)}\ J^{(j+1)}\cdots J^{(m)}] \end{aligned}$$

for $1\le i\le r$ and $1\le j\le m$, with the other elements set to be zero, where $J^{(i)}$ denotes the i-th column of the matrix J.

Remark 4.1 : For example, consider the 3-by-5 Jacobian matrix whose first three columns are nonsingular. Then the null space bases matrix $Z\in R^{2\times5}$ by the above Lemma is given by

$$Z=\begin{bmatrix}Z_{11} & Z_{12} & Z_{13} & Z_{14} & 0\\ Z_{21} & Z_{22} & Z_{23} & 0 & Z_{25}\end{bmatrix}$$

Table 1. DH parameters of the right arm.

#	1	2	3	4	5	6	7	8	9
$a(m)$	0	l_2	0	0	0	0	0	0	l_9
$d(m)$	l_1	0	0	0	l_3	0	l_4	0	0
$\alpha(\circ)$	-90	90	-90	-90	90	90	-90	90	0
$\theta(\circ)$	-45	0	45	0	-45	90	0	-90	0

Table 2. DH parameters of the left arm

#	1	2	3	4	5	6	7	8	9
$a(m)$	0	l_2	0	0	0	0	0	0	l_9
$d(m)$	$-l_1$	0	0	0	$-l_3$	0	$-l_4$	0	0
$\alpha(\circ)$	-90	90	-90	90	90	90	-90	-90	0
$\theta(\circ)$	45	0	45	0	-45	-90	0	90	0

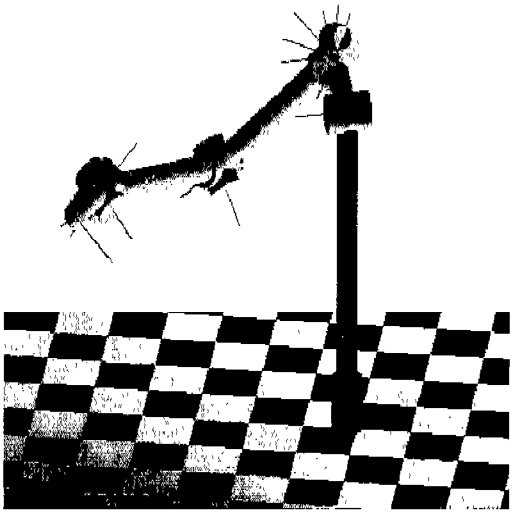
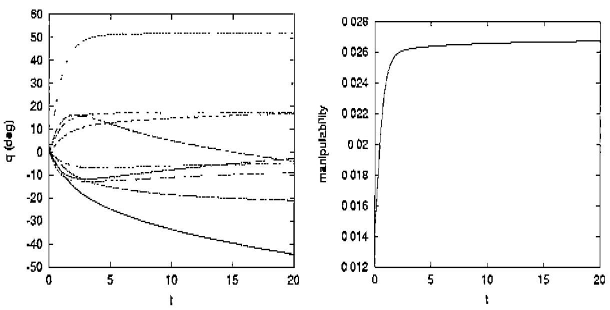


Fig. 3. A 9 dof manipulator.



(a) Joint trajectory (b) Manipulability variation

Fig. 4. Self-motion of a 9 dof arm.

where

$$\begin{aligned}
Z_{14} &= Z_{25} = -\det\{[J^{(1)} \quad J^{(2)} \quad J^{(3)}]\} \\
Z_{11} &= \det\{[J^{(4)} \quad J^{(2)} \quad J^{(3)}]\} \\
Z_{12} &= \det\{[J^{(1)} \quad J^{(4)} \quad J^{(3)}]\} \\
Z_{13} &= \det\{[J^{(1)} \quad J^{(2)} \quad J^{(4)}]\} \\
Z_{21} &= \det\{[J^{(5)} \quad J^{(2)} \quad J^{(3)}]\} \\
Z_{22} &= \det\{[J^{(1)} \quad J^{(5)} \quad J^{(3)}]\} \\
Z_{23} &= \det\{[J^{(1)} \quad J^{(2)} \quad J^{(5)}]\}.
\end{aligned} \tag{36}$$

V. Numerical example

A nine degrees of freedom manipulator shown in Fig. 3 whose DH parameters are summarized in Table 1 is simulated. The arm is commanded to do self-motion using (33) to focus on the ability of manipulability optimization. It was shown that the null motion generate by the gradient projection method drives the arm toward a local extremum asymptotically[11]. The manipulability was shown in Fig. 4, which verifies that the arm converges to a (local) maximum of the manipulability as shown in Fig. 4.

Now a 9 dof left manipulator was added to coordination task in relative kinematics method. The DH parameters are shown in Table 2. Two manipulators, the left of which grips the sphere object and the right of which is to execute some task, are shown in Fig. 5 at their initial configuration. To find a maximum manipulability configuration, the self-motion using the gradient of the manipulability was taken. Hence, currently the desired relative task velocity is zero, i.e. $\dot{r}_d = 0$ Thanks to the analytic recursive feature of the proposed computational method, no difficulty arises in implementing relative inverse kinematics of the dual manipulators system. The results are shown in Fig. 6.

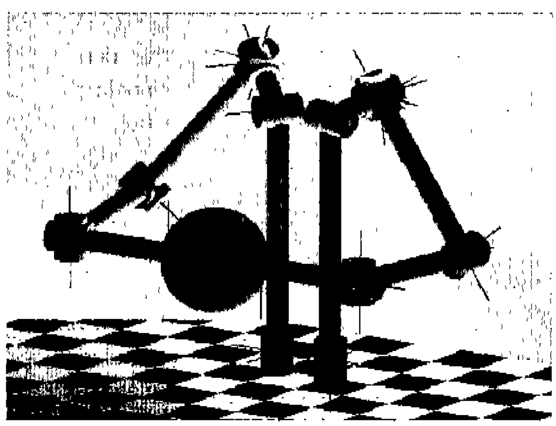
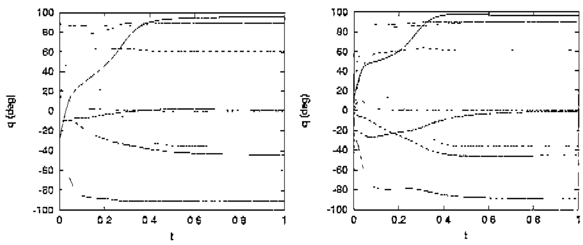
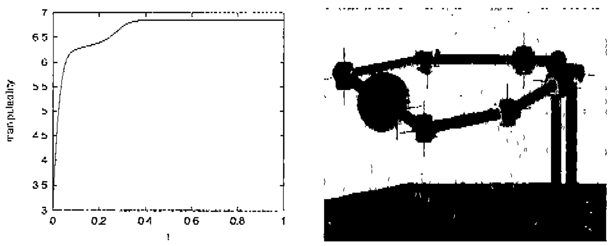


Fig. 5. A dual manipulator system employing two 9 dof manipulator.



(a) Joint trajectory of the right one (b) Joint trajectory of the left one



(c) Relative manipulability variation (d) Optimal configuration

Fig. 6. Self-motion of a dual manipulator system employing two 9 dof manipulators.

VI. Conclusion

Till now, for general type of redundant manipulators with arbitrary degrees of redundancy, calculation of the gradient of various dexterity measures, for example, the manipulability measure, is much difficult, since there are no methods available to compute it analytically. We have developed an algorithm to recursively compute the Jacobian and the derivative of Jacobian, and the gradient of the manipulability mea sure is automatically computed using them. The algorithm is shown to be easily extended to relative kinematics of dual manipulator system. Based on the algorithm, the gradient of a different dexterity measure can be obtained. Using these algorithms, the inverse kinematics of any redundant manipulator system can be executed automatically with the capability of performance measure optimization. A simulation with a nine degrees of freedom spatial manipulator and a dual manipulator system employing two 9 degrees of freedom spatial manipulators exhibited the expected ease in implementing the inverse kinematic task.

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Appendix : A. Proof of Lemma 2.1 and 2.2

Using the derivative formula of the determinant function

$$\frac{\partial \det\{A(x)\}}{\partial x} = \text{trace}\left\{\frac{\partial A}{\partial x}\,\text{adj}\,\{A\}\right\}$$

it is easy to show that

$$\begin{aligned}
&\frac{\partial \det\{J_{\text{MAN}}J^T_{\text{MAN}}\}}{\partial q_k}\\
&= \text{trace}\left\{\frac{\partial J_{\text{MAN}}J^T_{\text{MAN}}}{\partial q_k}\,\text{adj}\,\{J_{\text{MAN}}J^T_{\text{MAN}}\}\right\}\\
&= \det\{J_{\text{MAN}}J^T_{\text{MAN}}\}\,\text{trace}\left\{\frac{\partial J_{\text{MAN}}J^T_{\text{MAN}}}{\partial q_k}(J_{\text{MAN}}J^T_{\text{MAN}})^{-1}\right\}\\
&= \det\{J_{\text{MAN}}J^T_{\text{MAN}}\}\,\text{trace}\left\{\frac{\partial J_{\text{MAN}}}{\partial q_k}J^T_{\text{MAN}}(J_{\text{MAN}}J^T_{\text{MAN}})^{-1}\right.\\
&\quad\left.+J_{\text{MAN}}\frac{\partial J^T_{\text{MAN}}}{\partial q_k}(J_{\text{MAN}}J^T_{\text{MAN}})^{-1}\right\}\\
&= 2\det\{J_{\text{MAN}}J^T_{\text{MAN}}\}\,\text{trace}\left\{\frac{\partial J_{\text{MAN}}}{\partial q_k}J^T_{\text{MAN}}(J_{\text{MAN}}J^T_{\text{MAN}})^{-1}\right\}
\end{aligned} \tag{37}$$

where trace $\{AB\}$ = trace $\{BA\}$ and trace$\{A+B\}$ = trace $\{A\}$ + trace $\{B\}$ are used.

Note that

$$\begin{aligned}
&\text{trace}\left\{\frac{\partial J_{\text{MAN}}}{\partial q_k}J^T_{\text{MAN}}(J_{\text{MAN}}J^T_{\text{MAN}})^{-1}\right\}\\
&= \text{trace}\left\{T\frac{\partial J_{\text{HAND}}}{\partial q_k}J^+_{\text{HAND}}T^T\right\} + \text{trace}\left\{\frac{\partial T}{\partial q_k}T^T\right\}\\
&= \text{trace}\left\{\frac{\partial J_{\text{HAND}}}{\partial q_k}J^+_{\text{HAND}}\right\}
\end{aligned} \tag{38}$$

where $T=\begin{bmatrix} {}^{\text{ref}}R_{\text{tool}}(q) & 0 \\ 0 & {}^{\text{ref}}R_{\text{tool}}(q)\end{bmatrix}$, since

$$\text{trace}\left\{\frac{\partial T}{\partial q_k}T^T\right\} = \text{trace}\left\{\frac{\partial T}{\partial q_k}\,\text{adj}\,\{T\}\right\} = \frac{\partial \det\{T\}}{\partial q_k} = 0,$$

and tracer$\{\cdot\}$ is invariant under the similarity transform, as well as det$\{\cdot\}$. From (37) and (38), we have

$$\begin{aligned}
&\frac{\partial \det\{J_{\text{MAN}}J^T_{\text{MAN}}\}}{\partial q_k}\\
&= 2\det\{J_{\text{HAND}}J^T_{\text{HAND}}\}\,\text{trace}\left\{\frac{\partial J_{\text{HAND}}}{\partial q_k}J^+_{\text{HAND}}\right\}\\
&= \frac{\partial \det\{J_{\text{HAND}}J^T_{\text{HAND}}\}}{\partial q_k}
\end{aligned}$$

which is the proof of Lemma 2.1.

Since

$$\begin{aligned}
\frac{\partial m}{\partial q_k} &= \frac{1}{2\sqrt{\det\{J_{\text{MAN}}J^T_{\text{MAN}}\}}}\frac{\partial \det\{J_{\text{MAN}}J^T_{\text{MAN}}\}}{\partial q_k}\\
&= \sqrt{\det\{J_{\text{HAND}}J^T_{\text{HAND}}\}}\,\text{trace}\left\{\frac{\partial J_{\text{HAND}}}{\partial q_k}J^+_{\text{HAND}}\right\}
\end{aligned}$$

it proves Lemma 2.2.

Appendix : B. Proof of Lemma 4.1

It is obvious for $Z_{i,m+i}$ from (3). The proof of $Z_{i,j}$ is easy by careful manipulation of $J_2^T \operatorname{adj}\{J_1^T\}$. Observe that

$$(J_2^T)_{p,j} = J_{j,m+p}$$

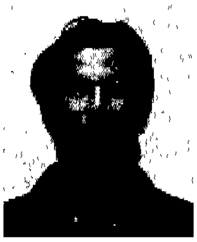
$$(\operatorname{adj}\{J_1^T\})_{j,q} = (-1)^{j+q}\det\{\hat{J}_{j,q}\}$$

where $\hat{J}_{j,q}$ is the matrix obtained by deleting j-th row and q-th column of J_1. Then for $1 \le p \le r$ and $1 \le q \le m$

$$Z_{p,q} = \sum_{j=1}^{m}(-1)^{j+q} J_{j,m+p} \det\{\hat{J}_{j,q}\}$$

$$= \det\begin{bmatrix} J_{1,1} & \cdots & J_{1,q-1} & J_{1,m+p} & J_{1,q+1} & \cdots & J_{1,m} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ J_{j-1,1} & \cdots & J_{j-1,q-1} & J_{j-1,m+p} & J_{j-1,q+1} & \cdots & J_{j-1,m} \\ J_{j,1} & \cdots & J_{j,q-1} & J_{j,m+p} & J_{j,q+1} & \cdots & J_{j,m} \\ J_{j+1,1} & \cdots & J_{j+1,q-1} & J_{j+1,m+p} & J_{j+1,q+1} & \cdots & J_{j+1,m} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ J_{m,1} & \cdots & J_{m,q-1} & J_{m,m+p} & J_{m,q+1} & \cdots & J_{m,m} \end{bmatrix}$$

which concludes Lemma 4.1.



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