

# 견실한 $H_\infty$ FIR 필터를 이용한 불확실성 기동표적의 추적

論 文

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## Maneuvering Target Tracking in Uncertain Parameter Systems Using Robust $H_\infty$ FIR Filters

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**Abstract** - This paper deals with the maneuver detection and target tracking problem in uncertain parameter systems using a robust  $H_\infty$  FIR filter to improve the unacceptable tracking performance due to the parameter uncertainty. The tracking filter used in the current paper is based on the robust  $H_\infty$  FIR filter proposed by Kwon *et al.* [1,2] to estimate the state signal in uncertain systems with parameter uncertainty, and the basic scheme of the proposed method is the input estimation approach. Tracking performance of the maneuver detection and target tracking method proposed is compared with other techniques, Bogler algorithm [4] and FIR tracking filter [2], via some simulations to exemplify the good tracking performance of the proposed method over other techniques.

**Key Words** :Robust  $H_\infty$  FIR filter; maneuvering target tracking; parameter uncertainty; input estimation; Kalman filter.

### 1. Introduction

The tracking problem for maneuvering targets has many military and civilian applications and has received considerable attention in the literature. There exist several approaches for the tracking problem, for example, the limited memory filter [5,6] which weights late measurement data, the maneuver compensation method which uses maneuver decision logic, and the filter-bank method which uses Bayesian sum of sub-filter outputs. Among them, the most popular is the compensation method using decision logic, and it includes the Q compensation technique [7] which compensates process noise covariance, the variable dimension filtering technique [8,9] which convert the target model after the maneuver detection, and the input estimation technique [1, 2, 4, 10, 11] which estimates the magnitude of the maneuver input.

In conventional methods, the Kalman filter has been used to track the maneuvering target. Using Kalman filter, there are basic problems posed by the maneuvering target. The problem is that the system model of the target moving with constant velocity in a straight line is different from that of the actual target moving with

acceleration or maneuvering, that is, there exists a mismatch between the modeled target dynamics and the actual one. If there exist modeling errors in the target model, the estimation error will diverge. To solve the divergence problem, Kwon and Yoo [2] have suggested a tracking algorithm for maneuvering targets using FIR filtering techniques [15, 16] based on the input estimation approach, which are known to be robust to modeling errors, in order to overcome the filter divergence problem under model uncertainties. However, this method may suffer from tracking performance for uncertain systems with all parameter uncertainties, although this method was known to nice tracking performance for uncertain systems with only measurement errors.

As a solution of poor tracking performance for systems with all parameter uncertainties, we presents here a  $H_\infty$  estimation techniques which has been no divergence performance to worst case estimation problems. There exists a vast literature on  $H_\infty$  estimation, see for example [17]-[22]. However, the  $H_\infty$  filters proposed so far are mainly restricted to pure estimation problem for time-invariant perfect mathematical model systems. As one of them the robust  $H_\infty$  FIR filter in the sense that it was an  $H_\infty$  filter with the FIR structure for uncertain systems proposed by Kwon *et al.* [14]. It was shown that the robust  $H_\infty$  FIR filter always has a solution if the standard  $H_\infty$  filter exists on the finite horizon  $[j-N, j]$ . It

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was noted that the filter proposed works on the general time-varying systems with parameter uncertainties by moving the horizon.

Therefore, this paper deals with the issue of the new tracking a maneuvering target tracking in parameter uncertain systems using the robust  $H_\infty$  FIR filter. It will be shown that the robust  $H_\infty$  FIR tracking filter has a good target tracking performance if the maneuvering target systems simultaneously having the model and measurement uncertainty. This point will be one of the main contributions of the current paper. The input estimation approach will be used here for maneuver detection and target tracking. The target system is described by parameter uncertain linear discrete time-varying state-space model and the maneuver is represented by an abrupt step input whose size is unknown. The robust  $H_\infty$  FIR filter is used for state estimator which yields the residual for maneuver detection and the least squares technique is used for input estimation. Maneuver detection is performed by  $\chi^2$  test based on the distribution of the estimated input under a nonmaneuvering condition. When maneuver is detected by the robust  $H_\infty$  FIR filter, then the target tracking is performed by compensating for the maneuvering input. Various computer simulations show that the superior performance of the proposed method compared with Bogler algorithm [4] and optimal FIR filter algorithm [1, 2]. This paper will use the pseudo input acceleration target model which has been proposed by Sung [3].

This paper is organized as follows: In Section 2, the robust  $H_\infty$  FIR filter and system description are presented. The maneuver detection method based on the robust  $H_\infty$  FIR filtering and the associated target tracking scheme are presented in Section 3. Various computer simulations and results are shown in Section 4. Conclusions are summarized in Section 5.

## 2. The Robust $H_\infty$ FIR Filter and Target System Models

### 2.1 Target system model

Let us consider discrete-time uncertain linear time-varying systems with state-space model of the form

$$x(i+1) = [A(i) + \Delta A(i)]x(i) + B(i)w(i) \quad (1)$$

$$y(i) = [C(i) + \Delta C(i)]x(i) + D(i)w(i) \quad (2)$$

$$z(i) = L(i)x(i) \quad (3)$$

where  $x(i) \in R^n$  is the state,  $w(i) \in R^r$  is the noise which belongs to  $l_2[0, \infty)$ ,  $y(i) \in R^m$  is the measured output,  $z(i) \in R^p$  is a linear combination of the state variables to be estimated,  $A, B, C, D$  and  $L$  are known time-varying real bounded matrices that describe the nominal system and that satisfies the condition

$$D(i)B^T(i) = 0, \quad D(i)D^T(i) > 0 \quad \forall i \quad (4)$$

and  $\Delta A(i)$  and  $\Delta C(i)$  represent the time-varying parameter uncertainties. These uncertainties are assumed to be in the following structure:

$$\begin{bmatrix} \Delta A(i) \\ \Delta C(i) \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F(i)E \quad (5)$$

with  $F(i) \in R^{r \times s}$  being an unknown real time-varying matrix function an satisfying

$$F^T(i)F(i) \leq I \quad \forall i \quad (6)$$

where  $H_1, H_2$  and  $E$  are known real constant matrices with appropriate dimensions. Here, the superscript ' $T$ ' denotes the transpose,  $I$  denotes the identity matrix with appropriate dimension, and the notation  $X \geq Y (X > Y)$  means that  $X - Y$  is positive semidefinite (respectively, positive definite).

### 2.2 The robust $H_\infty$ FIR filter

The nominal model in the target tracking problem and the robust  $H_\infty$  FIR filtering problems are solved in this subsection. Using the matching condition (5), the parameterized system which corresponds to the system (1)-(3), under a nonmaneuvering condition is derived as follows:

$$x(i+1) = \hat{A}x(i) + \hat{B}\hat{w}(i) \quad (7)$$

$$y(i) = \hat{C}x(i) + \hat{D}\hat{w}(i) \quad (8)$$

$$z(i) = Lx(i) \quad (9)$$

where  $\hat{A} = A + \gamma_i^{-2} \hat{B} \hat{B}^T Q(i, 0)A$

$$\hat{B} = \bar{B}[I - \gamma_i^{-2} \bar{B}^T Q(i, 0)\bar{B}]^{-1/2}$$

$$\hat{C} = C + \gamma_i^{-2} \hat{D} \hat{B}^T Q(i, 0)A$$

$$\hat{D} = \bar{D}[I - \gamma_i^{-2} \bar{B}^T Q(i, 0)\bar{B}]^{-1/2}$$

$$\bar{B} = \begin{bmatrix} B & \gamma_i & H_1 \\ \epsilon_i & & \end{bmatrix}, \quad \bar{D} = \begin{bmatrix} D & \gamma_i & H_2 \\ \epsilon_i & & \end{bmatrix}$$

and  $Q(i, 0)$  is the solution of the following difference Riccati equation

$$Q(i, n-1) = A^T Q(i, n)A + \gamma_i^{-2} A^T Q(i, n) \bar{B} [I - \gamma_i^{-2} \bar{B}^T Q(i, n)\bar{B}]^{-1} \bar{B}^T Q(i, n)A + \epsilon_i^2 E^T E \quad (10)$$

$$Q(i, N) = 0, \quad 1 \leq n \leq N$$

such that  $I - \gamma_i^{-2} \bar{B}^T Q(i, n)\bar{B} > 0$  for all  $n$ . And

$\hat{w}(i) \in R^{q+r}$  is a noise signal which belongs to  $l_2[0, \infty)$ ,  $\epsilon_i > 0$  is a scaling parameter to be chosen on each horizon  $[i-N, i]$ , and  $\gamma_i > 0$  is the disturbance attenuation level to be achieved for the robust  $H_\infty$  estimation problem.

The robust  $H_\infty$  FIR filter for the state  $x(\cdot)$  in eq.(7)-(9) is defined by the form

$$\hat{x}(i | N) = \sum_{k=i-N}^i H(i, k, N)y(k) \quad (11)$$

where  $H(i, \cdot; N)$  is the finite impulse response with the finite duration  $N$ . The filter of (11) and (14) is a one-step-ahead predictor since it estimates the state at the time point  $i+1$  based on the observation on  $[i-N, i]$ . The estimation error is here defined by

$$e(i+1) = z(i+1) - \hat{z}(i+1 | i, N) \quad (12)$$

Then the robust  $H_\infty$  FIR filtering problem is formulated as follows: Given the system (1)-(3) and a prescribed level of noise attenuation  $\gamma_i > 0$  on each horizon  $[i-N, i]$ , find an estimation for  $z(i)$  of the FIR structure (11)-(14) such that the estimation error is quadratically stable and satisfies the  $H_\infty$  performance  $\|e(i+1)\|^2 < \gamma_i^2 [\|w\|_{l_2}^2 + x_0^T P_0^{-1} x_0]$  for any non-zero  $w(\cdot) \in l_2[0, \infty)$  and for all admissible uncertainties  $F(\cdot)$

where  $\|e\|^2 = e^T e$ ,  
 $x_0 = x(i-N)$ ,  
 $P_0 = P(t-T, t-T) = Cov[x(i-N)]$

denotes the usual  $l_2$ -norm on the horizon  $[i-N, i]$ .

Assume that the system of (7)-(9) is uniformly completely observable and the system matrix  $\hat{A}$  is nonsingular. Then the robust  $H_\infty$  FIR filter (11) becomes time-invariant. In this case, the filter can be represented as:

$$\hat{x}(i+1 | i, N) = \sum_{k=i-N}^i H(i-k, N)y(k) \quad (13)$$

$$\hat{z}(i+1 | i, N) = L(i+1) \hat{x}(i | N) \quad (14)$$

And the impulse response of the robust  $H_\infty$  FIR filter is determined as follows:

$$H(i, j, N) = S^{-1}(i, N+1)L(i, j, N), \quad i-N \leq j \leq i \quad (15)$$

$$L(i, j, n) = S(i, n+1) \bar{S}^{-1}(i, n) \hat{A}^{-T} L(i, j, n-1), \quad (16)$$

$$0 \leq N-i+j+1 \leq n \leq N$$

$$L(i, j, N-i+j) = S(i, N-i+j+1) \hat{A} S^{-1}(i, N-i+j) \hat{C}^T [\hat{C} S^{-1}(i, N-i+j) \hat{C}^T + \hat{D} \hat{D}^T]^{-1}$$

$$S(i, n+1) = \bar{S}(i, n) - \bar{S}(i, n) \hat{B} [I + \hat{B}^T \bar{S}(i, n) \hat{B}]^{-1} \hat{B}^T \bar{S}(i, n) - \gamma_i^{-2} L^T L \quad (17)$$

$$S(i, -1) = \beta^{-2} I - \gamma_i^{-2} L^T L, \quad -1 \leq n \leq N$$

where

$$\bar{S}(i, n) = \hat{A}^{-T} [S(i, n) + \hat{C}^T (\hat{D} \hat{D}^T)^{-1} \hat{C}] \hat{A}^{-1}$$

It is noted that  $S(N)$  in Eq.(17) is the estimation error covariance of the robust  $H_\infty$  FIR filter in the worst case, i.e.,

$$\hat{P}(i+1) \equiv E[x(i+1) - \hat{x}(i+1 | N)] [x(i+1) - \hat{x}(i+1 | N)]^T \leq S(i, N)$$

It would be impractical to assume the initial condition on each moving horizon. Even if the initial state covariance  $P(t-T, t-T)$  is unknown, the robust  $H_\infty$  FIR filter is still applicable to this case by taking  $P(t-T, t-T) = \beta^2 I$  with  $\beta$  as a design parameter.

Since the robust  $H_\infty$  FIR filter (11) or (13) has been designed for the system (7)-(9) under the nonmaneuvering condition, it must be modified to account for maneuver. The target system model, under a maneuvering condition, can be described as follows:

$$x(i+1) = \hat{A}x(i) + \hat{B}\hat{w}(i) + Gu(i) \quad (18)$$

$$y(i) = \hat{C}x(i) + \hat{D}\hat{w}(i) \quad (19)$$

$$z(i) = Lx(i) \quad (20)$$

where  $Gu(i)$  is the maneuver input. The robust  $H_\infty$  FIR filter for maneuvering target system is described in the following result:

Corollary 2.1 [14] Assume that the system of (18), (19) and (20) are uniformly completely observable and the system matrix  $\hat{A}$  is nonsingular. Then there exists a solution to the  $H_\infty$  FIR filtering problem with the initial state covariance  $P(t-T, t-T) = \beta^2 I$ , if the following conditions are satisfied: (a) There exists a solution  $Q(i, n) \geq 0$  to the difference Riccati equation (10) such that  $I - \gamma_i^{-2} \bar{B}^T Q(i, n) \bar{B} > 0$  for all  $n$ ; (b) There exist a bounded symmetric matrix  $S(i, n) > 0$  for all  $n \geq 0$  which satisfies the Riccati difference equation,

$$S(i, n+1) = \bar{S}(i, n) - \bar{S}(i, n) \hat{B} [I + \hat{B}^T \bar{S}(i, n) \hat{B}]^{-1} \hat{B}^T \bar{S}(i, n) - \gamma_i^{-2} L^T L \quad (21)$$

$$S(i, -1) = \beta^{-2} I - \gamma_i^{-2} L^T L, \quad -1 \leq n \leq N$$

where  $\bar{S}(i, n) = \hat{A}^{-T} [S(i, n) + \hat{C}^T (\hat{D} \hat{D}^T)^{-1} \hat{C}] \hat{A}^{-1}$

Then FIR filter is determined as follows:

$$\hat{x}(i | N) = \sum_{k=1}^i [H(i, k, N) + H_u(i, k, N)]y(k) \quad (22)$$

$$\hat{z}(i+1 | i, N) = L(i)\hat{x}(i | N)$$

where the impulse responses  $H(i, \cdot; N)$  and  $H_u(i, \cdot; N)$  are calculated by

$$H(i, j, N) = S^{-1}(i, N+1)L(i, j, N), \quad i-N \leq j \leq i \quad (23)$$

$$L(i, j, n) = S(i, n+1) \bar{S}^{-1}(i, n) \hat{A}^{-T} L(i, j, n-1), \quad (24)$$

$$0 \leq N-i+j+1 \leq n \leq N$$

$$L(i, j, N-i+j) = S(i, N-i+j+1) \hat{A} S^{-1}(i, N-i+j) \hat{C}^T [\hat{C} S^{-1}(i, N-i+j) \hat{C}^T + \hat{D} \hat{D}^T]^{-1}$$

and

$$H_u(i, k, N) = \sum_{j=i-M}^k H(i, j, N) \hat{C} \Phi(j, k) G \quad (25)$$

where  $\Phi(\cdot, \cdot)$  is the state transition matrix of  $\hat{A}$ .

The robust  $H_\infty$  FIR filter problem in the above corollary is similarly formulated as that of Kwon et al. [15,16]. Namely, under assumptions given above, it is to find the optimal estimate, which has the FIR structure and the minimum variance criterion, for the state  $x(\cdot)$  of the system (18) at the current time  $i$  using the observation data  $z[i-N, i]$  and the control input data  $u[i-N, i]$  over the finite preceding interval. Note that, in Corollary 2.1 the width  $N$  of the observation interval can be taken any finite value greater than or equal to the observability index  $l_0$ . The estimation error covariance of the filter (22) is  $S(i, N)^{-1}$  which is obtained from (21). Nonsingularity of  $S(i, N)$  is guaranteed by the uniform complete observability of the system (18), (19) and (20).

The robust  $H_\infty$  FIR filter (18) of Corollary 2.1 does not require any statistical information about the initial state  $x(i-N)$ . Therefore, we can use it in the case there is no information about the initial state by setting  $P(i-N, i-N) = \beta^2$ . By the way, since it is not practical to assume that we have the information about the initial state at each interval  $[i-N, i]$ , it can be said that the robust  $H_\infty$  FIR filter (22) of Corollary 2.1 has the advantage that it always becomes time-invariant whenever the system of (18), (19) and (20) is time-invariant. The impulse response of the robust  $H_\infty$  FIR filter (22) is determined by computing on the interval  $[0, N]$  only once, it has the very simple form.

### 3. Maneuver Detection and Target Tracking

In this section, we present the maneuver detection method based on the the robust  $H_\infty$  FIR filter. We also

describe the associated target tracking method which employs maneuvering input compensation.

#### 3.1 Maneuver detection method

The residual value of the nominal robust  $H_\infty$  FIR filter (13) is defined, at time  $i$  under the nonmaneuvering condition, as follows:

$$r(i) \equiv z(i) - \hat{z}(i) = z(i) - L(i)\hat{x}(i+1 | i, N) \quad (26)$$

If the target began a maneuver at time  $[i-M-1]$  and if the maneuvering input were known and we used the robust  $H_\infty$  FIR filter (13), the residual value, under the maneuvering condition, can be represented by

$$\begin{aligned} r_m(i) &= z(i) - L(i)\hat{x}_m(i | N) \\ &= r(i) - L(i) \sum_{k=i-M}^i H_u(i-k, N) u_1 \\ &= r(i) - \varphi_M u_1 \end{aligned} \quad (27)$$

$$\text{where } \varphi_M = L(i) \sum_{k=0}^M H_u(k, N)$$

$M$  is the width of data window for maneuver detection and taken here as  $M \leq N$ . The second equality of (27) comes from the assumption that the input is constant over the time interval  $[i-M-1, i]$ , i.e.,

$$u(k) = u_1(k) \equiv \begin{cases} 0, & k < i-M-1 \\ u_1, & k \geq i-M-1 \end{cases} \quad (28)$$

Since (27) holds on the interval  $[i-M, i]$ , we have

$$r(k) = \varphi_M u_1 + r_m(k), \quad i-M \leq k \leq i \quad (29)$$

The residual  $r_m(\cdot)$  is a zero-mean white sequence. Therefore, from (29), we obtain the least squares estimate for the maneuvering input  $u$  as follows:

$$\hat{u}_1 = [\Psi^T \Psi]^{-1} \Psi^T R_r(i) \quad (30)$$

$$\text{where } \Psi \equiv \begin{bmatrix} \varphi_M \\ \varphi_{M-1} \\ \vdots \\ \varphi_0 \end{bmatrix}, \quad R_r(i) = \begin{bmatrix} r(i) \\ r(i-1) \\ \vdots \\ r(i-M) \end{bmatrix} \quad (31)$$

Based on the estimate  $\hat{u}_1$  of (30), we can detect the maneuver by the test variable

$$T_1(i) \equiv \hat{u}_1^T \{E[\hat{u}_1 \hat{u}_1^T]\}^{-1} \hat{u}_1 \quad (32)$$

Lemma 3.1 Under the nonmaneuvering condition, the estimated input  $u_1$  has zero mean and covariance

$$E[\hat{u}_1 \hat{u}_1^T] = [\Psi^T \Psi]^{-1} \Pi_r [\Psi^T \Psi]^{-1} \quad (33)$$

$$\text{where } \Pi_r \equiv \varphi_M^T \Gamma_r \varphi_M + \dots + \varphi_0^T \Gamma_r \varphi_0$$

$$\Gamma_r \equiv E[r(k)r(k)^T] = L(i)S^{-1}(N)L(i)^T + R \quad (34)$$

Proof : If there were no maneuver, the residual  $r(\cdot)$  forms a zero-mean white sequence with covariance (34) and we have from (30)

$$E[\hat{u}_1 \hat{u}_1^T] = [\Psi^T \Psi]^{-1} \{ \Psi^T E[R, R^T] \Psi \} \quad (35)$$

Under the nonmaneuvering condition, we have

$$\Psi^T E[R, R^T] \Psi = \varphi_M^T \Gamma_r \varphi_M + \dots + \varphi_0^T \Gamma_r \varphi_0 = \Pi_r \quad (36)$$

Hence the proof is completed.

By lemma 3.1, the test variable  $T_1(i)$  can be also calculated as follows:

$$T_1(i) = R_r^T \Psi \Pi_r^{-1} \Psi^T R_r \quad (37)$$

which is derived from substituting (30) and (33) into (32). The test variable (32) is based on the comparison between the estimated value of the maneuver input and the expected value under the nonmaneuvering condition. If  $T_1(i)$  is greater than a threshold value, we can say that a maneuver occurred at time  $(i-M-1)$ . But if not, it is said that there was no maneuver. When the target model is perfect, we can find the threshold value from table with  $n$  degrees of freedom since  $T_1(i)$  forms  $\chi^2$  distribution. However, if the target model had modeling errors, the threshold value would be different from that of  $\chi^2$  table and should be selected via prior experiments.

### 3.2 Target tracking method

After detecting the maneuver, we should compensate the maneuvering input to track the target. If the maneuver is detected at time  $i$ , the robust  $H_\infty$  FIR filter is switched to (38) using the maneuvering input estimate  $\hat{u}_1$  of (30):

$$\begin{aligned} \hat{x}_c(i | N) &= \sum_{k=i-N}^i [H(i-k, N)y(k) \\ &+ H_u(i-k, N) \hat{u}_1(k-1)] \\ &= \hat{x}(i | N) + \sum_{k=i-N}^i H_u(i-k, N) \hat{u}_1(k-1) \end{aligned} \quad (38)$$

where

$$\hat{u}_1(k) = \begin{cases} 0, & k < i-M-1 \\ \hat{u}_1, & k \geq i-M-1 \end{cases} \quad (39)$$

In (38),  $\hat{x}_c$  denotes the compensating filter using the input estimate  $\hat{u}_1$  instead of the unknown input  $u$  in (25). The residual value of the compensating filter (38) is defined by

$$r_c(j) \equiv z(j) - \hat{z}(j) = z(j) - L(j) \hat{x}_c(j | N), \quad j \geq i+1 \quad (40)$$

If the target began another maneuver at time  $(j-M-1)$ , the residual value, under another maneuvering condition, can be represented by

$$r_c(k) = \varphi_{M-k+j} u_c + r_m(k), \quad j-M \leq k \leq j \quad (41)$$

where  $u(k) = u_1(k) + u_c(k)$  and

$$u_c(k) = \begin{cases} 0, & k < j-M-1 \\ u_c, & k \geq j-M-1 \end{cases} \quad (42)$$

Using the least squares estimate for  $u_c$

$$\hat{u}_c = [\Psi^T \Psi]^{-1} \Psi^T R_c \quad (43)$$

$$\text{where } R_c \equiv \begin{bmatrix} r_c(j) \\ r_c(j-1) \\ \vdots \\ r_c(j-M) \end{bmatrix}$$

we can determine, by the test variable  $T_c$ , whether there was another maneuver or not:

$$T_c(j) \equiv \hat{u}_c^T \{ E[\hat{u}_c \hat{u}_c^T] \}^{-1} \hat{u}_c = R_c^T \Psi \Pi_c^{-1} \Psi^T R_c \quad (44)$$

where  $\Pi_c \equiv \varphi_M^T \Gamma_c \varphi_M + \dots + \varphi_0^T \Gamma_c \varphi_0$

$$\Gamma_c \equiv E[r_c(k)r_c(k)^T] = \Gamma_r \varphi_M (\Psi^T \Psi)^{-1} \Pi_r (\Psi^T \Psi)^{-1} \varphi_M$$

Eq. (44) is derived similarly as (37) in lemma 3.1.

If the test variable  $T_c(j)$  is greater than a threshold value  $\lambda_c$ , we can say that another maneuver occurred at time  $(j-M-1)$ . In that case, the maneuvering input estimate can be updated as follows:

$$\hat{u}(k) = \begin{cases} \hat{u}_1(k), & k < j-M-1 \\ \hat{u}_2, & k \geq j-M-1 \end{cases} \quad (45)$$

where

$$\hat{u}_2 = [\Psi^T \Psi]^{-1} \Psi^T R_r(j) \quad (46)$$

Repeating the procedure presented above, we can detect subsequent maneuvers and thus continue to track the maneuvering target continuously.

Bogler [4] and Chan et al. [10] have proposed target tracking methods using input estimation. Their methods are similar in concept as that presented here. But they are different in that the latter utilizes only the single robust  $H_\infty$  FIR filter and uses the input estimate directly as the test variable for the maneuver detection whereas the former methods use Kalman filter bank and are based on the residual statistics. Note that their methods are not robust in uncertain systems since they do not account for modeling errors. However, the proposed method here is not only suitable for real-time applications owing to its simple scheme but also robust to modeling errors due to the FIR structure.

## 4. Simulation

The performance of the robust  $H_\infty$  FIR tracking filter is here exemplified, via some simulation examples, in

comparison with Bogler algorithm [4] and FIR tracking filter [2], which are a typical tracking method with the input estimation procedure. It has been applied to the target moves in a 3-dimensional space and its dynamics are given by Sung [3] which has the nominal system matrices as follows:

$$A = \begin{bmatrix} I_3 & TI_3 & \tau^2(-1 + T/\tau + e^{-T/\tau})I_3 \\ 0_3 & I_3 & \tau(1 - e^{-T/\tau})I_3 \\ 0_3 & 0_3 & e^{-T/\tau}I_3 \end{bmatrix}, \quad B = \begin{bmatrix} 0_3 \\ 0_3 \\ I_3 \end{bmatrix},$$

$$G = \begin{bmatrix} [T^2/2 - \tau^2(-1 + T/\tau + e^{-T/\tau})]I_3 \\ [T - \tau(1 - e^{-T/\tau})]I_3 \\ (1 - e^{-T/\tau})I_3 \end{bmatrix}$$

$$C = [I_3 \ 0_3 \ 0_3], \quad L = [I_3 \ 0_3 \ 0_3]$$

where the sampling time  $T$  is taken as 1sec, and  $I_n$  and  $0_n$  denote  $n \times n$  identity and zero matrix, respectively. The system noise covariance and measurement noise covariance are taken as follows:

$$Q = 2\sigma_m^2 \tau \begin{bmatrix} q_{11}I_3 & q_{12}I_3 & q_{13}I_3 \\ q_{12}I_3 & q_{22}I_3 & q_{23}I_3 \\ q_{13}I_3 & q_{23}I_3 & q_{33}I_3 \end{bmatrix}, \quad R = I_9$$

$$q_{11} = \frac{\tau^5}{2} \left( 1 + \frac{2T}{\tau} - \frac{2T^2}{\tau^2} + \frac{2T^3}{3\tau^3} - e^{-2T/\tau} - \frac{4T}{\tau} e^{-T/\tau} \right)$$

$$q_{12} = \frac{\tau^4}{2} \left( 1 - \frac{2T}{\tau} + \frac{T^2}{\tau^2} + e^{-2T/\tau} - 2e^{-T/\tau} + \frac{2T}{\tau} e^{-T/\tau} \right)$$

$$q_{13} = \frac{\tau^3}{2} \left( 1 - e^{-2T/\tau} - \frac{2T}{\tau} e^{-T/\tau} \right)$$

$$q_{22} = \frac{\tau^3}{2} \left( -3 + \frac{2T}{\tau} - e^{-T/\tau} + 4e^{-T/\tau} \right)$$

$$q_{23} = \frac{\tau^2}{2} \left( 1 + e^{-2T/\tau} - 2e^{-T/\tau} \right)$$

$$q_{33} = \frac{\tau^2}{2} \left( 1 - e^{-2T/\tau} \right)$$

The deviation  $\sigma_m$  and the maneuver time constant  $\tau$  are taken as  $\sigma_m = \sqrt{5}m/sec^2$  and  $\tau = 10sec$ , respectively. Its velocity components along the  $x$ ,  $y$ , and  $z$  axes are assumed to be  $x_i = y_i = z_i = 0.0m/sec$  and the initial position of the target is assumed to be  $x_0 = y_0 = z_0 = 0.0m/sec$ .

The nominal system to be estimated is stable. The parameter uncertainty is taken as follows:

$$\Delta A = H_1 FE, \quad \Delta C = H_2 FE$$

$$H_1 = \begin{bmatrix} 000 & 111 & 111 \\ 111 & 111 & 000 \\ 111 & 000 & 111 \end{bmatrix}^T, \quad H_2 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.01 & 0.02 & 0.03 & 0.02 & 0.03 & 0.04 & 0.03 & 0.04 & 0.05 \\ 0.01 & 0.02 & 0.03 & 0.02 & 0.03 & 0.04 & 0.03 & 0.04 & 0.05 \\ 0.01 & 0.02 & 0.03 & 0.02 & 0.03 & 0.04 & 0.03 & 0.04 & 0.05 \end{bmatrix}$$

and  $F = 0_3$  for certain systems or  $F = I_3$  for uncertain systems. The design parameters are chosen as  $\gamma = 0.5176$ ,  $\epsilon = 0.2561$ ,  $\beta = 0.9500$  and data observation window  $N = 10$  for the robust  $H_\infty$  FIR filter proposed here.

Fig. 1 shows that the target begins to maneuver at 40 seconds during 20 seconds. Fig. 2 and Fig. 3 show the RMS(Root Mean Square) errors of state estimation which are computed via 30 Monte Carlo simulations using the FIR tracking filter, Bogler algorithm [4] and the robust  $H_\infty$  FIR filter, respectively. It is shown in the simulation results that the robust  $H_\infty$  FIR filter, the FIR tracking filter and Bogler algorithm [4] have a good tracking performance when there is no parameter uncertainty. However, when there is parameter uncertainty, the FIR tracking filter and Bogler algorithm [4] fail to detect the maneuver and cannot track the target owing to divergence problem. On the other hand, the proposed method using the robust  $H_\infty$  FIR filter shows satisfactory tracking performance even under parameter uncertainty.

### 5. Conclusions

This paper has proposed the maneuver detection method using the robust  $H_\infty$  FIR filter for the target tracking in parameter uncertain systems. The robust  $H_\infty$  FIR filter given by Eq. (22) is the extension of previous works Kown et al. [14] which is the pure state estimation problem. The maneuver detection and the target tracking presented here are based on  $\chi^2$  test and input estimation. Since the Bogler algorithm and the FIR tracking filter have the divergence problem under parameter uncertainties, most previous maneuver detection and target tracking methods based on Kalman filter will not work well in parameter uncertain systems. The FIR tracking filter can be an alternative but the problem, in this case, is also the divergence problem. On the contrary, since the robust  $H_\infty$  FIR filter enables us to overcome the divergence problems, the proposed method has a satisfactory performance in maneuver detection and target tracking even under parameter uncertainties, which has been shown via computer simulations

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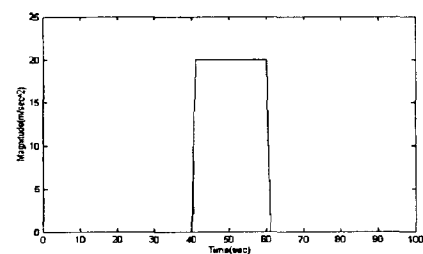
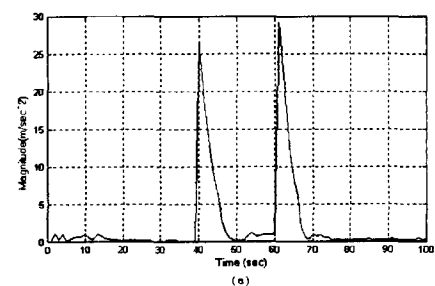
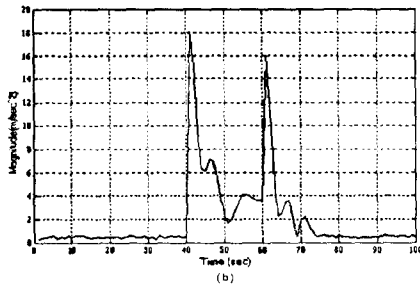


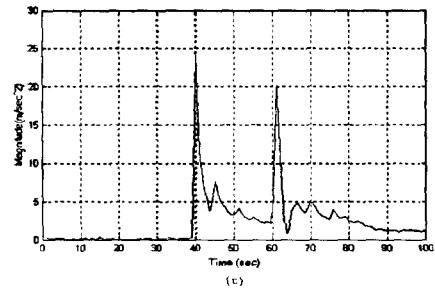
Figure 1. The Manuever input



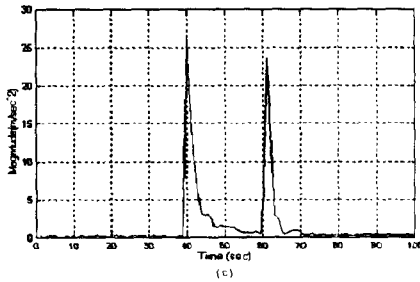
( a )



( b )



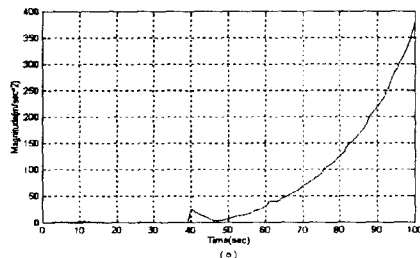
( c )



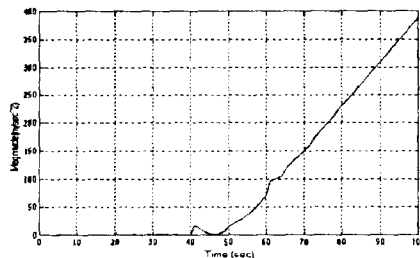
( c )

Figure 2. Tracking performance with no parameter uncertainty :

( a ) Bogler algorithm, ( b ) FIR tracking filter, and ( c ) Robust  $H_{\infty}$  FIR filter



( a )



( b )

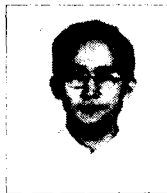
Figure 3. Tracking performance with parameter uncertainty :  
( a ) Bogler algorithm, ( b ) FIR tracking filter, and ( c ) Robust  $H_{\infty}$  FIR filter

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