Comm. Korean Math. Soc. **14** (1999), No. 2, pp. 405–423

PERFORMANCE ANALYSIS OF A STATISTICAL MULTIPLEXER WITH THREE-STATE BURSTY SOURCES

BONG DAE CHOI AND YONG WOOK JUNG

ABSTRACT. We consider a statistical multiplexer model with finite buffer capacity and a finite number of independent identical 3-state bursty voice sources. The burstiness of the sources is modeled by describing both two different active periods (at the rate of one packet per slot) and the passive periods during which no packets are generated.

Assuming a mixture of two geometric distributions for active period and a geometric distribution for passive period, we derive the recursive algorithm for the probability mass function of the buffer contents (in packets). We also obtain loss probability and the distribution of packet delay. Numerical results show that the system performance deteriorates considerably as the variance of the active period increases. Also, we see that the loss probability of 2-state Markov models is less than that of 3-state Markov models.

1. Introduction

We consider a statistical multiplexer model with finite buffer capacity M and a finite number N of independent identical three-state voice sources (see Fig. 1). This statistical multiplexer system is modeled as a discrete-time single server queueing system where input traffic is the superposition of N independent and identical three-state Markov models. In the slotted system, time is divided into fixed length intervals, called slot, such a way that one slot suffices to transmit exactly one packet from the multiplexer.

Received February 14, 1998. Revised February 5, 1999.

1991 Mathematics Subject Classification: 60K25, 60K30, 90B22.

Key words and phrases: statistical multiplexer, bursty sources, performance analysis.

This paper was in part supported by KOSEF (1995) and the research program from Ministry of Information and Communication.

The service discipline is assumed to be FCFS. Each source generates packets according to a 3-state Markov chain, where in any slot Markov chain is in one of three different states. There are two types of active periods, say A_1 and A_2, and one type of passive period. The time durations spent in the active period A_i and in the passive period are assumed to have geometrically distributed with parameter α_i and β, respectively $(i = 1, 2)$. The one packet per slot is generated in the active periods and no packets are generated in the passive period. At the end of each passive period, the next active period is of type A_1 with probability p and of type A_2 with probability $1 - p$, and at the end of each active period, the next period is passive period.

Extensive studies have been done on the statistical multiplexer with bursty sources, but most of studies have been limited to models with 2-state Markov models where each source alternates between one active period and one passive period [1, 2, 3, 5, 6, 7, 8, 9, 11, 12]. Experimental results [4, 10] showed that the probability mass function of a talkspurt duration is a mixture of two geometric distributions rather than one geometric distribution. This is why we assume that the duration of active period is a mixture of two geometric distribution functions in this paper.

Recently, Steyaert et al. [15] and Sohraby[12] have studied the statistical multiplexer model with infinite buffer capacity and a finite number of independent 3-state Markov models. Steyaert et al. [15] derived the mean buffer contents and mean packet delay. Sohraby [12] derived formula for a geometric tail approximation of the buffer contents. As far as a practical use, our assumption for finite capacity of buffer is more realistic than infinite capacity.

Our main purpose is to find the distribution of the buffer contents, loss probability and the distribution of the packet delay in the multiplexer.

The rest of this paper is organized as follows. In section 2, we analyze the arrival model. In section 3, we derive the distribution of the buffer contents. In section 4, we derive the loss probability. In section 5, we obtain the distribution of packet delay. Finally numerical examples are given in section 6.

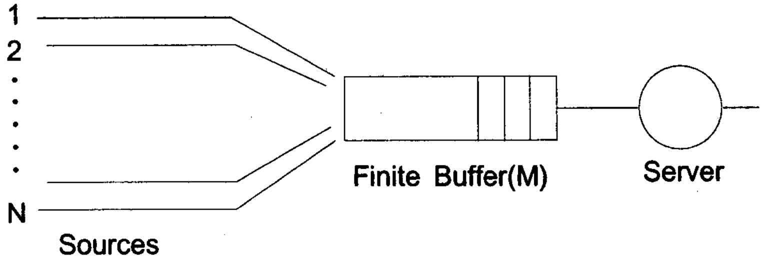


FIGURE 1. Statistical multiplexer

2. Arrival model

At any slot each source is in one of the active period of type A_1, the active period of type A_2 and passive period. Since the time durations spent in the active period of type $A_i (i = 1, 2)$ and in the passive period are assumed to have geometrically distributed with parameter $\alpha_i (i = 1, 2)$ and β, respectively, we have, for $n \geq 1$,

$$P\{\text{active period of type } A_1 \text{ is } n \text{ slots}\} = (1 - \alpha_1)\alpha_1^{n-1},$$
$$P\{\text{active period of type } A_2 \text{ is } n \text{ slots}\} = (1 - \alpha_2)\alpha_2^{n-1},$$
$$P\{\text{passive period is } n \text{ slots}\} = (1 - \beta)\beta^{n-1}.$$

At the end of each passive period, the next state is the active period of type A_1 with probability p and the active period of type A_2 with probability $1-p$, and at the end of each active period, the next period is passive. The state transition probability diagram is given in Fig. 2.

Note that the 3-state Markov chain for voice source is determined by four parameters α_1, α_2, β and p.

Among the N sources, we denote by $a_i(k)$ the random variable describing the number of active sources of type A_i at the k-th slot. Then the pair $(a_1(k), a_2(k))$ forms a homogeneous Markov chain. Let us define $a(j_1, j_2 | i_1, i_2)$ as the one-step transition probability that there are j_1 sources of active period (A_1) and j_2 sources of active period (A_2) at the k-th slot, if there were i_1 sources of active period (A_1) and i_2 sources of active periods (A_2) at the previous slot. With the definition in the previous

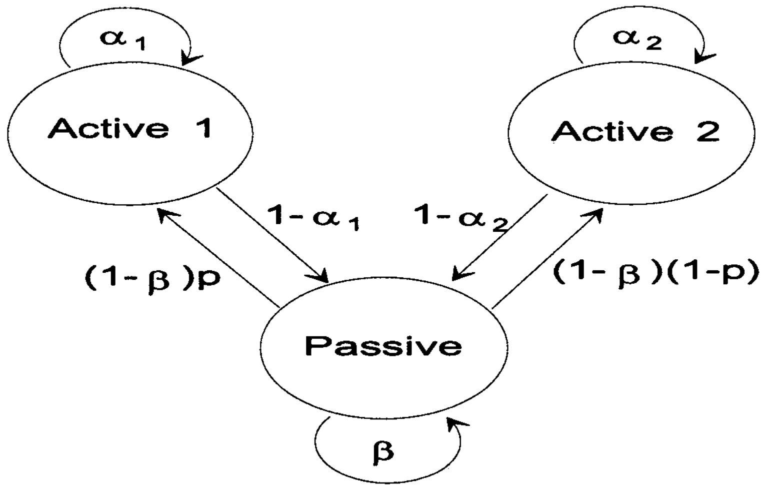


FIGURE 2. State transition probability diagram of a source

section, it is clear that

$$
\begin{aligned}
a(j_1, j_2|i_1, i_2) \; = \; & \sum_{k=(i_1+i_2+j_1-N)^+}^{(i_1,j_1)^-} \; \sum_{l=(i_1+i_2+j_1+j_2-N-k)^+}^{(i_2,j_2)^-} {}_{i_1}C_k \alpha_1^k (1-\alpha_1)^{i_1-k} \\
& \times {}_{N-i_1-i_2}C_{j_1-k}[(1-\beta)p]^{j_1-k} {}_{i_2}C_l \alpha_2^l (1-\alpha_2)^{i_2-l} \\
& \times {}_{N-i_1-i_2-j_1+k}C_{j_2-l}[(1-\beta)(1-p)]^{j_2-l} \beta^{N-i_1-i_2-j_1-j_2+k+l}, \qquad (1)
\end{aligned}
$$

where $(\cdot,\cdot)^- = \min(\cdot,\cdot)$, $(\cdot)^+ = \max(0,\cdot)$ and ${}_pC_q = \frac{p!}{q!(p-q)!}$.

Now, let us denote by $\pi(j_1, j_2)$ the steady-state probability that there are j_1 sources of active period(A_1) and j_2 sources of active period(A_2). Then $\pi(j_1, j_2)$ can be obtained from the following global balance equations.

$$
(2) \quad \pi(j_1, j_2) \; = \; \sum_{i_2=0}^{N} \sum_{i_1=0}^{N-i_2} \pi(i_1, i_2) a(j_1, j_2|i_1, i_2),
$$

$$
\text{for} \quad 0 \le j_1 + j_2 \le N, \quad j_1 \ge 0 \quad \text{and} \quad j_2 \ge 0.
$$

Together with the normalization condition $\sum_{\substack{(j_1,j_2)\\ 0\le j_1+j_2\le N}} \pi(j_1,j_2)=1$, it is easy to check that the solution of (2) is given by

$$\pi(j_1,j_2) = \frac{N!}{j_1!j_2!(N-j_1-j_2)!}\sigma_1^{j_1}\sigma_2^{j_2}(1-\sigma_1-\sigma_2)^{N-j_1-j_2}, \tag{3}$$

where

$$\sigma_1 = \frac{\frac{p}{1-\alpha_1}}{\frac{p}{1-\alpha_1}+\frac{1-p}{1-\alpha_2}+\frac{1}{1-\beta}}, \quad \sigma_2 = \frac{\frac{1-p}{1-\alpha_2}}{\frac{p}{1-\alpha_1}+\frac{1-p}{1-\alpha_2}+\frac{1}{1-\beta}}.$$

Note that $\frac{p}{1-\alpha_1}+\frac{1-p}{1-\alpha_2}$ and $\frac{1}{1-\beta}$ are the mean lengths of an active period and a passive period, respectively. Thus the quantity σ_1 and σ_2 are equal to the probability that any source is in active period of type A_1 and active period of type A_2 at an arbitrary slot, respectively.

3. Distribution of the buffer contents

In order to characterize the state of the multiplexer described in the section 1, a 3-dimensional state description is required. Let us denote by $v(k)$ the random variable indicating the buffer contents just after the k-th slot(i.e., at the beginning of the $(k+1)$-th slot). Then $(a_1(k),a_2(k),v(k))$ forms a Markov chain. We have the system equation as following.

$$v(k) = ((v(k-1)-1)^+ + a_1(k)+a_2(k), M)^-. \tag{4}$$

Defining the one-step transition probabilities of the Markov chain $\{(a_1(k), a_2(k), v(k))|k=1,2,\cdots\}$ as

$$\begin{aligned} p(j_1,j_2,n|i_1,i_2,m) \triangleq\ & P\{a_1(k)=j_1, a_2(k)=j_2, v(k)=n| \\ & a_1(k-1)=i_1, a_2(k-1)=i_2, v(k-1)=m\}. \end{aligned}$$

By applying the conditional probability, we have

$$\begin{aligned} (5)\ & p(j_1,j_2,n|i_1,i_2,m) \\ =\ & P\{v(k)=n|a_1(k)=j_1,a_2(k)=j_2,a_1(k-1)=i_1,a_2(k-1)=i_2, \\ & v(k-1)=m\}P\{a_1(k)=j_1,a_2(k)=j_2|a_1(k-1)=i_1, \\ & a_2(k-1)=i_2,v(k-1)=m\} \\ =\ & P\{v(k)=n|a_1(k)=j_1,a_2(k)=j_2,a_1(k-1)=i_1,a_2(k-1)=i_2 \\ & v(k-1)=m\}a(j_1,j_2|i_1,i_2). \end{aligned}$$

Using (4), we can rewrite (5) as

$$p(j_1, j_2, n|i_1, i_2, m)$$
$$= a(j_1, j_2|i_1, i_2)\delta[((m-1)^+ + j_1 + j_2, M)^- - n],$$

where $\delta(\cdot)$ is the kronecker-delta function.

Define the steady state distribution of $(a_1(k), a_2(k), v(k))$ as follows

$$\pi(j_1, j_2, n) \triangleq \lim_{k\to\infty} P[a_1(k) = j_1, a_2(k) = j_2, v(k) = n].$$

Then $\pi(j_1, j_2, n)$ satisfies the following the balance equations

$$(6) \qquad \pi(j_1, j_2, n) = \sum_{\substack{(i_1,i_2,m) \\ 0\le i_1+i_2\le(m,N)^- \\ 0\le m\le M}} p(j_1, j_2, n|i_1, i_2, m)\pi(i_1, i_2, m).$$

Together with the normalization condition, this is a set of linear equations for $\{\pi(j_1, j_2, n)\}$. In the following, we will develop a recursive algorithm to solve this set of equations, starting from the initial probability $\pi(0,0,0)$.

Combination of (6) and (7) yields

$$(7)\ \pi(j_1, j_2, n)$$
$$= a(j_1, j_2|0,0)\delta[(j_1 + j_2, M)^- - n]\pi(0,0,0)$$
$$+ \sum_{\substack{(i_1,i_2,m) \\ 0\le i_1+i_2\le(m,N)^- \\ 1\le m\le M}} a(j_1, j_2|i_1, i_2)\delta[(m - 1 + j_1 + j_2, M)^- - n] \times \pi(i_1, i_2, m),$$
$$\text{for } 0 \le n \le M, \quad 0 \le j_1 + j_2 \le (n, N)^-.$$

From this set of balance equations, we will derive a recursive algorithm for the determination of all $\pi(j_1, j_2, n)$. Specifically, our goal is to obtain $\pi(j_1, j_2, n)$ recursively from $\pi(i_1, i_2, m)$ for $m < n$, $0 \le i_1 + i_2 \le (m, N)^-$. First, we consider the case of $n = 1$. A set of three linear equations for the three unknown quantities $\pi(0,0,1)$, $\pi(1,0,1)$ and $\pi(0,1,1)$ is obtained by setting (j_1, j_2, n) being equal to $(1,0,1)$,$(0,1,1)$ and $(0,0,1)$ in equation

(8) as follows:

$$\begin{aligned}\pi(1,0,1) &= a(1,0|0,0)\pi(0,0,0)+a(1,0|0,0)\pi(0,0,1)\\ &\quad +a(1,0|1,0)\pi(1,0,1)+a(1,0|0,1)\pi(0,1,1),\end{aligned}$$

$$\begin{aligned}\pi(0,1,1) &= a(0,1|0,0)\pi(0,0,0)+a(0,1|0,0)\pi(0,0,1)\\ &\quad +a(0,1|1,0)\pi(1,0,1)+a(0,1|0,1)\pi(0,1,1),\end{aligned}$$

$$\begin{aligned}\pi(0,0,0) &= a(0,0|0,0)\pi(0,0,0)+a(0,0|0,0)\pi(0,0,1)\\ &\quad +a(0,0|1,0)\pi(1,0,1)+a(0,0|0,1)\pi(0,1,1).\end{aligned}$$

Solving these equations yields

$$\pi(0,0,1)=\frac{1}{A}\pi(0,0,0)$$

$$\times\begin{vmatrix} 1-a(0,0|0,0) & a(0,0|1,0) & a(0,0|0,1)\\ -a(1,0|0,0) & a(1,0|1,0)-1 & a(1,0|0,1)\\ -a(0,1|0,0) & a(0,1|1,0) & a(0,1|0,1)-1\end{vmatrix}, \tag{9a}$$

$$\pi(1,0,1)=\frac{1}{A}\pi(0,0,0)$$

$$\times\begin{vmatrix} a(0,0|0,0) & 1-a(1,0|0,0) & a(0,0|0,1)\\ a(1,0|0,0) & -a(1,0|0,0) & a(1,0|0,1)\\ a(0,1|0,0) & -a(0,1|0,0) & a(0,1|0,1)-1\end{vmatrix}, \tag{9b}$$

$$\pi(0,1,1)=\frac{1}{A}\pi(0,0,0)$$

$$\times\begin{vmatrix} a(0,0|0,0) & a(0,0|1,0) & 1-a(1,0|0,0)\\ a(1,0|0,0) & a(1,0|1,0)-1 & -a(1,0|0,0)\\ a(0,1|0,0) & a(0,1|1,0) & -a(0,1|0,0)\end{vmatrix}, \tag{9c}$$

where

$$A=\begin{vmatrix} a(0,0|0,0) & a(0,0|1,0) & a(0,0|0,1)\\ a(1,0|0,0) & a(1,0|1,0)-1 & a(1,0|0,1)\\ a(0,1|0,0) & a(0,1|1,0) & a(0,1|0,1)-1\end{vmatrix}.$$

Next, we consider the case: $2 \le n \le M-1$.

Case 1. $2 \le j_1 + j_2 \le (n, N)^-$

From equation (8), we have

$$\begin{aligned}
\pi(j_1, j_2, n) = {} & a(j_1, j_2|0,0)\delta[(j_1 + j_2, M)^- - n]\pi(0,0,0) \\
& + \sum_{\substack{(i_1,i_2,m) \\ 0 \le i_1+i_2 \le (m,N)^- \\ 1 \le m \le M+1-(j_1+j_2)}} a(j_1, j_2|i_1, i_2)\delta(m - 1 + j_1 + j_2 - n)\pi(i_1, i_2, m) \\
& + \sum_{\substack{(i_1,i_2,m) \\ 0 \le i_1+i_2 \le (m,N)^- \\ M+1-j_1-j_2 < m \le M}} a(j_1, j_2|i_1, i_2)\delta(M - n)\pi(i_1, i_2, m) \qquad (10) \\
= {} & a(j_1, j_2|0,0)\delta[(j_1 + j_2, M)^- - n]\pi(0,0,0) \\
& + \sum_{\substack{(i_1,i_2) \\ 0 \le i_1+i_2 \le (n+1-j_1-j_2,N)^-}} a(j_1, j_2|i_1, i_2)\pi(i_1, i_2, n + 1 - j_1 - j_2).
\end{aligned}$$

Equation (10) shows that $\pi(j_1, j_2, n)$ can be written by $\pi(i_1, i_2, m)$, $m \le n - 1$.

Case 2. $0 \le j_1 + j_2 \le 1$.

By setting (j_1, j_2) being equal to $(1,0)$ and $(0,1)$ in equation (8), we easily obtain two linear equations for $\pi(0,0,n)$, $\pi(1,0,n)$ and $\pi(0,1,n)$ in terms of the quantities that have already been determined with the equation (10):

$$\begin{aligned}
& a(1,0|0,0)\pi(0,0,n) + (a(1,0|1,0) - 1)\pi(1,0,n) \\
& \quad + a(1,0|0,1)\pi(0,1,n) = -B, \qquad (11a) \\
& a(0,1|0,0)\pi(0,0,n) + a(0,1|1,0)\pi(1,0,n) \\
& \quad + (a(0,1|0,1) - 1)\pi(0,1,n) = -C, \qquad (11b)
\end{aligned}$$

where

$$B \triangleq \sum_{\substack{(i_1,i_2) \\ 2 \le i_1+i_2 \le (n,N)^-}} a(1,0|i_1, i_2)\pi(i_1, i_2, n),$$

$$C \triangleq \sum_{\substack{(i_1,i_2) \\ 2 \le i_1+i_2 \le (n,N)^-}} a(i_1, i_2|0,1)\pi(i_1, i_2, n).$$

Finally, the third expression for these quantities can be obtained by substituting $(0,0,n-1)$ for (j_1,j_2,n) in equation (8), We thus obtain

$$\begin{aligned}&a(0,0|0,0)\pi(0,0,n)+a(0,0|1,0)\pi(1,0,n)+a(0,0|0,1)\pi(0,1,n)\\ &\qquad=\pi(0,0,n-1)-D,\end{aligned} \tag{11c}$$

where

$$D \triangleq \sum_{\substack{(i_1,i_2)\\ 2\le i_1+i_2\le (n,N)^-}} a(0,0|i_1,i_2)\pi(i_1,i_2,n).$$

It can be easily verified that the solution of (11a,b,c) is given by

$$\text{(12a)}\quad \pi(0,0,n)=\frac{1}{A}\begin{vmatrix}\pi(0,0,n-1)-D & a(0,0|1,0) & a(0,0|0,1)\\ -B & a(1,0|1,0)-1 & a(1,0|0,1)\\ -C & a(0,1|1,0) & a(0,1|0,1)-1\end{vmatrix},$$

$$\text{(12b)}\quad \pi(1,0,n)=\frac{1}{A}\begin{vmatrix}a(0,0|0,0) & \pi(0,0,n-1)-D & a(0,0|0,1)\\ a(1,0|0,0) & -B & a(1,0|0,1)\\ a(0,1|0,0) & -C & a(0,1|0,1)-1\end{vmatrix},$$

$$\text{(12c)}\quad \pi(0,1,n)=\frac{1}{A}\begin{vmatrix}a(0,0|0,0) & a(0,0|1,0) & \pi(0,0,n-1)-D\\ a(1,0|0,0) & a(1,0|1,0)-1 & -B\\ a(0,1|0,0) & a(0,1|1,0) & -C\end{vmatrix}.$$

From (9a,b,c), (10) and (12a,b,c), $\pi(j_1,j_2,l)$ can be written as

$$\pi(j_1,j_2,l)=\alpha(j_1,j_2,l)\pi(0,0,0),\quad 1\le l\le M-1. \tag{13}$$

On the other hand,

$$\pi(j_1,j_2)=\sum_{k=j_1+j_2}^{M}\pi(j_1,j_2,k). \tag{14}$$

By substituting $(0,0)$ for (j_1,j_2) in above equation (14) and using $\pi(0,0,M)=0$, we obtain

$$\pi(0,0)=\sum_{k=0}^{M}\pi(0,0,k)=\sum_{k=0}^{M-1}\pi(0,0,k). \tag{15}$$

From (3),(13) and (15), we can obtain the value $\pi(0,0,0)$.

From (3), (13) and (14), we can obtain all $\pi(j_1,j_2,M)$ for $0\le j_1+j_2\le M$.

Therefore we obtain all $\pi(j_1, j_2, n)$, $0 \le n \le M$, $0 \le j_1 + j_2 \le (n, N)^-$. The distribution of the buffer contents v at the steady state at the end of a slot is obtained

$$P\{v = n\} = \sum_{\substack{(j_1, j_2) \\ 0 \le j_1 + j_2 \le (N, n)^-}} \pi(j_1, j_2, n), \quad 0 \le n \le M.$$

4. Loss probability

To find loss probability, we need to calculate the probability distribution of the buffer contents at arrival instant of tagged packet.

Let $a_i^*(i = 1, 2)$ denote the total number of sources with active period(A_i) in the slot during which the tagged packet is arriving to the multiplexer buffer, and v^* the buffer contents that the arriving tagged packet sees.

Then

$$(16) \qquad r(i_1, i_2) \triangleq P\{a_1^* = i_1, a_2^* = i_2\} = \frac{(i_1 + i_2)\pi(i_1, i_2)}{N(\sigma_1 + \sigma_2)}.$$

Moreover, we have

$$\begin{aligned}
(17)\ & P\{v^* = k | a_1^* = i_1, a_2^* = i_2\} \\
&= P\{v(n) = k | a_1(n+1) = i_1, a_2(n+1) = i_2\} \\
&= \frac{P\{v(n) = k, a_1(n+1) = i_1, a_2(n+1) = i_2\}}{P\{a_1(n+1) = i_1, a_2(n+1) = i_2\}} \\
&= \frac{1}{P\{a_1(n+1) = i_1, a_2(n+1) = i_2\}} \\
&\times \sum_{\substack{(j_1, j_2) \\ 0 \le j_1 + j_2 \le (k, N)^-}} P\{a_1(n+1) = i_1, a_2(n+1) = i_2 | v(n) = k, a_1(n) = j_1, a_2(n) = j_2\} \\
&\times P\{a_1(n) = j_1, a_2(n) = j_2, v(n) = k\} \\
&= \frac{1}{\pi(i_1, i_2)} \sum_{\substack{(j_1, j_2) \\ 0 \le j_1 + j_2 \le (k, N)^-}} a(i_1, i_2 | j_1, j_2)\pi(j_1, j_2, k).
\end{aligned}$$

Let $P\{\text{served}\}$ be the probability that an arriving tagged packet is served. Then we have

$$P\{\text{served} | a_1^* = i_1, a_2^* = i_2, v^* = k\} = \frac{(i_1 + i_2, M - (k-1)^+)^-}{i_1 + i_2}. \tag{18}$$

Hence

$$P\{\text{served}\} = \sum_{k=0}^{M} \sum_{\substack{(i_1,i_2) \\ 1 \le i_1+i_2 \le N}} P\{\text{served} | a_1^* = i_1, a_2^* = i_2, v^* = k\} \times P\{v^* = k | a_1^* = i_1, a_2^* = i_2\} P\{a_1^* = i_1, a_2^* = i_2\}. \tag{19}$$

From (16), (17), (18) and (19), we obtain

$$P\{\text{served}\} = \frac{1}{N(\sigma_1 + \sigma_2)} \times \sum_{k=0}^{M} \sum_{\substack{(i_1,i_2) \\ 1 \le i_1+i_2 \le N}} \sum_{\substack{(j_1,j_2) \\ 0 \le j_1+j_2 \le (k,N)^-}} (i_1 + i_2, M - (k-1)^+)^- \times a(i_1, i_2 | j_1, j_2) \pi(j_1, j_2, k). \tag{20}$$

The loss probability of an arriving packet is given by

$$\begin{aligned} P\{\text{loss}\} &= 1 - P\{\text{served}\} \\ &= 1 - \frac{1}{N(\sigma_1 + \sigma_2)} \sum_{k=0}^{M} \sum_{\substack{(i_1,i_2) \\ 1 \le i_1+i_2 \le N}} \sum_{\substack{(j_1,j_2) \\ 0 \le j_1+j_2 \le (k,N)^-}} (i_1 + i_2, M - (k-1)^+)^- \\ &\times a(i_1, i_2 | j_1, j_2) \pi(j_1, j_2, k). \end{aligned}$$

5. Distribution of the packet delay

We define the packet delay W of a tagged packet as the number of slots between the end of the slot of arrival of this packet in the buffer and the end of the slot during which it is transmitted.In the following, we assume the service discipline is FCFS. We assume that the service order among the cells arriving during the same slot in the buffer is random.

For $(k-1)^+ + 1 \le l \le (k-1)^+ + (i_1 + i_2, M - (k-1)^+)^-$, we have

(21)

$$P\{W = l | a_1^* = i_1, a_2^* = i_2, v^* = k, \text{served}\} = \frac{1}{(i_1 + i_2, M - (k-1)^+)^-}.$$

$$(22)\ P\{W=l|\text{served}\} = \frac{1}{P\{\text{served}\}} \sum_{1\le i_1+i_2\le N} \sum_{k=0}^{M} P\{a_1^*=i_1, a_2^*=i_2\}$$
$$\times P\{W=l|a_1^*=i_1, a_2^*=i_2, v^*=k, \text{served}\}$$
$$\times P\{\text{served}|a_1^*=i_1, a_2^*=i_2, v^*=k\}$$
$$\times P\{v^*=k|a_1^*=i_1, a_2^*=i_2\}.$$

From (16), (17), (18), (21) and (22), we obtain

$$(23)\quad P\{W=l|\text{served}\}$$
$$= \frac{1}{N(\sigma_1+\sigma_2)P(\text{served})} \sum_{1\le i_1+i_2\le N} \sum_{(k-1)^+=(l-(i_1+i_2,M-(k-1)^+)^-)^+}^{l-1}$$
$$\sum_{0\le j_1+j_2\le (k,N)^-} a(i_1,i_2|j_1,j_2)\pi(j_1,j_2,k).$$

6. Numerical examples

Let us investigate the influence of the variance of the distribution of the active period and the influence of the parameters of a source on the buffer behavior. In order to do so, the following burstiness factors K and L are used [15].

$$K \triangleq (1-\sigma)\left(\frac{p}{1-\alpha_1}+\frac{1-p}{1-\alpha_2}\right),$$

$$L \triangleq \frac{(1-\sigma)^2}{K(K+\sigma-1)}\left[p(1-p)\left(\frac{1}{1-\alpha_1}-\frac{1}{1-\alpha_2}\right)^2 + p\frac{\alpha_1}{(1-\alpha_1)^2} + (1-p)\frac{\alpha_2}{(1-\alpha_2)^2}\right],$$

where $\sigma = \sigma_1+\sigma_2$.

Note that K is the ratio of the mean length of an active period in our model, to the mean length of an active period in the case that each source is active with probability σ and passive with probability $1-\sigma$. L is the ratio of the variance of an active period in our model, to the variance of a geometrically distributed active period with the same mean length $\frac{p}{1-\alpha_1} + \frac{1-p}{1-\alpha_2}$.

The impact of variance of active period(L) on the system performance measures such as loss probability and mean packet delay is illustrated in Fig. 3. Fig. 3 displays the loss probability, mean packet delay and tail distribution of buffer contents for given $\sigma = 0.1, p = 0.2, M = 5$ and $K = 5$ and varying L and number N of sources. It is shown that the influence of variance of active period is far from negligible. For increasing values of L, the system performance deteriorates considerably.

Fig. 4 illustrates the influence of K on the behavior of buffer. Fig. 4 displays the loss probability, mean packet delay and tail distribution of buffer contents for given $\sigma = 0.1, p = 0.2, M = 5$ and $L = 7$. Fig. 4 shows that loss probability and mean packet delay are less sensitive to K than L, but the influence of K on the performance measures is not negligible.

Note that 3-state Markov model is determined by exactly four parameters α_1, α_2, β and p. Next examples show how much the four parameters have an effect on performance of the system.

In Fig. 5, we investigate the variation of the performance measures as the buffer capacity increases. The parameters used in Fig. 5 are given as following:

- $(\alpha_1, \alpha_2, \beta, p) = (0.8, 0.4, 0.95, 0.1)$

In Fig. 5, we can easily find an efficient buffer capacity needed to satisfy a given QoS. Fig. 5(a) shows that buffer capacity 16 is enough for satisfying the loss probability less than 10^{-4} for 3 sources, and buffer capacity 17 is enough to satisfy the loss probability less than 10^{-3} for 5 sources. On the other hand, the buffer capacity must be less than 12 in order to satisfy the mean packet delay less than 3 for 7 sources.

Note that 2-state Markov model is a special case of 3-state Markov model with $p = 0$. The performance measures for 2-state Markov models and 3-state Markov models are compared in Fig. 6 and Fig. 7. The parameters used in Fig. 6– Fig. 7 are given as followings:

- $N = 3$
- $(\alpha_1, \alpha_2, \beta, p) = (0.8, 0.4, 0.95, 0.1)$ for 3-state Markov models
- $(\alpha_2, \beta) = (0.5, 0.95)$ for 2-state Markov models, where α_2 was chosen such a way that mean active period for both models is the same.

Fig. 6 displays a comparison of loss probabilities between 2-state Markov models and 3-state Markov models. As an example, Fig. 6 shows that buffer capacity for 2-state Markov model is 10 to satisfy the loss probability less than 10^{-4}, but buffer capacity for 3-state Markov model is 15 to satisfy the loss probability less than 10^{-4}

Fig. 7 displays a comparison of the mean packet delay between 2-state Markov models and 3-state Markov models and shows that mean packet delay is not significant.

References

[1] F. Bonomi, S. Montagna and R. Paglino, *A further look at statistical multiplexing in ATM networks*, Computer networks and ISDN system 26 (1993), 119-138.

[2] H. Bruneel, *Queueing behavior of statistical multiplexers with correlated inputs*, IEEE Trans. Commun. vol. Som-36 (12) (1988), 1339-1341.

[3] K. M. Elsayed, *On the superposition of discrete-time Markov renewal processes and Application to statistical multiplexing of bursty traffic sources*, IEEE GLOBECOM'94 No. 28 (1994), 1113-1117.

[4] J. G. Gruber, *A comparison of measured and calculated speech temporal parameters relevant to speech activity detection*, IEEE Trans. Commun. vol. Com-34 No. 4 (4) (1982), 728-737.

[5] H. Heffes and D. M. Lucantoni, *A Markov modulated characterization of packetized voice and data traffic and related statistical multiplexer performance*, IEEE J. Select. Areas Commun. vol. SAC-4 No. 6 (9) (1986), 856-868.

[6] G. U. Hwang and B. D. Choi, *Closed-form expressions on the geometric tail behavior of statistical multiplexers with heterogeneous traffic*, IEEE Transaction on Comm. vol. 46, No. 12 (1998), 1575-1579.

[7] I. Ide, *Superposition of interrupted poisson process and its application to packetized voice multiplexer*, ITC-12 (1989), 1399-1405.

[8] Y. H. Kim and C. K. Un, *Performance Analysis of statistical multiplexing for heterogeneous bursty traffic in an ATM network*, IEEE Trans. on Comm. 42 (2/3/4) (1994), 745-753.

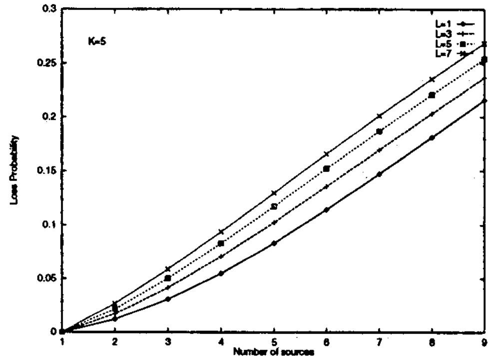
[9] D. S. Lee and S. Q. Li, *Transient analysis of a switched poisson arrival queue under overload control*, performance Evaluation 17 (1993), 13-29.

[10] H. H. Lee and C. K. Un, *A study of on-off characteristic of conversational speech*, IEEE Trans. Commun. vol. Com-34 No. 6(7) (1986), 630-637.

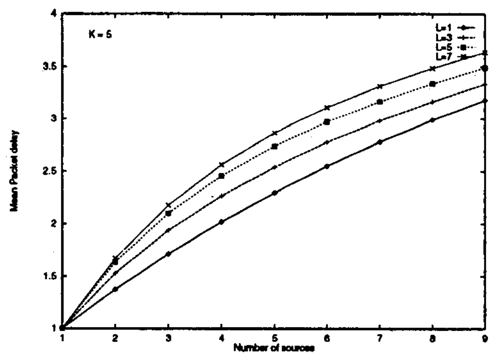
[11] N. M. Marafih, R. L. Pickholtz and Y. Q. Zhang, *A new approximation for analyzing the multiplexing of bursty sources in ATM networks*, Telecommun. Syst. 4 (1995), 3-31.

[12] K. Sohraby, *On the asymptotic behavior of heterogeneous statistical multiplexer with applications*, proceedings INFOCOM'92, 839-847.

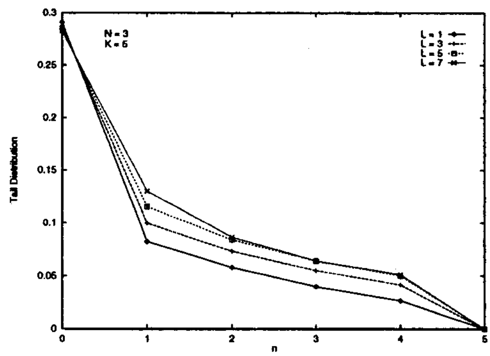
[13] ______, *Highly-bursty sources and their admission control in ATM networks*, Asynchronous Transfer Mode Networks, Y. viniotis and R. O. onvural (editors), New York: Plenum, 1993.



(a) Loss Probability

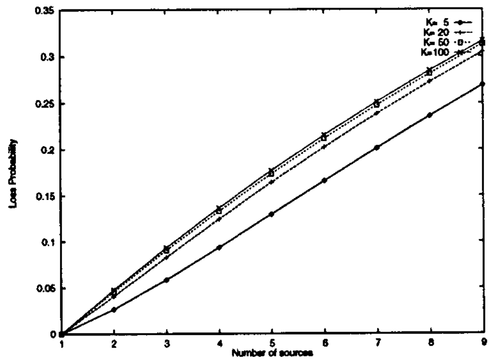


(b) Mean Packet Delay

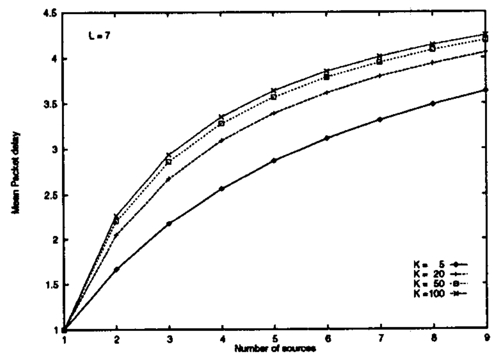


(c) Tail distribution

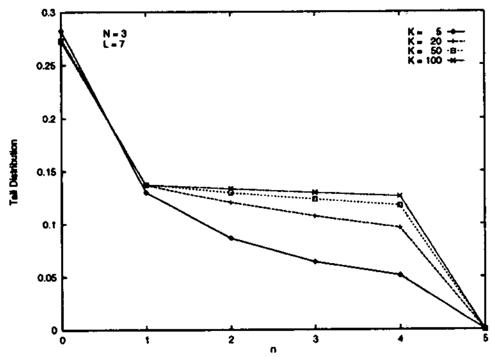
FIGURE 3. Influence of L for given $\sigma = 0.1, p = 0.2, M = 5, K = 5$



(a) Loss Probability

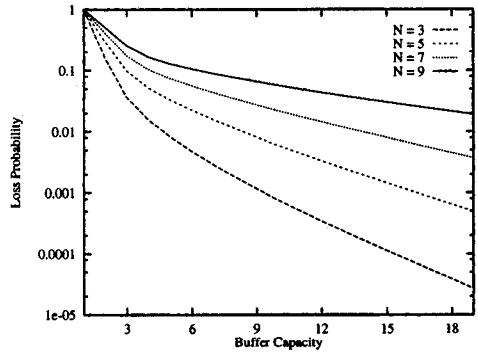


(b) Mean Packet Delay

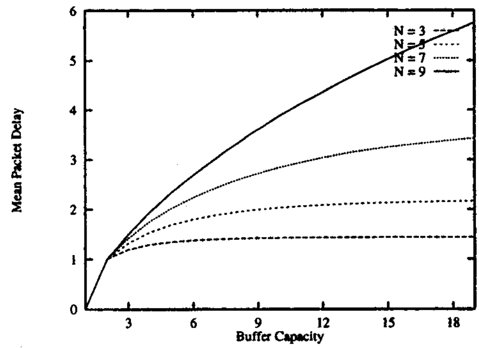


(c) Tail distribution

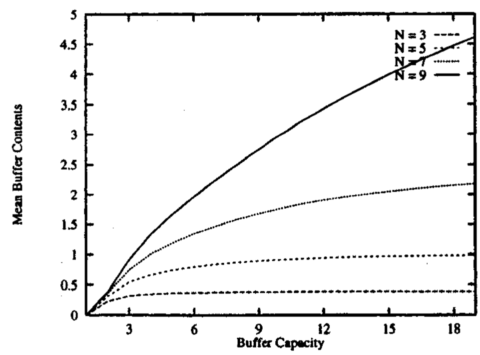
FIGURE 4. Influence of K for given $\sigma = 0.1, p = 0.2, M = 5, L = 7$



(a) Loss Probability v.s. Buffer Capacity



(b) Mean Packet Delay v.s. Buffer Capacity



(c) Mean Buffer Contents v.s. Buffer Capacity

FIGURE 5. $(\alpha_1, \alpha_2, \beta, p) = (0.8, 0.4, 0.95, 0.1)$

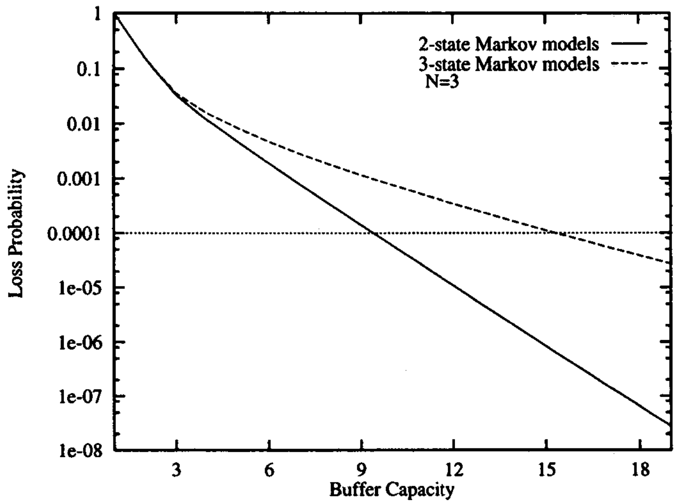


FIGURE 6. A Comparison of Loss probabilities between 2-state Markov models and 3-state Markov models

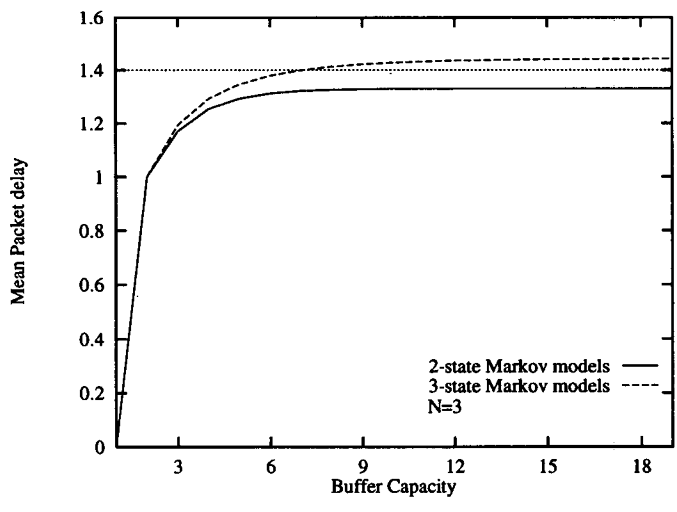


FIGURE 7. A Comparison of Packet delay between 2-state Markov models and 3-state Markov models

[14] B. Steyaert and H. Brunnel, *An effective algorithm to calculated the distribution of the buffer contents and the packet delay in a multiplexer with bursty sources,* IEEE GLOBECOM'91 (1991), 471-475.

[15] ———, *On the performance of multiplexers with three-state bursty sources: Analytic results*, IEEE Trans. Commun. vol. 43 No. 2/3/4 (2/3/4) (1995), 1299-1303.
[16] A. M. Viterbi, *Approximate analysis of time-synchronous packet networks*, IEEE J. Select. Areas Commun. vol. SAC-4 (9) (1986), 879-890.

Bong Dae Choi
Department of Mathematics
Korea University
1, Aanam-dong, Sungbuk-gu
Seoul 136-701, Korea
E-mail: bdchoi@semi.korea.ac.kr

Yong Wook Jung
Department of Mathematics
Kyonggi Institute of Technology
Shihung 429-450, Korea