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ON FUZZY T_2-AXIOMS

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ABSTRACT. Some fuzzy T_2-axioms are characterized in terms of the notion of fuzzy closure and the relationship between those fuzzy T_2-axioms are obtained. Also, finite fuzzy topological spaces satisfying one of those axioms are studied.

1. Introduction

Several fuzzy T_2-axioms have been defined in different ways and investigated by many authors, such as Hutton and Reilly [2], Ganguly and Saha [1], Sinha [3] and Srivastava [4]. In particular, Sinha characterized and compared some of them. The aim of this paper is to provide another equivalent conditions for these axioms and to study finite fuzzy topological spaces satisfying one of these axioms.

2. Basic definitions

Let X be a set of points and I the unit interval $[0,1]$. A *fuzzy set* A in X is a mapping from X into I. For $x \in X$ and $\lambda \in (0,1]$, a fuzzy set x_λ, defined by

$$x_\lambda(y) = \begin{cases} \lambda, & y = x \\ 0, & y \neq x \end{cases}$$

is called a *fuzzy point* in X.

Let 0_X and 1_X be, respectively, the constant fuzzy sets taking the values 0 and 1 on X.

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DEFINITION 2.1. A collection τ of fuzzy sets in X is called a *fuzzy topology* on X if

(1) 0_X and 1_X belong to τ,
(2) any union of members of τ is in τ and
(3) any finite intersection of members of τ is in τ.

Members of a fuzzy topology on X are called *fuzzy open sets* of X and their complements *fuzzy closed sets* of X. The intersection of all fuzzy closed sets containing A is called the *fuzzy closure* of A and denoted by $Cl(A)$.

Throughout this paper, a "fts X" means a set X equipped with a suitable fuzzy topology.

A fuzzy open set A in a fts X is called a *fuzzy open nbd* of a fuzzy point x_λ if $\lambda \leq A(x)$. A fuzzy set A in X is said to be *q-coincident* with a fuzzy set B, denoted by A_qB, if there exists $x \in X$ such that $A(x) + B(x) > 1$. For a fuzzy point x_λ in a fts X, a fuzzy set A is called a *fuzzy q-nbd* of x_λ if there exists a fuzzy open set U in X such that $x_{\lambda_q}U$ and $U(z) \leq A(z)$ for all $z \in X$.

DEFINITION 2.2 ([1]). A fts X is said to be *fuzzy GS-T_2* if for every pair of distinct fuzzy points x_α and y_β, the following conditions are satisfied:

(1) if $x \neq y$, then x_α and y_β have fuzzy open nbds which are not q-coincident.

(2) if $x = y$ and $\alpha < \beta$ (say), then x_β has a fuzzy q-nbd A and x_α has a fuzzy open nbd U such that $A_{\not q}U$.

DEFINITION 2.3. ([3]) A fts X is said to be *fuzzy S-T_2* if for any fuzzy point x_α,

$$x_\alpha = \bigcap_{U \in \mathcal{U}} Cl(U)$$

where $\mathcal{U}$ is the collection of fuzzy open nbds of x_α.

DEFINITION 2.4 ([2]). A fts X is said to be *fuzzy HR-T_2* if for every fuzzy set A in X, there exists a collection $\{U_{ij} | i \in I, j \in J_i\}$ of fuzzy open sets in X such that

$$A = \bigcup_{i \in I}\Big(\bigcap_{j \in J_i} U_{ij}\Big) = \bigcup_{i \in I}\Big(\bigcap_{j \in J_i} Cl(U_{ij})\Big).$$

DEFINITION 2.5 ([4]). An fts X is said to be *fuzzy SS-T_2* if for any distinct fuzzy points x_α and y_β with $x \neq y$, there exist fuzzy open nbds U and V of x_α and y_β, respectively, such that $U \cap V = 0_X$.

For definitions and notations which are not explained here, we refer to [1], [3] and [4].

3. Main results

THEOREM 3.1. *For a fts X, the following are equivalent:*

1. *X is fuzzy GS-T_2.*
2. *for any two distinct fuzzy points x_α and y_β: (1) if $x \neq y$, then there exist fuzzy open nbds U and V of x_α and y_β, respectively, such that $Cl(V) \leq 1_X - U$ and $Cl(U) \leq 1_X - V$; (2) if $x = y$ and $\alpha < \beta$ (say), then there exists a fuzzy open nbd U of x_α such that $x_\beta \notin Cl(U)$.*

PROOF. (1⇒2) Assume $x \neq y$. By condition (1) of Definition 2.2, there exist fuzzy open nbds U and V of x_α and y_β, respectively, such that $U_{\not q}V$. It then follows from $U_{\not q}V$ that $U(z) \leq 1 - V(z)$ and $V(z) \leq 1 - U(z)$ for all $z \in X$. Since $1_X - U$ and $1_X - V$ are fuzzy closed, we have $Cl(V) \leq 1_X - U$ and $Cl(U) \leq 1_X - V$.

Now, assume $x = y$. By condition (1) of Definition 2.2, there exist a fuzzy q-nbd A of x_β and a fuzzy open nbd U of x_α such that $A_{\not q}U$. Let V be a fuzzy open set in X such that $x_{\beta_q}V$ and $V(x) \leq A(x)$. Then, it follows from $\beta > 1 - V(x) = (Cl(1 - V))(x)$, $V(x) \leq A(x)$ and $U(x) \leq 1 - A(x)$ that $\beta > Cl(U)(x)$. That is, $x_\beta \notin Cl(U)$.

(2⇒1) Let x_α and y_β be distinct fuzzy points in X.

First, assume $x \neq y$. By condition (1) of 2, there exist fuzzy open nbds U and V of x_α and y_β, respectively, such that $Cl(V) \leq 1_X - U$. This inequality implies that for all $z \in X$, $U(z) + V(z) \leq (Cl(V))(z) + U(z) \leq 1$. That is, $U_{\not q}V$.

Now, assume $x = y$ and $\alpha < \beta$. By condition (2) of 2, there exists a fuzzy open nbd U of x_α such that $x_\beta \notin Cl(U)$. Let $A = 1_X - Cl(U)$. Since for all $z \in X$, $A(z) + U(z) \leq 1$, we conclude that $A_{\not q}U$. Furthermore, A is a fuzzy open set and $\beta + A(x) > \alpha + A(x) \geq 1$. Thus, A is a fuzzy q-nbd of x_β such that $A_{\not q}U$. □

THEOREM 3.2. *For a fts X, the following are equivalent:*

1. *X is fuzzy S-T_2*

2. *for any two distinct fuzzy points x_α and y_β: (1) if $x \neq y$, then there exist fuzzy open sets U_1, U_2, V_1, V_2 in X such that $x_\alpha \in U_1, y_{\beta q} V_1, U_{1\not q} V_1$ and $y_\beta \in V_2, x_{\alpha q} U_2, U_{2\not q} V_2$; (2) if $x = y$ and $\alpha < \beta$ (say), then there exist fuzzy open sets U and V in X such that $x_\alpha \in U, x_{\beta q} V$ and $U_{\not q} V$.*

3. *for any two distinct fuzzy points x_α and y_β: (1) if $x \neq y$, then there exist fuzzy open nbds U and V of x_α and y_β, respectively, such that $x_\alpha \notin Cl(V)$ and $y_\beta \notin Cl(U)$; (2) if $x = y$ and $\alpha < \beta$ (say), then there exists a fuzzy open nbd U of x_α such that $x_\beta \notin Cl(U)$.*

PROOF. (1⇔2) Theorem 2.4 of [3].

(2⇒3) Assume $x \neq y$. By condition (1) of 2, there exist fuzzy open sets U, V, W_1, W_2 in X such that $x_\alpha \in U, y_{\beta q} W_1, U_{\not q} W_1$ and $y_\beta \in V, x_{\alpha q} W_2, V_{\not q} W_2$. This implies that $x_\alpha \in U, \beta > (1_X - W_1)(y), U \leq 1_X - W_1$ and $y_\beta \in V, \alpha > (1_X - W_2)(x), V \leq 1_X - W_2$. Therefore, U and V are nbds of x_α and y_β, respectively, such that $Cl(U)(y) \leq [Cl(1_X - W_1)](y) = (1_X - W_1)(y) < \beta$ and $Cl(V)(x) \leq [Cl(1_X - W_2)](x) = (1_X - W_2)(x) < \alpha$.

Now, assume that $x = y$. By condition (2) of 2, there exist fuzzy open sets U and V in X such that $x_\alpha \in U, x_{\beta q} V$ and $U_{\not q} V$. Then $x_\alpha \in U, \beta > (1_X - V)(x)$ and $U \leq 1_X - V$. Consequently, U is a fuzzy open nbd of x_α such that $Cl(U)(x) \leq [Cl(1_X - V)](x) = (1_X - V)(x) < \beta$.

(3⇒2) Assume $x \neq y$. By condition (1) of 3, there exist fuzzy open nbds U_1 and V_2 of x_α and y_β, respectively, such that $x_\alpha \notin Cl(V_2)$ and $y_\beta \notin Cl(U_1)$. Let $V_1 = 1_X - Cl(U_1)$ and $U_2 = 1_X - Cl(V_2)$. Then V_1 and U_2 are fuzzy open sets in X such that $\beta + V_1(y) = \beta + [1 - Cl(U_1)(y)] > \beta + (1 - \beta) = 1, \alpha + U_2(y) = \alpha + [1 - Cl(V_2)(x)] > \alpha + (1 - \alpha) = 1, V_1 = 1_X - Cl(U_1) \leq 1_X - U_1, U_2 = 1_X - Cl(V_2) \leq 1_X - V_2$. That is, $x_\alpha \in U_1, y_{\beta q} V_1, U_{1\not q} V_1$ and $y_\beta \in V_2, x_{\alpha q} U_2, U_{2\not q} V_2$.

Now, assume $x = y$. By condition (2) of 3, there exists a fuzzy open nbd U of x_α such that $x_\beta \notin Cl(U)$. Let $V = 1_X - Cl(U)$. Then V is a fuzzy open set in X such that $V \leq 1_X - U$ and $\beta + V(x) = \beta + [1 - Cl(U)(x)] > \beta + (1 - \beta) = 1$. That is, $x_\alpha \in U, x_{\beta q} V$ and $U_{\not q} V$.□

THEOREM 3.3. *For an fts X, the following are equivalent:*

1. *X is fuzzy HR-T_2.*

2. *if* $\alpha \in (0,1)$ *and* $\mathcal{U}$ *is the collection of all fuzzy open nbds of* x_α*, then* $x_\alpha = \cap_{U \in \mathcal{U}} Cl(U)$.

3. *for any* $x, y \in X$ *and any* $\alpha, \beta \in (0,1)$*: (1) if* $x \neq y$*, then there exist fuzzy open nbds* U *and* V *of* x_α *and* y_β*, respectively, such that* $y_\beta \notin Cl(U)$ *and* $x_\alpha \notin Cl(V)$*; (2) if* $x = y$ *and* $\alpha < \beta$ *(say), then there exists a fuzzy open nbd* U *of* x_α *such that* $x_\beta \notin Cl(U)$.

4. *for any two distinct fuzzy points* x_α *and* y_β *with* $x \neq y$ *or else* $x = y$ *and* $\frac{1}{2} < \alpha < \beta$*, there exist fuzzy open sets* U *and* V *such that* $x_\alpha q U$*,* $x_\beta q V$ *and* $U \cap V \leq \frac{1}{2}$.

5. $\Delta = 1_X - \sup\{A \times B | A$ *and* B *are fuzzy sets and* $A \leq 1_X - B\}$ *is fuzzy closed in* $X \times X$.

PROOF. (1⇒2) By hypothesis, there exists a collection $\{V_{lk} | l \in L, k \in K_l\}$ of fuzzy open sets in X such that

$$1_X - x_\alpha = \bigcup_{l \in L} \Big(\bigcap_{k \in K_l} Cl(V_{lk}) \Big).$$

For every $l \in L$ and every $k \in K_l$, let $U_{lk} = 1_X - Cl(V_{lk})$. Then

$$x_\alpha = \bigcap_{l \in L} \Big(\bigcup_{k \in K_l} U_{lk} \Big).$$

Since for each $l \in L$, $\cup_{k \in K_l} U_{lk}$ is a fuzzy open neighborhood of x_α, we have

$$x_\alpha \in \bigcap_{U \in \mathcal{U}} U \leq \bigcap_{l \in L} \Big(\bigcup_{k \in K_l} U_{lk} \Big) = x_\alpha.$$

Thus,

$$x_\alpha = \bigcap_{U \in \mathcal{U}} U.$$

First, we shall prove that $(\cap_{U \in \mathcal{U}} Cl(U))(x) = \alpha$. To show this, assume to the contrary that $(\cap_{U \in \mathcal{U}} Cl(U))(x) = \beta > \alpha$. Then for all $U \in \mathcal{U}$, $(Cl(U))(x) \geq \beta$. Choose $\gamma \in (\alpha, \beta)$. By hypothesis, there exists a collection $\{V_{ij} | i \in I, j \in J_i\}$ of fuzzy open sets in X such that $x_\gamma = \cup_{i \in I}(\cap_{j \in J_i} V_{ij}) = \cup_{i \in I}(\cap_{j \in J_i} Cl(V_{ij}))$. Then there exists $i_0 \in I$ such that $\alpha \leq (\cap_{j \in J_{i_0}} V_{i_0 j})(x)$. This means that $\{V_{i_0 j} | j \in J_{i_0}\} \subset \mathcal{U}$. Since

$(\bigcap_{j\in J_{i_0}} Cl(V_{i_0j}))(x) \le \gamma$, there exists $j_0 \in J_{i_0}$ such that $(Cl(V_{i_0j_0}))(x) < \beta$. Consequently, we have an obvious contradiction that

$$\beta = \left(\bigcap_{U\in\mathcal{U}} Cl(U)\right)(x) \le \left(Cl(V_{i_0j_0})\right)(x) < \beta.$$

Thus

$$\left(\bigcap_{U\in\mathcal{U}} Cl(U)\right)(x) = \alpha.$$

Now, we shall prove $(\bigcap_{U\in\mathcal{U}} Cl(U))(y) = 0$ for all $y \in X - \{x\}$. By hypothesis, there exists a collection $\{U_{ij} | i \in I, j \in J_i\}$ of fuzzy open sets in X such that

$$x_1 = \bigcup_{i\in I}\left(\bigcap_{j\in J_i} U_{ij}\right) = \bigcup_{i\in I}\left(\bigcap_{j\in J_i} Cl(U_{ij})\right).$$

This means that $(\bigcap_{j\in J_i} Cl(U_{ij}))(y) = 0$ for all $i \in I$ and all $y \in X - \{x\}$ and that, for every $\beta \in [\alpha, 1)$, there exists $i_\beta \in I$ such that $\beta < (\bigcap_{j\in J_{i_\beta}} U_{i_\beta j})(x) \le 1$, i.e, $\{U_{i_\beta j} | j \in J_{i_\beta}\} \subset \mathcal{U}$. Let y be a point in $X - \{x\}$. For each positive integer n, choose $j_n \in J_{i_\beta}$ such that $(Cl(U_{i_\beta j_n}))(y) < 1/n$. Notice that each $U_{i_\beta j_n}$ is a member of $\mathcal{U}$ and $(\bigcap_n Cl(U_{i_\beta j_n}))(y) = 0$. Since y is arbitrary in $X - \{x\}$ and $\{U_{i_\beta j_n}\}$ is a subcollection of $\mathcal{U}$, we have

$$\left(\bigcap_{U\in\mathcal{U}} Cl(U)\right)(y) = 0 \text{ for all } y \in X - \{x\}.$$

(2⇒3) Clear.

(3⇒4) Assume that $x \ne y$. By condition (1) of 3, there exist fuzzy open nbds W and V of $x_{\frac{\alpha}{2}}$ and $y_{1-\frac{\beta}{2}}$, respectively, such that $(Cl(V))(x) < \frac{\alpha}{2}$ and $(Cl(W))(y) < 1 - \frac{\beta}{2}$. Let $U = 1_X - Cl(V)$. Then $x_{\alpha q} U$ and $y_{\beta q} V$. Furthermore, since $U_q V$, we have $U \cap V \le \frac{1}{2}$.

Now, assume that $x = y$ and $\frac{1}{2} < \alpha < \beta$. Choose $\epsilon > 0$ such that $\alpha < \beta - \epsilon$. By condition (2) of 3, there exists a fuzzy open nbd U of x_α such that $(Cl(U))(x) < \beta$. Let $V = 1_X - Cl(U)$. Then $x_{\alpha q} U$ and $y_{\beta q} V$. Also, $U \cap V \le \frac{1}{2}$ because $U_q V$.

(4⇒5) Theorem 2.14 of [3].

(5⇒1) Proposition 7 of [2]. □

THEOREM 3.4. *For any fts X, the following are equivalent:*

1. *X is fuzzy GS-T_2.*

2. *X is fuzzy S-T_2 and for $x, y \in X$ with $x \neq y$, there exists a fuzzy open nbd U of x_1 such that $Cl(U)(y) = 0$.*

3. *X is fuzzy HR-T_2 and for $x, y \in X$ with $x \neq y$, there exists a fuzzy open nbd U of x_1 such that $Cl(U)(y) = 0$.*

4. *for $x, y \in X$: (1) if $x \neq y$, then there exists a fuzzy open nbd U of x_1 such that $Cl(U)(y) = 0$; (2) if $x = y$ and $\alpha < \beta$, then there exists a fuzzy open nbd U of x_α such that $x_\beta \notin Cl(U)$.*

PROOF. (1$\Leftrightarrow$2) Theorem 2.11 of [3].

(2$\Rightarrow$3) Theorem 2.15 of [3].

(3$\Rightarrow$2) Let $x \in X$ and let $\mathcal{U}$ be the collection of all fuzzy open nbds of x_1. For each $y \in X - \{x\}$, choose a fuzzy open nbd U_y of x_1 such that $Cl(U_y)(y) = 0$. Then $x_1 = \cap_{y \in X - \{x\}} Cl(U_y)$. Since $\{U_y | y \in X - \{x\}\} \subset \mathcal{U}$, we have

$$\bigcap_{y \in X - \{x\}} Cl(U_y) = x_1 \leq \bigcap_{U \in \mathcal{U}} Cl(U) \leq \bigcap_{y \in X - \{x\}} Cl(U_y).$$

Therefore, $x_1 = \cap_{U \in \mathcal{U}} Cl(U)$. By combining this with Theorem 3.3, we obtain X is fuzzy S-T_2.

(3$\Leftrightarrow$4) Obvious by the definition of fuzzy HR-T_2 spaces. □

THEOREM 3.5. *For a fts X, the following are equivalent:*

1. *For every $x \in X$, x_1 is fuzzy closed.*

2. *For $x, y \in X$ with $x \neq y$, there exists a fuzzy open nbd U of x_1 such that $U(y) = 0$.*

PROOF. (1$\Rightarrow$ 2) Let x, y be distinct points of X and let $U = 1_X - y_1$. Then $U(x) = 1$ and $U(y) = 0$. Since y_1 is fuzzy closed in X, U is fuzzy open.

(2$\Rightarrow$1) Let $x \in X$. For every $y \neq x$, choose a fuzzy open set V_y such that $V_y(x) = 0$ and $V_y(y) = 1$. Let $V = \cup_{y \neq x} V_y$. Then

$$V(z) = \begin{cases} 0, & z = x \\ 1, & z \neq x. \end{cases}$$

Since V is fuzzy open in X, we have $x_1 = 1_X - V$ is fuzzy closed in X.□

THEOREM 3.6. *If X is fuzzy SS-T_2, then for every $x \in X$, x_1 is fuzzy closed in X.*

PROOF. Similar to the proof of (2⇒1) of Theorem 3.5. □

For a fuzzy set A in X,

$$A_0 = \{x \in X | A(x) > 0\}$$

is called the *support* of A. Note that for any fuzzy set A in X, $X - A_0 \subset (1_X - A)_0$.

In general, the converse of Theorem 3.6 does not hold as is seen from the following example.

EXAMPLE 3.7. Let X be an infinite set. Suppose τ consist of 0_X and all those fuzzy sets in X whose complements have finite supports. Then τ is a fuzzy topology on X. For each $x \in X$, let U_x be a fuzzy set defined by

$$U_x(y) = \begin{cases} 0, & y = x \\ 1, & y \neq x. \end{cases}$$

Then $1_X - U_x = x_1$, and hence $(1_X - U_x)_0 = \{x\}$. Thus x_1 is fuzzy closed in X. But this fts is not fuzzy SS-T_2. To show this, assume to the contrary that for any distinct fuzzy points x_α and y_β with $x \neq y$, there exist fuzzy open nbds U and V of x_α and y_β, respectively, such that $U \cap V = 0_X$. Then $U_0 \subset X - V_0 \subset (1_X - V)_0$ because $U \cap V = 0_X$. Since $(1_X - V)_0$ is finite, U_0 is finite. This contradicts to the fact that U is fuzzy open in X.

We end this paper with some remarks for a finite fts.

If X is a finite fts, then the converse of Theorem 3.6 holds. In fact, we have the following theorem.

THEOREM 3.8. *For a finite fts X, the following are equivalent:*

1. *X is fuzzy SS-T_2.*
2. *For every $x \in X$, x_1 is fuzzy closed in X.*
3. *For $x, y \in X$ with $x \neq y$, there exists a fuzzy open nbd U of x_1 such that $U(y) = 0$.*

PROOF. (1⇒2) Theorem 3.6.

(2⇔3) Theorem 3.5.

(3⇒1) Let x_α and y_β be distinct fuzzy points. For every $z \neq x$, choose a fuzzy open nbd U_z of x_1 such that $U_z(z) = 0$. Since X is finite, the fuzzy set $U = \cap_{z \neq x} U_z$ is fuzzy open. Also,

$$U(z) = \begin{cases} 1, & z = x \\ 0, & z \neq x. \end{cases}$$

Similarly, we have a fuzzy open set V satisfying

$$V(z) = \begin{cases} 1, & z = y \\ 0, & z \neq y. \end{cases}$$

Clearly, U and V are fuzzy open nbds of x_α and y_β, respectively, such that $U \cap V = 0_X$. □

As an application of Theorem 3.2, we obtain the following theorem.

THEOREM 3.9. *Let X be a finite fts. Then X is fuzzy S-T_2 if and only if X is fuzzy discrete space.*

PROOF. (⇐) Clear.

(⇒) By Definition 2.3, every fuzzy point in X is fuzzy closed. Now, let x_α be a fuzzy point in X. Assume $\gamma < \alpha$. By Theorem 3.2, there exists a fuzzy open nbd U_γ of x_γ such that $x_\alpha \notin Cl(U_\gamma)$. Note that $\gamma \leq U_\gamma(x) < \alpha$. Let $U = \cup_{0<\gamma<\alpha} U_\gamma$. Then U is a fuzzy open set and $U(x) = \alpha$. Let $V = (1_X - \cup_{y \neq x} y_1) \cap U$. Since X is finite, V is a fuzzy open set in X. Furthermore, $V = x_\alpha$. □

But there is a finite fuzzy HR-T_2 space which is not a discrete space as is seen from the following example.

EXAMPLE 3.10. Let $X = \{x, y\}$ and let $A_{\lambda\mu}$ be the fuzzy set in X defined by

$$A_{\lambda\mu}(z) = \begin{cases} \lambda & \text{for } z = x \\ \mu & \text{for } z = y. \end{cases}$$

Let $\tau = \{A_{\lambda\mu} | 0 < \lambda \leq 1, 0 < \mu \leq 1\} \cup \{A_{00}\}$. It is clear that τ is a fuzzy topology on X. Since $x_1 = A_{10}$ and $y_1 = A_{01}$ are not fuzzy open under the fuzzy topology τ, X is not a fuzzy discrete space. Now, since $A_{\lambda\mu}$ is both fuzzy open and fuzzy closed for each $\lambda, \mu \in (0,1)$, it is easy to show that for any $\alpha, \beta \in (0,1]$,

$$x_\alpha = \bigcup_{0<\lambda<\alpha} \left(\bigcap_{0<\mu<1} A_{\lambda\mu} \right) = \bigcup_{0<\lambda<\alpha} \left(\bigcap_{0<\mu<1} Cl(A_{\lambda\mu}) \right)$$

and

$$y_\beta = \bigcup_{0<\mu<\beta} \left(\bigcap_{0<\lambda<1} A_{\lambda\mu} \right) = \bigcup_{0<\mu<\beta} \left(\bigcap_{0<\lambda<1} Cl(A_{\lambda\mu}) \right).$$

Thus, X is fuzzy HR-T_2.

Since the fts given in Example 3.10 is not a fuzzy discrete space, X is fuzzy S-T_2 and hence, from Theorem 3.9, we have that a fuzzy HR-T_2 space may fail to be a fuzzy S-T_2.

COROLLARY 3.11. *For a finite fts X, the following are equivalent:*
1. *X is fuzzy GS-T_2.*
2. *X is fuzzy S-T_2 and fuzzy SS-T_2.*

PROOF. (1⇒2) By Theorem 3.4, X is fuzzy S-T_2 and for $x, y \in X$ with $x \neq y$, there exists a fuzzy open nbd U of x_1 such that $Cl(U)(y) = 0$. By Theorem 3.9, X is fuzzy discrete and hence $Cl(U) = U$. Thus by Theorem 3.7, X is fuzzy SS-T_2.

(2⇒1) By Theorem 3.9, Theorem 3.8 and Theorem 3.4, it is clear. □

COROLLARY 3.12. *Let X be a finite fts. Then X is fuzzy GS-T_2 if and only if X is fuzzy discrete space.*

PROOF. (⇒) It follows from Theorem 3.4 and Theorem 3.9.

(⇐) For any distinct fuzzy points x_α and y_β with $x \neq y$, let $U = x_\alpha$ and $V = y_\beta$. Since X is fuzzy discrete, U and V are fuzzy open nbds of x_α and y_β, respectively, such that $U \cap V = 0_X$. This shows that X is fuzzy SS-T_2. Thus by Theorem 3.9 and Corollary 3.11, X is fuzzy GS-T_2. □

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