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APPROXIMATE FIBRATIONS ON MANIFOLD DECOMPOSITIONS

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1. Introduction

Coram and Duvall [2] introduced an approximate fibration as a map having the approximate homotopy lifting property for every space, which is a generalization of a Hurewicz fibration and a cell-like map.

A proper map $p : M \to B$ between locally compact ANR's is called an *approximate fibration* if it has the following approximate homotopy lifting property: given an open cover ε of B, an arbitrary space X, and two maps $g : X \to M$ and $F : X \times I \to B$ such that $p \circ g = F_0$, there exists a map $G : X \times I \to M$ such that $G_0 = g$ and $p \circ G$ is ε-close to F.

If a proper map $p : M \to B$ is an approximate fibration, not only are the point inverses homotopy equivalent but also there exists an exact homotopy sequence between M, B and fibers of p as follows;

$$\cdots \to \pi_{i+1}(B) \to \pi_i(p^{-1}b) \to \pi_i(M) \to \pi_i(B) \to \cdots$$

It is very essential to examine whether a given decomposition map is an approximate fibration, for then, above exact homotopy sequence provides us structural informations about any one object by means of their interrelations with the rests.

A closed n-manifold N is called a *codimension k fibrator* if whenever there is a usc decomposition G of an arbitrary $(n+k)$-manifold M such that each element of G is homotopy equivalent to N and $dim M/G < \infty$, then $p : M \to M/G$ is an approximate fibration.

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QUESTION. *Which manifolds N are codimension k fibrators?*

Liem [16] proved that S^n $(n \geq 2)$ is a codimension 1 fibrator, and Daverman [4] showed that if G is a decomposition of an $(n+1)$-manifold M into continua having the shape of arbitrary closed n-manifolds then M/G is a 1-dimensional manifold, furthermore, if each element of G is locally flat in M, then p is an approximate fibration.

The main problem is to determine which manifolds N are codimension 2 fibrators.

Simply connected manifolds, closed manifolds N with $\pi_1(N) \cong Z_2$ (for example, real projective n-spaces $(n > 1)$), closed manifolds with finite (or abelian) fundamental group and nonzero Euler characteristic are known to be codimension-2 fibrators ([1], [5] and [6]).

And closed hopfian manifolds with hopfian fundamental group and nonzero Euler characteristic as well as closed hopfian manifolds with hyperhopfian fundamental group are known to be codimension-2 fibrators ([5] and [14]). But there are some non-codimension-2 fibrators (see [5], [6], and [7]).

The question of whether the collection of codimension 2 fibrators is closed under the Cartesian product operator remains unsolved. In [11] and [12], Im showed that any product of finitely many orientable surfaces of genus at least two is a codimension 2 (orientable) fibrator. Recently Kim [16] generalized Im's results to the orientation-free version. To determine whether any product of finitely many codimension 2 fibrators is a codimension 2 fibrator, one may confront the question, which is widely open, whether the collection of hopfian manifolds is closed under the Cartesian product operator.

In this paper, we first show that the product $F \times A$ of a closed hopfian n-manifold F and a closed orientable aspherical m-manifold A is a hopfian manifold under either solvable $\pi_1(F)$ and $\chi(A) \neq 0$, or finite $\pi_1(F)$. Then using those facts we investigate the conditions under which products of codimension 2 fibrators are again codimension 2 fibrators.

2. Preliminaries

We assume all spaces are locally compact, metrizable ANR's, and all manifolds are connected and boundaryless. A manifold M is said to be *closed* if M is compact and boundaryless.

Let N and N' be closed n-manifolds and $f : N \longrightarrow N'$ be a map. If both N and N' are orientable, then the *degree of* f is the nonnegative integer d such that the induced endomorphism of $f_* : H_n(N; Z) \cong Z \longrightarrow H_n(N'; Z) \cong Z$ amounts to multiplication by d, up to sign. In general, the *degree mod 2 of* f is the nonnegative integer d such that the induced endomorphism of $f_* : H_n(N; Z_2) \cong Z_2 \longrightarrow H_n(N'; Z_2) \cong Z_2$ amounts to multiplication by d.

Suppose that N is a closed n-manifold and a proper map $p : M \longrightarrow B$ is N-like. Let G be the set of all fibers, i.e , $G = \{p^{-1}(b) : b \in B\}$. Put $C = \{p(g) \in B : g \in G$ and there exist a neighborhood U_g of g in M and a retraction $R_g : U_g \longrightarrow g$ such that $R_g \mid g' : g' \longrightarrow g$ is a degree one map for all $g' \in G$ with $g' \in G$ in $U_g\}$, and $C' = \{p(g) \in B : g \in G$ and there exist a neighborhood U_g of g in M and a retraction $R_g : U_g \longrightarrow g$ such that $R_g \mid g' : g' \longrightarrow g$ is a degree one mod 2 map for all $g' \in G$ with $g' \in G$ in $U_g\}$. Call C the *continuity set of* p and C' the *mod 2 continuity set of* p D. Coram and P. Duvall [3] showed that C and C' are dense, open subsets of B.

Coram and Duvall [3] gave several characterizations for an approximate fibration. One of them is that a proper map $p : M \to B$ is an approximate fibration if and only if it is k-movable for all k (for details, see [3]), since then, this criterion has been the most used to check under which conditions inverse images of p are homotopy equivalent. The following terms help a lot for looking into these conditions

A closed orientable manifold N is called *hopfian* if every degree one map $N \to N$ which induces a π_1-isomorphism is a homotopy equivalence. A group H is *hopfian* if every epimorphism $\Theta : H \to H$ is necessarily an isomorphism, while a finitely presented group H is *hyperhopfian* if every homomorphism $\Psi : H \to H$ with $\Psi(H)$ normal and $H/\Psi(H)$ cyclic is an automorphism.

The symbol χ is used to denote *Euler characteristic.*

A group H is said to be *residually finite* if for each $e_H \neq h \in H$, there exists a finite group A and a homomorphism $\Phi : H \to A$ with $\Phi(h) \neq e_A$.

In the study of a decomposition map $p : M^{n+k} \to B$ from $(n+k)$-manifold M^{n+k}, codimension 2 is much more advantageous and accessible than other dimensions on account of following result.

THEOREM 2.1 [8]. *If G is a usc decomposition of an orientable $(n+2)$-manifold M into closed, orientable n-manifolds, then the decomposition space $B = M/G$ is a 2-manifold and $D = B \setminus C$ is locally finite in B, where C represents the continuity set of the decomposition map $p : M \longrightarrow B$. If either M or some elements of G are nonorientable, B is a 2-manifold with boundary (possibly empty) and $D' = (int\ B) \setminus C'$ is locally finite in B, where C' represents the mod 2 continuity set of p.*

The next results give useful information connecting hopfian manifolds and hopfian fundamental groups.

THEOREM 2.2 [6]. *A closed orientable n-manifold N is a hopfian manifold if any one of the following conditions holds:*
1) $n \leq 4$;
2) $\pi_1(N)$ contains a nilpotent subgroup of finite index;
3) $\pi_i(N)$ is trivial for $1 < i < n-1$.

THEOREM 2.3 [6]. *A closed hopfian manifold N with either hopfian $\pi_1(N)$ and $\chi(N) \neq 0$ or hyperhopfian $\pi_1(N)$ is a codimension 2 fibrator.*

3. Products of Hopfian manifolds

In this section we discuss conditions under which products of hopfian manifolds are hopfian manifolds.

LEMMA 3.1. *Let Γ_1 and Γ_2 be groups and let $\phi : \Gamma_1 \times \Gamma_2 \to \Gamma_1 \times \Gamma_2$ be an isomorphism.*
Then, $\phi(\Gamma_1 \times 1) = \Gamma_1 \times 1$ under one of the following conditions
1) Γ_1 is solvable and Γ_2 has no nontrivial abelian normal subgroups;
2) Γ_1 is finite and Γ_2 is torsion free.

Proof. 1) Since Γ_1 is solvable, by taking successive derived subgroups, we have a derived series

$$\Gamma_1 \geq \Gamma_1' \geq \Gamma_1'' \geq \cdots \Gamma_1^{(k-1)} \geq \Gamma_1^{(k)} = 1$$

for some k. From the fact that $\Gamma_1^{(i)} \times \Gamma_2^{(i)} = (\Gamma_1 \times \Gamma_2)^{(i)}$ is a characteristic subgroup of $\Gamma_1 \times \Gamma_2$, we have an isomorphism $\phi \mid: \Gamma_1^{(i)} \times \Gamma_2^{(i)} \to \Gamma_1^{(i)} \times \Gamma_2^{(i)}$ for each i. Note that $\Gamma_1^{(k)} = 1$ implies that $\Gamma_1^{(k-1)}$ is abelian. By hypothesis of Γ_2, $\Gamma_2^{(k-1)}$ has no nontrivial abelian normal subgroup so that $\phi(\Gamma_1^{(k-1)} \times 1) = \Gamma_1^{(k-1)} \times 1 \subset \Gamma_1^{(k-2)} \times \Gamma_2^{(k-2)}$

Consider the induced map $\overline{\phi} \mid \Gamma_1^{(k-2)}/\Gamma_1^{(k-1)} \times \Gamma_2^{(k-2)} : \Gamma_1^{(k-2)}/\Gamma_1^{(k-1)} \times \Gamma_2^{(k-2)} \to \Gamma_1^{(k-2)}/\Gamma_1^{(k-1)} \times \Gamma_2^{(k-2)}$. Again since $\Gamma_2^{(k-2)}$ has no nontrivial abelian normal subgroup, $\overline{\phi} \mid (\Gamma_1^{(k-2)}/\Gamma_1^{(k-1)} \times 1) = \Gamma_1^{(k-2)}/\Gamma_1^{(k-1)} \times 1$. Hence we have that $\phi(\Gamma_1^{(k-2)} \times 1) = \Gamma_1^{(k-2)} \times 1$. By the inductive step we have the conclusion.

2) is obvious.

THEOREM 3 2 *Let F be a closed hopfian $n-$manifold and let A be a closed orientable aspherical $m-$manifold. If either 1) $\pi_1(F)$ is solvable and $\chi(A) \neq 0$, or 2) $\pi_1(F)$ is finite, then $F \times A$ is a hopfian manifold*

Proof Proof of 1) Let $f . F \times A \to F \times A$ be a degree one map which induces a π_1-isomorphism.

Now since $\chi(A) \neq 0$ implies that $\pi_1(A)$ has no nontrivial abelian normal subgroup [17], by Lemma 3.1, we have a map $f^* : F \times A^* \to F \times A^*$ satisfying the following commutative diagram

$$\begin{array}{ccc}
F & \xrightarrow{f^{**}} & F \\
{\scriptstyle \jmath}\big\downarrow & & \big\uparrow{\scriptstyle pr} \\
F \times A^* & \xrightarrow{f^*} & F \times A^* \\
{\scriptstyle Id\times q}\big\downarrow & & \big\downarrow{\scriptstyle Id\times q} \\
F \times A & \xrightarrow{f} & F \times A,
\end{array}$$

where $\jmath$ is the inclusion, pr is the projection, $q : A^* \to A$ is the universal covering map, and $f^{**} = pr \circ f^* \circ \jmath$. Note that j and pr are homotopy equivalences, for A^* is contractible. The fact of $deg\ f = 1$ implies that $f_* : H_k(F \times A) \to H_k(F \times A)$ is an isomorphism for every $k \geq$

0. In particular, $f_* : H_n(F \times A) \to H_n(F \times A)$ is an isomorphism so that $f_* \mid$ *free part of* $H_n(F \times A)$: *free part of* $H_n(F \times A)$ $\to$ *free part of* $H_n(F \times A)$ is an isomorphism. Here note $Z \cong H_n(F) \otimes H_0(A)$ is a subgroup of the *free part of* $H_n(F \times A)$. From the diagram, we see that $(f_* \mid)(H_n(F) \otimes H_0(A)) \subset H_n(F) \otimes H_0(A)$. But since $f_* \mid$ *free part of* $H_n(F \times A)$ is an isomorphism, we have an isomorphism $f_* \mid H_n(F) \otimes H_0(A) : H_n(F) \otimes H_0(A) \to H_n(F) \otimes H_0(A)$. Then by simple diagram chasing we see that the degree of f^{**} is one. The hopfianness of F implies that f^{**} is a homotopy equivalence, and so is f^*. Therefore since $f_\# : \pi_1(F \times A) \to \pi_1(F \times A)$ is an isomorphism, we have an isomorphism $f_\# : \pi_k(F \times A) \to \pi_k(F \times A)$ for all $k \geq 1$. By the Whitehead Theorem, f is a homotopy equivalence.

Proof of 2). Lemma 3.1 guarantees the existence of f^* as in the proof of 1), since $\pi_1(A)$ is torsion free. Now just copy the proof of 1)

COROLLARY 3.3. *Let F be a closed orientable $n-$manifold with nilpotent $\pi_1(F)$ and let A be a closed orientable aspherical $m-$manifold with $\chi(A) \neq 0$. Then $F \times A$ is a hopfian manifold.*

Proof. Hopfianness of F comes from Theorem 2.2.

LEMMA 3.4. *Let F and A be closed orientable $n-$manifolds. If $\pi_1(F)$ is finite and $\pi_1(A)$ is infinite, then any map $f : F \to A$ has degree zero.*

Proof. Consider the following

$$\begin{array}{ccc} \widetilde{F} & \xrightarrow{\tilde{f}} & \widetilde{A} \\ {\scriptstyle q_F}\downarrow & & \downarrow{\scriptstyle q_A} \\ F & \xrightarrow{f} & A, \end{array}$$

where q_F and q_A are the universal covering maps, and $\widetilde{f}$ is the lifting of $f \circ q_F$. Since $H_n(\widetilde{A}) = 0$, the homomorphism $(f \circ q_F)_* : H_n(\widetilde{F}) \to H_n(A)$ is trivial, so that $f \circ q_F$ has degree zero. It follows from degree of $q_F = [\pi_1(F) : \pi_1(\widetilde{F})] \geq 1$ that $f : F \to A$ has degree zero.

THEOREM 3.5. *Let F and A be closed orientable $n-$manifolds. If $\pi_1(F)$ is finite and $\pi_i(A)$ is trivial for $1 < i < n$, then $F \times A$ is hopfian.*

Proof. If $\pi_1(A)$ is finite, then $\pi_1(F) \times \pi_1(A)$ is finite, so that $F \times A$ is hopfian. Now assume that $\pi_1(F) \times \pi_1(A)$ is infinite. Let $f : F \times A \to F \times A$ be a degree one map which induces a π_1-isomorphism. Consider the following commutative diagram

$$\begin{array}{ccc} \widetilde{F} \times \widetilde{A} & \xrightarrow{\tilde{f}} & \widetilde{F} \times \widetilde{A} \\ {\scriptstyle q_F \times q_A}\downarrow & & \downarrow{\scriptstyle q_F \times q_A} \\ F \times A & \xrightarrow{f} & F \times A, \end{array}$$

where $q_F : \widetilde{F} \to F$ and $q_A : \widetilde{A} \to A$ are the universal covering maps, and $\widetilde{f}$ is the lifting of $f \circ (q_F \times q_A)$. Note that the degree of $\widetilde{f}$ is one. To show that f is a homotopy equivalence, it suffices to show that $\widetilde{f}$ is a homotopy equivalence.

Claim 1 : $g_F = pr_F \circ f \circ j_F$ has degree one, where $j_F : F \to F \times A$ is the inclusion and $pr_F : F \times A \to F$ is the projection.

The isomorphism $f_* : H_n(F \times A) \to H_n(F \times A)$ induces $2 \times 2-$invertible matrix

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

where a_{ij} corresponds to the coefficient in $f_* : H_n(F\times A) \to H_n(F\times A)$. By Lemma 3.3, $a_{12} = 0$ so that $a_{11} = a_{22} = 1$, i.e., degree of $g_F = 1$.

Claim 2: For any k , $(\widetilde{f})_* : H_k(\widetilde{F} \times \widetilde{A}) \to H_k(\widetilde{F} \times \widetilde{A})$ is an isomorphism.

By the Hurewicz Theorem, $H_i(\widetilde{A}) = 0$ for $1 \leq i < n$. Since $H_k(\widetilde{F} \times \widetilde{A}) \cong H_k(\widetilde{F}) \otimes H_0(\widetilde{A}) \cong H_k(\widetilde{F})$, we may regard the homomorphism $(\widetilde{f})_* : H_k(\widetilde{F} \times \widetilde{A}) \to H_k(\widetilde{F} \times \widetilde{A})$ as $(\widetilde{g}_F)_* : H_k(\widetilde{F}) \to H_k(\widetilde{F})$, where $\widetilde{g}_F$ is the lifting of $pr_F \circ f \circ j_F$.

By Claim 1, g_F has degree one, and so does $\widetilde{g}_F$ since $\widetilde{g}_F$ is the lifting of $g_F \circ q_F$. This implies that $(\widetilde{g}_F)_* : H_k(\widetilde{F}) \to H_k(\widetilde{F})$ is an isomorphism for all $k \geq 0$.

COROLLARY 3.6. *Let N be a closed orientable aspherical n manifold. Then $S^n \times N$ is hopfian.*

4. Products of fibrators

LEMMA 4.1. *Let G and K be hyperhopfian groups. If G is finite and K is torsion free, then $G \times K$ is hyperhopfian.*

Proof. Let $\phi : G \times K \to G \times K$ be a homomorphism with $\phi(G \times K)$ normal and $(G \times K)/\phi(G \times K)$ cyclic. Consider the following diagram

$$\begin{tikzcd} G \arrow[d, "i_G"'] & G \\ G \times K \arrow[r, "\phi"] & G \times K \arrow[u, "pr_G"'] \arrow[d, "pr_K"] \\ K \arrow[u, "i_K"] & K, \end{tikzcd}$$

where i_G, i_K are inclusions and pr_G, pr_K are projections. Then $K/pr_K \circ \phi(G \times K)$ is cyclic. From the fact that $pr_K \circ \phi \circ i_G(G)$ is trivial, $pr_K \circ \phi(G \times K) = pr_K \circ \phi \circ i_K(K)$ and $K/pr_K \circ \phi \circ i_K(K)$ is cyclic. Using the hyperhopficity of K, we have an isomorphism $pr_K \circ \phi \circ i_K : K \to K$. Similarly, we have an isomorphism $pr_G \circ \phi \circ i_G : G \to G$, because $pr_K \circ \phi \circ i_G(G)$ is trivial. As a result, it is easy to see that ϕ is an isomorphism.

The next recent result of Chinen gives us useful information about manifolds with hyperhopfian fundamental group being codimension 2 fibrators.

LEMMA 4.2. *[1, Theorem 5.3] Let N be a closed $n-$manifold with hyperhopfian fundamental group. If every $N-$like proper map $p : M^{n+2} \longrightarrow B^2$ from an $(n+2)-$manifold M onto a $2-$manifold B is an approximate fibration over C', where C' denotes the mod 2 continuity set of p, then N is a codimension-2 fibrator.*

THEOREM 4.3. *Let F be a closed $n-$manifold with finite $\pi_1(F)$ and let A be a closed aspherical $m-$manifold. If $\pi_1(F)$ and $\pi_1(A)$ are hyperhopfian, then $N \equiv F \times A$ is a codimension 2 fibrator.*

Proof. Case I: N is orientable.

It follows from Theorem 2.3, Theorem 3.2 and Lemma 4.1 that N is a codimension 2 fibrator.

Case II: N is non-orientable.

Let a proper map $p : M^{n+2} \to B^2$ be $N-$like. By Lemma 4.1 and 4.2, it suffices to show that p is an approximate fibration over the mod 2 continuity set C' of p. Assume that $B^2 = C'$. Fix $g_0 \in G$ with $p(g_0) \in C'$. Take a neighborhood $U(\subset C')$ of $p(g_0)$ such that $p^{-1}(U)$ retracts to g_0, and take a smaller connected neighborhood V of $p(g_0)$ such that $p^{-1}(V)$ deformation retracts to g_0 in $p^{-1}(U)$. Call this retraction $R : p^{-1}(V) \longrightarrow g_0$. Take the covering map $q : M^* \to p^{-1}(V)$ corresponding to $R_{\#}^{-1}(H)$, where H is the intersection of all index 2 subgroups of $\pi_1(N)$. Since $[\pi_1(p^{-1}(V)) : R_{\#}^{-1}(H)] = [\pi_1(g_0) \cdot H] < \infty$, q is finite. Then we have that for all $g \in G$ with $p(g) \in C'$, $q^{-1}(g) \equiv g^*$ is connected and has the homotopy type of N_H, where N_H is the covering space of N corresponding to H (see [15] for the detailed proof). Set $G^* = \{g^* : g \in G \text{ with } p(g) \in V\}$ and let $p^* = p \circ q : M^* \to M^*/G^*$ be the decomposition map. Here note $M^*/G^* = V$.

Claim: p^* is an approximate fibration over the continuity set $C(p^*)$ of p^*.

Fix $g_b^* \in G^*$ with $p^*(g_b^*) = p(g_b) = b \in C(p^*)$. Carefully take a small neighborhood $W(\subset C(p^*))$ of b and a retraction $R_b : p^{-1}(W) \to g_b$. Let $R_b^* : W^* \equiv p^{-1}(W) \to g_b^*$ be the lifting of R_b. For any $a \in W$ consider the following

$$\begin{array}{ccccc} g_a^* & \longrightarrow & W^* & \xrightarrow{R_b^*} & g_b^* \\ {\scriptstyle q|}\downarrow & & {\scriptstyle q|}\downarrow & & {\scriptstyle q|}\downarrow \\ g_a & \longrightarrow & p^{-1}(W) & \xrightarrow{R_b} & g_b. \end{array}$$

We regard this diagram as the commutative diagram

$$\begin{array}{ccc} N_H & \xrightarrow{\bar{f}} & N_H \\ {\scriptstyle q}\downarrow & & {\scriptstyle q}\downarrow \\ N & \xrightarrow{f} & N, \end{array}$$

where $N \sim g_b = F \times A$ and $N_H \sim g_b^*$.

Construct the covering $q_1 \times q_2 : \tilde{F} \times \tilde{A} \to F \times A$ corresponding to $H^{'} \times H^{''}$, where $H^{'}$ and $H^{''}$ are the intersections of all index 2 subgroups of $\pi_1(F)$ and $\pi_1(A)$, respectively. Choose $\tilde{F} = F$ or $\tilde{A} = A$ in case F or A is orientable. It is easy to check that $H \subset H^{'} \times H^{''}$, for $H \subset H^{'} \times 1$ and $H \subset 1 \times H^{''}$. Hence we have the following commutative diagram

$$\begin{array}{ccc} N_H & \xrightarrow{\bar{f}} & N_H \\ {\scriptstyle q'}\downarrow & & \downarrow{\scriptstyle q'} \\ \tilde{F} \times \tilde{A} & \xrightarrow{\tilde{f}} & \tilde{F} \times \tilde{A} \\ {\scriptstyle q_1 \times q_2}\downarrow & & \downarrow{\scriptstyle q_1 \times q_2} \\ F \times A(= N) & \xrightarrow{f} & F \times A(= N), \end{array}$$

where $\tilde{f}$ is the lifting of f and $q' : N_H \to \tilde{F} \times \tilde{A}$ is the covering map. Since $\bar{f}$ has degree one, so does $\tilde{f}$. Then $\bar{f}, \tilde{f}$, and f induce π_1-epimorphisms. From the fact that $\pi_1(F \times A)$ is hopfian, $f_\#$ is a π_1-isomorphism, and so $\tilde{f}_\#$ is. Applying similarly to the argument of the proof of Theorem 3.2, we see that $\tilde{f}$ is a homotopy equivalence so that f is. The claim follows from the complete movability condition [3].

Now since the continuity set $C(p^*)$ of p^* is dense, open in V, and $V \setminus C(p^*)$ is locally finite, we can localize the situation so that V is an open disk containing $b_0 = p(g_0)$ and p is an approximate fibration over $V \setminus b_0$. Also we assume that $R : p^{-1}(V) \to g_0$ is a strong deformation retraction. From the fact that $\pi_1(N)$ is hyperhopfian, we have that $p : p^{-1}(V) \to V$ is an approximate fibration.

THEOREM 4.4. *Let F be a closed $n-$manifold with $\chi(F) \neq 0$ and let A be a closed aspherical $m-$manifold with $\chi(A) \neq 0$. Then $N \equiv F \times A$ is a codimension 2 fibrator if it satisfies one of the followings*

1) $\pi_1(F)$ is finite and $\pi_1(A)$ is hopfian;

2) $\pi_1(F)$ is nilpotent and $\pi_1(A)$ is residually finite;

3) F is hopfian with residually finite solvable $\pi_1(F)$ and residually finite $\pi_1(A)$.

Proof. First, note that $\pi_1(N)$ is hopfian, because any finitely presented group which has a finite index hopfian subgroup is hopfian [10] for 1), and every finitely generated nilpotent group is residually finite for 2).

Case I. N is orientable.

All facts from section 3 gives us that $F \times A$ is hopfian with hopfian $\pi_1(N)$ so that by Proposition 2.3 $F \times A$ is a codimension-2 fibrator.

Case II: N is non-orientable.

Let a proper map $p : M^{n+m+2} \to B^2$ be $N-$like. Applying to the argument of the proof in Theorem 4.3, we see that p is an approximate fibration over some dense open subset O of C' with locally finite $C' \setminus O$, where C' is the mod 2 continuity set of p. Using the fact $\chi(N) = \chi(F) \times \chi(A) \neq 0$ and the method of the proof in [14, Theorem 3.3], we see that $O = C'$. Moreover, the proof of $intB = C'$ and $\partial B = \emptyset$ is just copy of the proofs in [14,Lemma 3.2 and Theorem 3.3].

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