논문 99-8-3-01

Acoustic field simulation of a PZT4 disc projector using a coupled FE-BE method

S. S. Jarng

Abstract

This paper describes the application of a coupled finite element-boundary element method (FE-BEM) to obtain the steady-state response of a piezoelectric transducer. The particular structure considered is a PZT4 disc-typed projector. The projector is three-dimensionally simulated to transduce applied electric charge on axial surfaces of the piezoelectric disc to acoustic pressure in air or in water. The directivity pattern of the acoustic field formed from the projected sound pressure is also simulated. And the displacement of the disc caused by the externally applied electric charge is shown in temporal motion. The coupled FE-BE method is described in detail.

1. Introduction

In underwater acoustics, transducers and transducer materials are perhaps most essential area of research. Sonar transducers are the only sensor with which any kind of signal can be transmitted and received in deep ocean. There are several aspects of consideration in the design and fabrication of sonar transducers; acoustic power, sensitivity, directivity and frequency response etc.. The former two factors are mainly depending on types of material while the latter two factors are determined by structural design. Because of it, the development of sonar transducers are carried out in such two aspects; materials and structural design. The most frequently used material for sonar transducer is piezoelectric materials because of their stable performance and economical price at present. And in the structural aspect there are endless diversities in sonar transducer design. Also many different computational methods have been developed for effectively designing CAD tools. Electric circuit modelling is the most effective analytical tool for simple types of sonar transducer. And many numerical methods for simulating sonar transducers have been developed to analyze other complicated types of transducer structures for better performance. Since a sonar transducer is used in water, modelling of the sonar transducer must satisfy both internal materialistic transduction and externally radiating condition. In these aspects the finite element method (FEM) and the boundary element method (BEM) is perhaps the most suitable numerical techniques for the solution. Both methods were developed for the numerical solution of partial differential equations (PDE) with boundary conditions. Since both methods solve the PDEs by numerically elemental integration, they are compatible each other and therefore they can be coupled together [1,2]

Different types of in-air piezoelectric transducers have been simulated by the FEM

* 조선대학교 전기제어계측공학부(Dept. of Control & Instrumentation, Chonsun University)

<접수일자 : 1998년 7월 31일>

[3-5]. And also modified FEMs such as the mixed FE perturbation method [6] or the mixed FE plane-wave method [7] have been developed in order to simulate an array of transducers or composite sonar transducers. Further developments have been made so as to include the effects of infinite fluid loading on transducer surface. For example, Bossut et. al. [8] and Hamonic et. al. [9] used fluid finite elements as an extension to structural finite elements with the condition that outer boundary of the fluid elements represents continued radiation. Others used 'infinite' fluid elements for infinite acoustic radiation [10,11]. The BEM is probably accepted as the most suitable method for the radiation problem because the BEM directly solves the Helmholtz PDE with the radiation condition [1,2].

The main aim of this paper is to develope a coupled FE-BEM and to simulate the structural behaviour of the flooded piezoelectric disc when the sonar transducer is driven by externally applied electrical charge. This particular disc-typed transducer is chosen to verify the directivity pattern of the underwater acoustic radiation at a high frequency such as 200KHz. The presented disc transducer is often used in fishery sonars at 200KHz driving frequency. The directivity pattern of the projected acoustic pressure is also shown in temporal motion and compared with that of a theoretical results. Both directivity patterns in water as well as in air are presented with expected projector sensitivity.

2. Numerical Methods

2.1 Finite Element Method (FEM)

The following equation (1) is the integral formulation of the piezoelectric equations:

$$\begin{aligned} \{F\}+\{F_l\} &= [K_{uu}]\{a\}+[K_{u\phi}]\{\phi\} \\ &\quad -\omega^2[M]\{a\} + j\omega[R]\{a\} \\ -\{Q\} &= [K_{\phi u}]\{a\} + [K_{\phi\phi}]\{\phi\} \end{aligned} \tag{1}$$

where

$\{F\}$	Applied Mechanical Force
$\{F_l\}$	Fluid Interaction Force
$\{Q\}$	Applied Electrical Charge
$\{a\}$	Elastic Displacement
$\{\phi\}$	Electric Potential
$[K_{uu}]$	Elastic Stiffness Matrix
$[K_{u\phi}]$	Piezoelectric Stiffness Matrix $[K_{\phi u}] = [K_{u\phi}]'$
$[K_{\phi\phi}]$	Permittivity Matrix
$[M]$	Mass Matrix
$[R]$	Dissipation Matrix
ω	Angular Frequency

The isoparametric formulation for 3-dimensional structural elements is well documented by Allik H. et. al. [3]. Each 3-dimensional finite element is composed of 20 quadratic nodes and each node has nodal displacement (a_x, a_y, a_z) and electric potential (ϕ) variables. In local coordinates the finite element has 6 surface planes (±xy, ±yz, ±zx) which may be exposed to external fluid environment. The exposed surface is used as a boundary element which is composed of 8 quadratic nodes.

2.2 Boundary Element Method (BEM)

For sinusoidal steady-state problems, the Helmholtz equation, $\nabla^2\Psi + k^2\Psi = 0$, represents the fluid mechanics. Ψ is the acoustic pressure with time variation, $e^{j\omega t}$, and k(=ω/c) is the wave number. In order to solve the Helmholtz equation in an infinite fluid media, a solution to the equation must not only satisfy structural surface boundary condition (BC), $\frac{\partial\Psi}{\partial n} = \rho_f\,\omega^2\,a_n$, but also the radiation condition at infinity, $\lim_{|r|\to\infty}\oint_S(\frac{\partial\Psi}{\partial r}+jk\Psi)^2 dS = 0$. $\frac{\partial}{\partial n}$ represents differentiation along the outward normal to the boundary. ρ_f and a_n are the fluid density and the normal displacement on the structural surface.

The Helmholtz integral equations derived from Green's second theorem provides such a solution for radiating pressure waves;

$$\oint_S \left(\Psi(q) \frac{\partial G_k(p,q)}{\partial n_q} - G_k(p,q) \frac{\partial \Psi(q)}{\partial n_q} \right) dS_q = \beta(p)\ \Psi(p) - \Psi_{inc}(p) \tag{2}$$

where $G_k(p,q) = \frac{e^{-jkr}}{4\pi r}$, $r = |p-q|$

p is any point in either the interior or the exterior and q is the surface point of integration. β(p) is the exterior solid angle at p.

The acoustic pressure for the ith global node, $\Psi(p_i)$, is expressed in discrete form [12]:

$$(1 \le i \le ng)$$

$$\beta(p_i)\ \Psi(p_i) - \Psi_{inc}(p_i)$$

$$= \oint_S \left(\Psi(q) \frac{\partial G_k(p_i,q)}{\partial n_q} - G_k(p_i,q) \frac{\partial \Psi(q)}{\partial n_q} \right) dS_q \tag{3a}$$

$$= \sum_{m=1}^{nt} \int_{S_m} \left(\Psi(q) \frac{\partial G(p_i,q)}{\partial n_q} - G(p_i,q) \frac{\partial \Psi(q)}{\partial n_q} \right) dS_q$$

q∈S sub m (3b)

$$= \sum_{m=1}^{nt} \int_{S_m} \left(\begin{array}{c} \sum_{j=1}^{8} N_j(q)\, \Psi_{m,j} \frac{\partial G(p_i,q)}{\partial n_q} \\ - G(p_i,q) \sum_{j=1}^{8} N_j(q) \frac{\partial \Psi_{m,j}}{\partial n_q} \end{array} \right) dS_q \tag{3c}$$

$$= \sum_{m=1}^{nt} \sum_{j=1}^{8} \left(\int_{S_m} N_j(q) \frac{\partial G(p_i,q)}{\partial n_q} dS_q \right) \Psi_{m,j} - \rho_f\, \omega^2 \sum_{m=1}^{nt} \sum_{j=1}^{8} \left(\int_{S_m} N_j(q)\, G(p_i,q)\, n_q dS_q \right) a_{m,j} \tag{3d}$$

$$= \sum_{m=1}^{nt} \sum_{j=1}^{8} A^i_{m,j} \Psi_{m,j} - \rho_f\, \omega^2 \sum_{m=1}^{nt} \sum_{j=1}^{8} B^i_{m,j} a_{m,j} \tag{3e}$$

where nt is the total number of surface elements and $a_{m,j}$ are three dimensional displacements. Equation (3b) is derived from equation (3a) by discretizing integral surface. And equation (3c) is derived from equation (3b) since an acoustic pressure on an integral surface is interpolated from adjacent 8 quadratic nodal acoustic pressures corresponding the integral surface. Then equation (3d) is derived from equation (3c) by swapping integral notations with summing notations. Finally the parentheses of equation (3d) is expressed by upper capital notations for simplicity.

When equation (3e) is globally assembled, the discrete Helmholtz equation can be represented as

$$([A] - \beta[I]) \{ \Psi \} = + \rho_f\, \omega^2 [B] \{ a \} - \{ \Psi_{inc} \} \tag{4}$$

where [A] and [B] are square matrices of (ng by ng) size. ng is the total number of surface nodes.

Where the impedance matrices of equation (4), [A] and [B], are computed, two types of singularity arise [13]. One is that the Green's function of the equation, $G_k(p_i, q)$, becomes infinite as q approaches to p_i. This problem is solved by mapping such rectangular local coordinates into triangular local coordinates and again into polar local coordinates [14]. The other is that at certain wave number the matrices become ill-conditioned. These wave number are corresponding to eigenvalues of the interior Dirichlet problem [15]. One approach to overcome the matrix singularity is that [A] and [B] of equation (4) are modified to provide a unique solution for the entire frequency range [16-19]. The modified matrix equation referred to as the modified Helmholtz gradient formulation (HGF) [19] is obtained by adding a multiple of an extra integral equation to equation (4).

$$([A] - \beta[I] \oplus \alpha[C]) \{ \Psi \} = + \rho_f\, \omega^2 ([B] \oplus \alpha[D]) \{ a \} - (\Psi_{inc} \oplus \alpha \frac{\partial \Psi_{inc}}{\partial n_p}) \tag{5}$$

where

$$\alpha = - \frac{\sqrt{-1}}{k \cdot (\textit{Number of surface elements adjacent a surface node})}$$

[C] and [D] are rectangular matrices of (nt by ng) size. nt is the total number of surface elements. $\oplus$ symbol indicates that the rows of [C],[D] corresponding to surface elements adjacent a surface node are added to the row of [A],[B] corresponding to the surface node, that is,

$$\sum_{i=1}^{ng}\sum_{j=1}^{ng}A(i,j)=\sum_{i=1}^{ng}\sum_{j=1}^{ng}A(i,j)+\sum_{i=1}^{ng}\sum_{j=1}^{ng}\Big(\sum_{m=1}^{S(i)}\alpha\, C(m,j)\Big)$$
$$\sum_{i=1}^{ng}\sum_{j=1}^{ng}B(i,j)=\sum_{i=1}^{ng}\sum_{j=1}^{ng}B(i,j)+\sum_{i=1}^{ng}\sum_{j=1}^{ng}\Big(\sum_{m=1}^{S(i)}\alpha\, D(m,j)\Big) \tag{6}$$

where S(i) is the number of surface element adjacent a surface node. The derivation of the extra matrices [C],[D] are well described by Francis D.T.I.[19]. Equation (6) may be reduced in its formulation using superscript $\oplus$ for convenience;

$$A^{\oplus}\{\Psi\} = +\rho_f\,\omega^2 B^{\oplus}\{a\} - \Psi^{\oplus}_{inc} \tag{7}$$

where

$$([A]-\beta[I]\oplus\alpha[C]) \equiv A^{\oplus}$$
$$([B]\oplus\alpha[D]) \equiv B^{\oplus}$$
$$\Big(\Psi_{inc}\oplus\alpha\frac{\partial\Psi_{inc}}{\partial n_p}\Big) \equiv \Psi^{\oplus}_{inc}$$

Equation (7) can be written as

$$\{\Psi\} = +\rho_f\,\omega^2(A^{\oplus})^{-1}B^{\oplus}\{a\}-(A^{\oplus})^{-1}\Psi^{\oplus}_{inc} \tag{8}$$

2.3 Coupled FE-BE Method

The acoustic fluid loading on the solid-fluid interface generates interaction forces. These forces can be related to the surface pressures by a coupling matrix [L] [2,12];

$$\{F_f\} = -[L]\{\Psi\} \tag{9}$$

where $[L] = \int N^t n N dS$. N is a matrix of surface shape functions and n is an outward normal vector at the surface element. N^t is the transposed form of N matrixes.

Equations (8) and (9) indicate that the interaction force can be expressed by functions of elastic displacement instead of acoustic pressure. This relationship can be applied to equation (1) when the sonar transducer model is submerged into the infinite fluid media:

$$\begin{aligned}\{F\}+[L](A^{\oplus})^{-1}\Psi^{\oplus}_{inc} &= [K_{uu}]\{a\} + [\rho_f\,\omega^2[L](A^{\oplus})^{-1}B^{\oplus}]\{a\}\\ &\quad + [K_{u\phi}]\{\phi\} - \omega^2[M]\{a\} + j\omega[R]\{a\}\\ -\{Q\} &= [K_{\phi u}]\{a\} + [K_{\phi\phi}]\{\phi\}\end{aligned} \tag{10}$$

Since the present sonar transducer is modelled as a projector, the internal force vector, {F}, and the external incident pressure, $[L](A^{\oplus})^{-1}\Psi^{\oplus}_{inc}$, of equation (10) are removed. The only applied BC for the equation is the applied electrical charge vector, {Q}. The acoustic pressure in the far field is determined by β(p)=1 for given values of surface nodal pressure and surface nodal displacement;

$$\begin{aligned}\Psi(p_i) &= \sum_{m=1}^{nt}\sum_{j=1}^{8}A^i_{m,j}\Psi_{m,j}-\rho_f\,\omega^2\sum_{m=1}^{nt}\sum_{j=1}^{8}B^i_{m,j}a_{m,j}\\ &\quad -(A^{\oplus})^{-1}\Psi^{\oplus}_{inc}\end{aligned} \tag{11}$$

3. Results

The coupled FE-BE method has been programmed with Fortran language running at a supercomputer Cray C90. Calculation is done with double precision and the program is made for three dimensional structures. Because each structural node has 4 DOF, the size of the globally assembled coefficient matrices of the matrix equation are 4*ng by 4*ng. The particular structure considered is a flooded piezoelectric (PZT4) disc (Fig. 1). The diameter and the thickness of the disc are 5cm and 1cm respectively. The whole disc had been divided into 224 isoparametric elements (Fig. 2). Global node numbers are attributed at 20 nodes of each element. It is desired to have more elements representing smaller local regions for higher frequency analysis. However, calculation with more number of nodes cost more time. Therefore meshing of elements depends on the maximal limit of interest frequency. It is a common practice to have the size of the largest element to be less than $\lambda/3$. In this paper the interest frequency of the acoustic radiation is 200KHz, so that $\lambda/3$ is about 5.8mm.

Table 1 shows the material properties of the PZT4 piezoelectric ceramic. The actual ceramic shell is axially polarized and therefore the electrode is coated axially on upper and lower surfaces [20]. The present modelling of the sonar transducer is a sonar projector. So, the disc ceramic is three-dimensionally simulated to transduce an external electric charge on upper and lower surfaces of the disc to acoustic pressure in the far field. This electrical energy drives the piezoelectric disc as a transmitter. From equation (11) the acoustic pressure in the far field is calculated along the circle with the directivity angle ϕ (Fig. 1). The normalized value of the far filed pressure is used as the quantitative degree of the directivity.

Table 1. Material Properties of PZT4
(Axially Polarized Properties)

		Unit			Unit			Unit
ρ	7500	Kg/m^3	C_z^z	1.15E+11	N/m^2	$e_{p,z}^z$	15.1	(N/m^2)/(V/m)
C_x^x	1.39E+11	N/m^2	C_{yz}^{yz}	2.56E+10	N/m^2	$e_{p,z}^{yz}$	12.7	(N/m^2)/(V/m)
C_y^x	7.78E+10	N/m^2	C_{zx}^{zx}	2.56E+10	N/m^2	$e_{p,z}^{zx}$	12.7	(N/m^2)/(V/m)
C_z^x	7.43E+10	N/m^2	C_{xy}^{xy}	3.06E+10	N/m^2	ε_x^x	6.46E-9	F/m
C_y^y	1.39E+11	N/m^2	$e_{p,z}^x$	-5.2	(N/m^2)/(V/m)	ε_y^y	6.46E-9	F/m
C_z^y	7.43E+10	N/m^2	$e_{p,z}^y$	-5.2	(N/m^2)/(V/m)	ε_z^z	5.62E-9	F/m

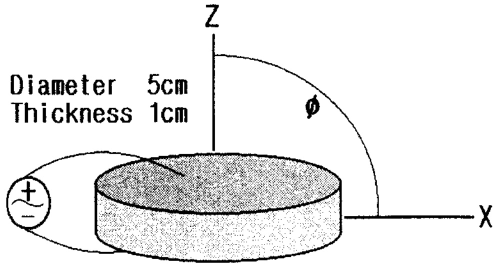

Figure 1 PZT4 disc-typed projector. The projector is three-dimensionally simulated to transduce applied electric charge on axial surfaces of the piezo-electric disc to acoustic pressure in air or in water.

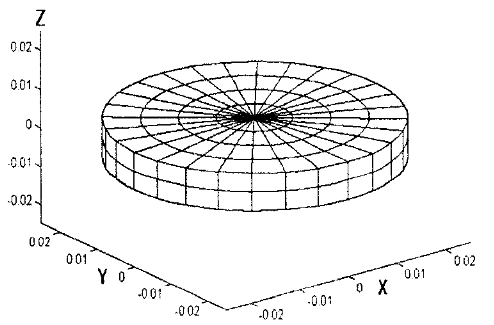

Figure 2 The disc structure is discretized into finite structural elements. The piezo-electric (PZT4) disc can be divided into either 224 elements.

Fig. 3 shows the directivity patterns of the disc ceramic in polar form at 200KHz input frequency. Fig. 3(a) and Fig. 3(b) are the coupled FE-BEM result and the theoretical result respectively [21]. They look very similar except side lobes. Since the theoretical result is analytically derived using only axial(Z axis) displacement while the coupled FE-BEM result is calculated from finite elements including three dimensional displacements, that difference could happen. Fig. 4 shows the same directivity patterns of the numerical result (continuous line) and the analytical result(dotted line) in rectangular form with logarithmic scale. The -3dB beam width of the numerical result is about 8.9° while that of the analytical result is about 8.82°. Because the difference between the maximum normalized pressure of the main lobe and that of the second lobe is more than 14dB, the difference in side lobes is in fact not significant. Fig. 5 shows the directivity pattern of the coupled FE-BEM result in three dimensional polar form at 200KHz.

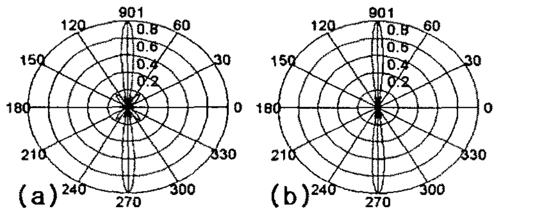

Figure 3 the directivity pattern of the coupled FE -BEM result in three dimensional polar form at 200KHz

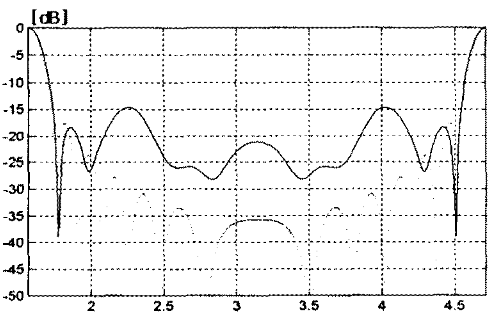

Figure 4 Directivity patterns of the disc ceramic in rectangular form with logarithmic scale at 200KHz; coupled FE-BEM result (continuous line), theoretical result (dotted line)

The projector sensitivity of the flooded ceramic disc at 200KHz is 188 dB re 1 μPa/V at the maximum peak of the directivity pattern. And Fig. 6 shows the three dimensional displacement of the disc at a constant temporal phase. In the figure the most significant displacement happens at the center of the disc in axial direction

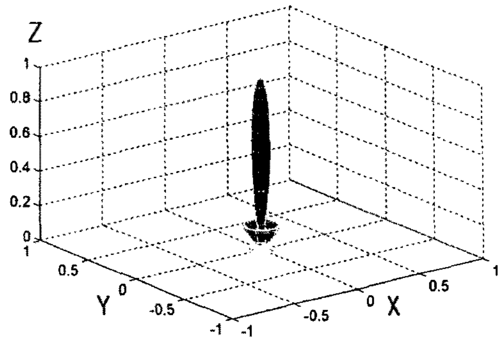

Figure 5 the directivity pattern of the coupled FE-BEM result in three dimensional polar form at 200KHz

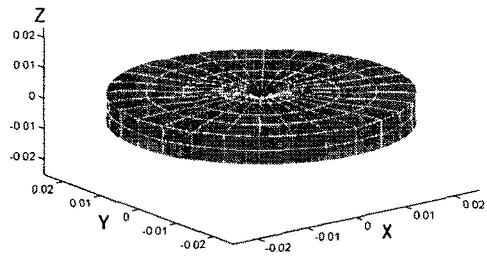

Figure 6 the three dimensional displacement of the disc at a constant temporal phase.

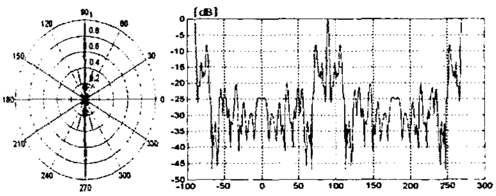

Figure 7 The directivity pattern of the ceramic disc in air at 200KHz in polar form (a) and in rectangular form with logarithmic scale (b).

Fig. 7 shows the directivity pattern of the ceramic disc in air at 200KHz in polar form (a) and in rectangular form with logarithmic scale (b). The projector sensitivity of the in-air ceramic disc at 200KHz is 139 dB re 1 μPa/V at the maximum peak of the directivity pattern. The directivity pattern of the ceramic disc in air is much narrower at the center than that in water while the projector sensitivity in air is about one tenth of that in water.

4. Conclusion

A coupled FE-BE method has been developed and applied to simulate a sonar transducer. The particular structure considered is a flooded piezoelectric disc. The transducer is three-dimensionally simulated to transduce external electrical charges on upper and lower surfaces of the disc to acoustic pressure in the far field. The acoustic field formed from the projected sound pressure is also simulated. And the displacement of the disc caused by the externally driven electrical charge is shown in temporal motion. The coupled FE-BE method is very useful for predicting the mechanical and the acoustical behaviour of the sonar transducer.

In general, as the frequency of external loading to the piezoelectric transducer is increased, more number of structural finite elements are necessarily required. Most of executing time of the coupled FE-BEM program is spent in matrix solution in which the size of the matrix is increased to 4ng by 4ng matrix as the number of global nodes are increased to ng. Therefore the present numerical method need to be linked to a faster matrix solver such as parallel processing for next work.

ACKNOWLEDGMENTS

This study was supported by research funds from Chosun University, 1998

References

[1] R.R. Smith, J.T. Hunt and D. Barach, "Finite element analysis of acoustically radiating structures with application to sonar transducers", J. Acoust. Soc. Am. vol. 54, No. 5, PP:1277-1288, 1973.

[2] O.C. Zienkiewicz, D. Kelly and P. Bettess, "The coupling of finite element method and boundary solutions procedures", Int. J. Num. Methods Eng., Vol. 11, PP:355-375, 1977.

[3] H. Allik and T.J.R. Hughes, "Finite element method for piezoelectric vibration", Int. J. Numer. Method Eng., Vol. 2, PP:151-157, 1970.

[4] Y. Kagawa and T. Yamabuchi, "Finite element approach for a piezoelectric circular rod", IEEE Trans. Sonics & Ultrasonics, Vol. su-23, No. 6, PP:379-385, 1976.

[5] N.F. Ivina, "Numerical analysis of the normal modes of circular piezoelectric plates of finite dimensions", Sov. Phys. Acoust., Vol. 35(4), PP:385-388, 1989.

[6] D. Boucher, M. Lagier and C. Maerfeld, "Computation of the vibrational modes for piezoelectric array transducers using a mixed finite element perturbation method", IEEE Trans. Sonics & Ultrasonics, Vol. su-28, No. 5, PP:318-330, 1981.

[7] J.N. Decarpigny, J.C. Debus, B. Tocquet and D. Boucher, "In-air analysis of piezoelectric tonpilz transducers in a wide frequency band using a mixed finite-plane wave method", J. Acoust. Soc. Am. Vol. 78, No. 5, PP:1499-1507, 1985.

[8] R. Bossut and J.N. Decarpigny, "Finite element modeling of radiating structures using dipolar damping elements", J. Acoust. Soc. Am. Vol. 86, No. 4, PP:1234-1244, 1989.

[9] B. Hamonic, J.C. Debus and J.N. Decarpigny, "Analysis of a radiating thin-shell sonar Transducer using the finite element method", J. Acoust. Soc. Am. Vol. 86, No. 4, PP:1245-1253, 1989.

[10] J.M.M.C. Marques and D.R.J. Owen, "Infinite elements in quasi-static materially nonlinear problems", Comp. & Struc., Vol. 18(4), PP: 739-751, 1984.

[11] J.P.E. Goransson and C.F. Davidsson, "A three dimensional infinite element for wave propagation", J. Sound & Vibration, Vol. 115(3), PP:558-559, 1987.

[12] L.G. Copley, "Integral equation method for radiation from vibrating bodies", J. Acoust. Soc. Am. Vol. 41, PP:807-816, 1967.

[13] L.G. Copley, "Fundamental results concerning integral representations in acoustic radiation", J. Acoust. Soc. Am. Vol. 44, PP:28-32, 1968.

[14] E. Skudrzyk, "The foundation of acoustics", (Springer-Verlag, New York, 1971), PP:408-409, Equation(76), 1971.

[15] D.T.I. Francis, "A boundary element method for the analysis of the acoustic field in three dimensional fluid-structure interaction problems", Proc. Inst. of Acoust., Vol. 12, Part 4, PP:76-84, 1990.

[16] H.A. Schenck, "Improved integral formulation for acoustic radiation problems", J. Acoust. Soc. Am. Vol. 44, PP:41-58, 1968.

[17] A.J. Burton and G.F. Miller, "The application of integral integration methods to the numerical solutions of some exterior boundary problems", Proc. R. Soc. London, Ser. A 323, PP:201-210, 1971.

[18] R.F. Kleinman and G.F. Roach, "Boundary integral equations for the three dimensional Helmholtz equation", SIAM Rev., Vol. 16, PP: 214-236, 1974.

[19] D.T.I. Francis, "A gradient formulation of the Helmholtz integral equation for acoustic radiation and scattering", J. Acoust. Soc. Am. Vol. 93(4) Part 1, PP:1700-1709, 1993.

[20] W.P. Mason, "Electromechanical transducer and wave filters", D.Van Nostrand Co. Inc., 2nd ed., New York, 1948.

[21] D. Stansfield, "Underwater electroacoustic transducers", Bath University Press and Institute of Acoustics, pp:159-163, 1991.

著 者 紹 介

장순석 (張淳晳)

『센서학회지 제7권 제6호』 논문98-7-6-01 p.1 참조

현재 조선대학교 전기제어계측공학부 부교수