논문 99-8-4-01

Simulation of magnetostrictive Terfenol-D rod dynamics using a coupled FE-BEM

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Abstract

This paper describes the application of the coupled FE-BEM (finite element-boundary element method) for the numerical harmonic analysis of the linear dynamic behaviour of a magnetostrictive Terfenol-D rod in water. The magnetostrictive rod is three-dimensionally simulated to transduce applied electric current in a helical coil around the rod to mechanical displacement. The theoretical derivation of the magnetostrictive matrix equation is described in detail. The steady-state resonance response of the displacement is shown. In addition, the directivity pattern and the radiation impedance are also shown.

1. Introduction

Recently, the sonar transducer of a new magnetostrictive material such as Terfenol-D, $Tb_{0.3}Dy_{0.7}Fe_2$, gains interest by many ocean scientists and engineers[1]. The features of this material is characterized as having giant strain, such that a maximum quantity of strain exceeds 2000 ppm, and a Young's modulus is as low as 20 GPa[2]. The sonar transducer of the particular magnetostrictive material is mainly designed for a low-frequency and high-power projector[3]. Magnetostrictive sonar projectors have been proved to produce 10 times higher power efficiency than the same size and weight of piezoelectric sonar projectors[4]. In order to design sonar transducers with magnetostrictive materials, modelling of the dynamic behaviour of magnetostrictive materials is required. Linearized finite element models, see e.g. Claeyssen[5], which have their origin in the modelling of piezoelectric materials, are still the most common method of modelling magnetostrictive materials[6]. Nonlinear dynamic models for Terfenol-D based on the finite difference method (FDM) are also suggested by Kvarnsjo and Engdahl[7]. Even though the requirements for developing a commercial program package for the sonar transducer design applicable to both piezoelectric and magnetostrictive materials are getting more interest, there are only a few packages available[8,9,10].

The main purpose of this paper is to simulate three dimensionally the magnetostrictive behaviour of the Terfenol-D rod using a coupled FE-BEM when the magnetostrictive transducer is driven in water by externally applied electric current in a helical coil around the rod. Particular emphasis is on the theoretical derivation of the magnetostrictive matrix equation such as virtual work principle instead of other variational principle and analysis of mechanical and acoustical characteristics.

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2. Finite element-boundary element modelling

2.1. Piezoelectric matrix equations

The following equation (1) is the integral formulation of the piezoelectric equations[11]:

$$[F]=[K'_{uu}][a]+[K'_{u\Phi}][\Phi]-\omega^2[M][a]+j\omega[R][a]$$
$$-[Q]=[K'_{\Phi u}][a]+[K'_{\Phi\Phi}][\Phi] \quad (1)$$

where

$[F]$ Applied mechanical force
$[Q]$ Applied electric charge
$[a]$ Elastic displacement
$[\Phi]$ Electric potential
$[K'_{uu}]$ Elastic stiffness matrix
$[K'_{u\Phi}]$ Piezoelectric stiffness matrix
$[K'_{\Phi u}]=[K'_{u\Phi}]^t$
$[K'_{\Phi\Phi}]$ Permittivity matrix
$[M]$ Mass matrix
$[R]$ Dissipation matrix
ω Angular frequency
ρ Fluid density
c Fluid sound speed

The formula of the piezoelectric matrix equations will be compared with those of the magnetostrictive matrix equations as shown in next section. Each quadratic three-dimensional element is composed of 20 nodes and each node has nodal displacement (a_x, a_y, a_z) and electric potential (Φ) variables[11].

2.2. Magnetostrictive matrix equations

Modelling of a magnetostrictive transducer in this paper is based on a virtual work principle in similarity with modelling of a piezoelectric transducer as shown in section 2.1. The active magnetostrictive material Terfenol-D is practically magnetized dynamically by a current in the driving coil and biased by either permanent magnets or DC offset currents, so that the dynamic magnetostrictive effects are assumed to be linear in modelling. The set of constitutive laws which govern the magnetostrictive transduction process can be modelled three-dimensionally by a pair of magnetostrictive equations applied to a single point in the bounded magnetostrictive continua in tensor form[12]:

$$T_{ij}=C^H_{ijkl}S_{kl}-e_{nij}H_n$$
$$B_m=e_{mkl}S_{kl}+\mu^S_{mn}H_n \quad (2)$$

where

T : Stress tensor
S : Strain tensor
B : Magnetic flux density vector [tesla]
H : Magnetic field intensity vector [A/m]
C^H : Constant-H stiffness
e : Piezomagnetic constant
μ^S : Constant-S permeability
$\mu_o=4\pi\cdot 10^{-7}$ $[H/m]$ in air

Also equation (2) can be expressed in matrix form

$$[T]=[C^H][S]-[e][H]$$
$$[B]=[e]^t[S]+[\mu^S][H] \quad (3)$$

And equation (4) shows the details of the matrix components of equation (3):

$$\begin{bmatrix}T_x\\T_y\\T_z\\T_{yz}\\T_{zx}\\T_{xy}\end{bmatrix}=\begin{bmatrix}C^x_x & C^x_y & C^x_z & C^x_{yz} & C^x_{zx} & C^x_{xy}\\ C^y_x & C^y_y & C^y_z & C^y_{yz} & C^y_{zx} & C^y_{xy}\\ C^z_x & C^z_y & C^z_z & C^z_{yz} & C^z_{zx} & C^z_{xy}\\ C^{yz}_x & C^{yz}_y & C^{yz}_z & C^{yz}_{yz} & C^{yz}_{zx} & C^{yz}_{xy}\\ C^{zx}_x & C^{zx}_y & C^{zx}_z & C^{zx}_{yz} & C^{zx}_{zx} & C^{zx}_{xy}\\ C^{xy}_x & C^{xy}_y & C^{xy}_z & C^{xy}_{yz} & C^{xy}_{zx} & C^{xy}_{xy}\end{bmatrix}\begin{bmatrix}S_x\\S_y\\S_z\\S_{yz}\\S_{zx}\\S_{xy}\end{bmatrix}-\begin{bmatrix}e^x_x & e^x_y & e^x_z\\ e^y_x & e^y_y & e^y_z\\ e^z_x & e^z_y & e^z_z\\ e^{yz}_x & e^{yz}_y & e^{yz}_z\\ e^{zx}_x & e^{zx}_y & e^{zx}_z\\ e^{xy}_x & e^{xy}_y & e^{xy}_z\end{bmatrix}\begin{bmatrix}H_x\\H_y\\H_z\end{bmatrix} \quad (4)$$

$$\begin{bmatrix}B_x\\B_y\\B_z\end{bmatrix}=\begin{bmatrix}e^x_x & e^y_x & e^z_x & e^{yz}_x & e^{zx}_x & e^{xy}_x\\ e^x_y & e^y_y & e^z_y & e^{yz}_y & e^{zx}_y & e^{xy}_y\\ e^x_z & e^y_z & e^z_z & e^{yz}_z & e^{zx}_z & e^{xy}_z\end{bmatrix}\begin{bmatrix}S_x\\S_y\\S_z\\S_{yz}\\S_{zx}\\S_{xy}\end{bmatrix}+\begin{bmatrix}\mu^x_x & \mu^x_y & \mu^x_z\\ \mu^y_x & \mu^y_y & \mu^y_z\\ \mu^z_x & \mu^z_y & \mu^z_z\end{bmatrix}\begin{bmatrix}H_x\\H_y\\H_z\end{bmatrix}$$

where x,y,z indicate the directions in the global cartesian coordinates system and the latter two superscripts of equation (3), H and S, are removed for convenience.

The magnetic field is expressed using the reduced-scalar-potential formulation which consists in a decomposition of the magnetic field into two parts [13];

$$[H]=[H^S]+[H^M] \quad (5)$$

where $[H^S]=[H^{S_o}]\cdot H_p$ is the magnetizing field, in which H_p is the source current in the coil p which creates $[H^{S_o}]$. $[H^{S_o}]$ is a term calculated analytically using the Biot-Savart law [14].

$$[H^{S_o}]=\frac{1}{4\pi}\int_\Omega \frac{[J]\times[r]}{|[r]|^3}d\Omega \quad (6)$$

where Ω is volume of current source, [J] is current source density vector at Ω, [r] is position vector from current source to node point. $[H^M]=-\nabla u_\phi \equiv -[D_\phi]\,u_\phi$ is the demagnetizing field, which can be expressed as the gradient of a reduced-scalar-potential u_ϕ [15] where.

$$[D_\phi]=\begin{bmatrix}\frac{\partial}{\partial x} & 0 & 0\\ 0 & \frac{\partial}{\partial y} & 0\\ 0 & 0 & \frac{\partial}{\partial z}\end{bmatrix}$$

Therefore equation (5) is given as equation (7) which follows Ampere' law neglecting eddy currents:

$$\begin{aligned}\nabla\times[H] &= \nabla\times([H^S]-\nabla u_\phi)\\ &= \nabla\times[H^S]-\nabla\times\nabla u_\phi=\nabla\times[H^S]=[J]\end{aligned} \quad (7)$$

where

$$\nabla\times\nabla u_\phi=\begin{vmatrix} i & j & k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \frac{\partial u_\phi}{\partial x} & \frac{\partial u_\phi}{\partial y} & \frac{\partial u_\phi}{\partial z}\end{vmatrix}$$

$$=i\left(\frac{\partial^2 u_\phi}{\partial y\partial z}-\frac{\partial^2 u_\phi}{\partial z\partial y}\right)+j\left(\frac{\partial^2 u_\phi}{\partial z\partial x}-\frac{\partial^2 u_\phi}{\partial x\partial z}\right)+k\left(\frac{\partial^2 u_\phi}{\partial x\partial y}-\frac{\partial^2 u_\phi}{\partial y\partial x}\right)=0$$

The magnetic flux density has to be conservative, that is,

$$\nabla\cdot[B] \equiv [D_\phi]^t[B]=0 \quad (8)$$

The strain is expressed in terms of the spatial derivatives of the displacement;

$$[S]=[D_u]\cdot[u]=\begin{bmatrix}\frac{\partial}{\partial x} & 0 & 0\\ 0 & \frac{\partial}{\partial y} & 0\\ 0 & 0 & \frac{\partial}{\partial z}\\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x}\\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0\end{bmatrix}\cdot[u] \quad (9)$$

The magnetostrictive equations are partial differential equations and their solution is therefore derived as an integral matrix formulation with boundary conditions. The elasticity of the magnetostrictive material obeys the Newton's law while the magnetics of the magnetic domain obeys the Maxwell's law as mentioned on next page. Since the basis of FEM is a direct integration process at the level of finitely discretized elements, the magnetostrictive equations are transformed to an integral form by means of various techniques such as the virtual work method, the variational method and the weighted residual method [16].

When some external energy is imposed on the magnetostrictive transducer, the applied energy is absorbed throughout the transducer and is transformed into another form. This relationship can be expressed in elemental matrix form as only one element is considered according to the virtual work principle [11]:

$$[\delta a]^t[F]=\int_V[\delta S]^t[T]dV+\int_V[\delta u]^t\rho[\ddot{u}]dV \quad (10a)$$

$$\begin{aligned}\int_V[\delta H]^t[B]dV &= \int_V[\delta H^M]^t[B]dV\\ &= \int_V[-[D_\phi]\delta u_\phi]^t[B]dV\\ &= \int_V-[\delta u_\phi]^t[D_\phi]^t[B]dV=0\end{aligned} \quad (10b)$$

since $[\delta H^S]=0$, $[H^M]=-[D_\phi]u_\phi$, $[D_\phi]^t\,[B]=0$ and ρ is the density of the material. In equation (10) δa and δu are a nodal displacement and a displacement in an element respectively. Equation (10a) is derived from Newton's law while Equation (10b) from Maxwell's law.

If equation(3) is applied to equation (10a),

$$
\begin{aligned}
[\delta a]^t\,[F] &= \int_V [\delta S]^t\,[T]dV + \int_V [\delta u]^t\,\rho\,[\ddot{u}]dV \\
&\cong \int_V [\delta S]^t\,[C^H][S]dV - \int_V [\delta S]^t\,[e][H]dV \\
&\quad + \int_V [\delta a]^t\,[N_u]^t\,\rho\,[N_u][\ddot{a}]dV \\
&\cong \int_V [\delta a]^t\,[A]^t\,[C^H][A][a]dV - \int_V [\delta a]^t\,[A]^t\,[e][H^M]dV \\
&\quad - \int_V [\delta a]^t\,[A]^t\,[e][H^S]dV - \omega^2 \int_V [\delta a]^t\,[N_u]^t\,\rho\,[N_u][a]dV \\
&\cong \int_V [\delta a]^t\,[A]^t\,[C^H][A][a]dV + \int_V [\delta a]^t\,[A]^t\,[e][D_M][\phi]dV \\
&\quad - \int_V [\delta a]^t\,[A]^t\,[e][H^S]dV - \omega^2 \int_V [\delta a]^t\,[N_u]^t\,\rho\,[N_u][a]dV
\end{aligned}
\tag{11}
$$

since

$[S]=[D_u][u]\cong[D_u][N_u][a]\equiv[A][a]$,

$[H^M]=-[D_\phi]u_\phi\cong-[D_\phi][N_\phi][\phi]\equiv-[D_M][\phi]$

Equation (11) is re-written without $[\delta a]^t$ as follows

$$
\begin{aligned}
&[F]+\int_V [A]^t[e][H^S]dV \\
&= \int_V [A]^t[C^H][A]dV[a] + \int_V [A]^t[e][D_M]dV[\phi] \\
&\quad - \omega^2 \int_V [N_u]^t\rho[N_u]dV[a]
\end{aligned}
\tag{12}
$$

Also when equation(3) is applied to equation (10b),

$$
\begin{aligned}
&\int_V -[\delta u_\phi]^t[D_\phi]^t[B]dV \\
&\cong \int_V -[\delta\phi]^t[N_\phi]^t[D_\phi]^t[e]^t[S]dV \\
&\quad + \int_V -[\delta\phi]^t[N_\phi]^t[D_\phi]^t[\mu^S][H]dV = 0 \\
&\Rightarrow \int_V [D_M]^t[e]^t[S]dV + \int_V [D_M]^t[\mu^S][H]dV = 0 \\
&\Rightarrow \int_V [D_M]^t[e]^t[A]dV[a] \\
&\quad + \int_V [D_M]^t[\mu^S][H^M+H^S]dV = 0
\end{aligned}
\tag{13}
$$

The last expression of equation (13) is re-written without $[\delta u_\phi]^t$;

$$
\begin{aligned}
&-\int_V [D_M]^t[\mu^S][H^S]dV \\
&= \int_V [D_M]^t[e]^t[A]dV[a] - \int_V [D_M]^t[\mu^S][D_M]dV[\phi]
\end{aligned}
\tag{14}
$$

The pairs of equation (12) and equation (14) are defined as magnetostrictive matrix equations and is given as short form as follows.

$$
\begin{aligned}
[F]+[I_u] &= [K_{uu}][a] + [K_{u\phi}][\phi] \\
&\quad - \omega^2[M][a] + j\omega[R][a] \\
-[I_\phi] &= [K_{\phi u}][a] + [K_{\phi\phi}][\phi]
\end{aligned}
\tag{15}
$$

where

$[I_u]$	Force induced by pre-determined magnetic field
$[I_\phi]$	Magnetic flux induced by pre-determined magnetic field
$[\phi]$	Reduced magnetic potential
$[K_{u\phi}]$	Magnetostrictive coupling matrix
$[K_{\phi u}]=[K_{u\phi}]^t$	
$[K_{\phi\phi}]$	Permeability (magnetic rigidity) matrix
$j\omega[R]$	Dissipation matrix

The volume integration of the magnetostrictive matrix equations is done in local coordinates by three-dimensional Gauss-qadrature formula and is independent of global shapes of individual elements [11,16]:

$$[I_u]=\int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1}[A]^t[e][H^S]|\,Ja\,|d\xi d\eta d\zeta$$

$$[I_\phi]=\int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1}[D_M]^t[\mu^S][H^S]|\,Ja\,|d\xi d\eta d\zeta$$

$$[K_{uu}]=\int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1}[A]^t[C^H][A]|\,Ja\,|d\xi d\eta d\zeta$$

$$[K_{u\phi}]=\int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1}[A]^t[e][D_M]|\,Ja\,|d\xi d\eta d\zeta$$

$$[K_{\phi u}] = \int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1}[D_M]^t\,[e]^t[A]|\,Ja\,|d\xi d\eta d\zeta$$

$$[K_{\phi\phi}] = -\int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1}[D_M]^t[\mu^S][D_M]|Ja|d\xi d\eta d\zeta$$

$$[M] = \int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1}[N_u]^t\rho[N_u]|\,Ja\,|d\xi d\eta d\zeta$$

$$[R] = \int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1}[N_u]^t\rho_R[N_u]|\,Ja\,|d\xi d\eta d\zeta$$

where ρ_R is a damping factor and ξ, η, ζ are local cartesian coordinates and Jacobian matrix [Ja] is derived as

$$[Ja] = \begin{bmatrix} \frac{\partial N^1}{\partial \xi} & \frac{\partial N^2}{\partial \xi} & \cdots & \frac{\partial N^n}{\partial \xi} \\ \frac{\partial N^1}{\partial \eta} & \frac{\partial N^2}{\partial \eta} & \cdots & \frac{\partial N^n}{\partial \eta} \\ \frac{\partial N^1}{\partial \zeta} & \frac{\partial N^2}{\partial \zeta} & \cdots & \frac{\partial N^n}{\partial \zeta} \end{bmatrix} \cdot \begin{bmatrix} x^1\; y^1\; z^1 \\ x^2\; y^2\; z^2 \\ \cdots \\ \cdots \\ \cdots \\ x^n\; y^n\; z^n \end{bmatrix} \quad (16)$$

where n=20 for 3 dimensional isoparametric elements and N is an isoparametric shape function. Any point in the global cartesian co-ordinates is mapped from the local coordinates;

$$x = [N^i][x^i]^t = [N^1, N^2, \ldots N^n][x^1, x^2, \ldots x^n]^t$$
$$y = [N^i][y^i]^t = [N^1, N^2, \ldots N^n][y^1, y^2, \ldots y^n]^t \quad (17)$$
$$z = [N^i][z^i]^t = [N^1, N^2, \ldots N^n][z^1, z^2, \ldots z^n]^t$$

The magnetostrictive matrix equations (15) are just analogy with the piezoelectric matrix equations (1). Because of this feature, the finite element code for the piezoelectric matrix equations could be used for solving of the magnetostrictive matrix equations for analysis of mechanical characteristics [5].

2.3. Boundary Element Method (BEM)

For sinusoidal steady-state problems, the Helmholtz equation, $\nabla^2\Psi + k^2\Psi = 0$, represents the fluid mechanics. Ψ is the acoustic pressure with time variation, $e^{j\omega t}$, and k(=ω/c) is the wave number. In order to solve the Helmholtz equation in an infinite fluid media, a solution to the equation must not only satisfy structural surface boundary condition (BC), $\frac{\partial\Psi}{\partial n} = \rho_f\,\omega^2 a_n$, but also the radiation condition at infinity, $\lim_{|r|\to\infty}\oint_S(\frac{\partial\Psi}{\partial r} + jk\Psi)^2 dS = 0$. $\frac{\partial}{\partial n}$ represents differentiation along the outward normal to the boundary. ρ_f and a_n are the fluid density and the normal displacement on the structural surface. The Helmholtz integral equations derived from Green's second theorem provides such a solution for radiating pressure waves;

$$\oint_S\left(\Psi(q)\frac{\partial G_k(p,q)}{\partial n_q} - G_k(p,q)\frac{\partial\Psi(q)}{\partial n_q}\right)dS_q = \beta(p)\;\Psi(p) - \Psi_{inc}(p) \quad (18)$$

where $G_k(p,q) = \frac{e^{-jkr}}{4\pi r}$, $r = |p-q|$

p is any point in either the interior or the exterior and q is the surface point of integration. β(p) is the exterior solid angle at p.

The acoustic pressure for the i^{th} global node, $\Psi(p_i)$, is expressed in discrete form[17]:

$$(\; 1 \le i \le ng \;)$$

$$\beta(p_i)\Psi(p_i) - \Psi_{inc}(p_i)$$
$$= \oint_S\left(\Psi(q)\frac{\partial G_k(p_i,q)}{\partial n_q} - G_k(p_i,q)\frac{\partial\Psi(q)}{\partial n_q}\right)dS_q \quad (19a)$$

$$= \sum_{m=1}^{nt}\int_{S_m}\left(\Psi(q)\frac{\partial G(p_i,q)}{\partial n_q} - G(p_i,q)\frac{\partial\Psi(q)}{\partial n_q}\right)dS_q$$

q∈S sub m (19b)

$$= \sum_{m=1}^{nt}\int_{S_m}\left|\begin{matrix}\sum_{j=1}^{8}N_j(q)\,\Psi_{m,j}\frac{\partial G(p_i,q)}{\partial n_q} \\ -\,G(p_i,q)\sum_{j=1}^{8}N_j(q)\frac{\partial\Psi_{m,j}}{\partial n_q}\end{matrix}\right|dS_q \quad (19c)$$

$$= \sum_{m=1}^{nt}\sum_{j=1}^{8}\left(\int_{S_m} N_j(q)\frac{\partial G(p_i,q)}{\partial n_q}dS_q\right)\Psi_{m,j} - \rho_f\,\omega^2\sum_{m=1}^{nt}\sum_{j=1}^{8}\left(\int_{S_m} N_j(q)\,G(p_i,q)n_q dS_q\right)a_{m,j} \tag{19d}$$

$$= \sum_{m=1}^{nt}\sum_{j=1}^{8}A^i_{m,j}\Psi_{m,j} - \rho_f\,\omega^2\sum_{m=1}^{nt}\sum_{j=1}^{8}B^i_{m,j}a_{m,j} \tag{19e}$$

where nt is the total number of surface elements and $a_{m,j}$ are three dimensional displacements. Equation (19b) is derived from equation (19a) by discretizing integral surface. And equation (19c) is derived from equation (19b) since an acoustic pressure on an integral surface is interpolated from adjacent 8 quadratic nodal acoustic pressures corresponding the integral surface. $N_j(q)$is a surface shape function. Then equation (19d) is derived from equation (19c) by swapping integral notations with summing notations. Finally the parentheses of equation (19d) is expressed by upper capital notations for simplicity.

When equation (19e) is globally assembled, the discrete Helmholtz equation can be represented as

$$([A]-\beta[I])\{\Psi\} = +\rho_f\,\omega^2[B]\{a\} - \{\Psi_{inc}\} \tag{20}$$

where [A] and [B] are square matrices of (ng by ng) size. ng is the total number of surface nodes.

Where the impedance matrices of equation (20), [A] and [B], are computed, two types of singularity arise[18]. One is that the Green's function of the equation, $G_k(p_i, q)$, becomes infinite as q approaches to p_i. This problem is solved by mapping such rectangular local coordinates into triangular local coordinates and again into polar local coordinates[19]. The other is that at certain wave number the matrices become ill-conditioned. These wave number are corresponding to eigenvalues of the interior Dirichlet problem[20]. One approach to overcome the matrix singularity is that [A] and [B] of equation (20) are modified to provide a unique solution for the entire frequency range[21-24]. The modified matrix equation referred to as the modified Helmholtz gradient formulation (HGF)[24] is obtained by adding a multiple of an extra integral equation to equation (20).

$$([A]-\beta[I]\oplus\alpha[C])\{\Psi\} = +\rho_f\,\omega^2([B]\oplus\alpha[D])\{a\} - (\Psi_{inc}\oplus\alpha\frac{\partial\Psi_{inc}}{\partial n_p}) \tag{21}$$

where

$$\alpha = -\frac{\sqrt{-1}}{k\cdot(\text{Number of surface elements adjacent a surface node})}$$

[C] and [D] are rectangular matrices of (nt by ng) size. nt is the total number of surface elements. ⊕ symbol indicates that the rows of [C],[D] corresponding to surface elements adjacent a surface node are added to the row of [A],[B] corresponding to the surface node, that is,

$$\sum_{i=1}^{ng}\sum_{j=1}^{ng}A(i,j) = \sum_{i=1}^{ng}\sum_{j=1}^{ng}A(i,j) + \sum_{i=1}^{ng}\sum_{j=1}^{ng}\left(\sum_{m=1}^{S(i)}\alpha C(m,j)\right)$$
$$\sum_{i=1}^{ng}\sum_{j=1}^{ng}B(i,j) = \sum_{i=1}^{ng}\sum_{j=1}^{ng}B(i,j) + \sum_{i=1}^{ng}\sum_{j=1}^{ng}\left(\sum_{m=1}^{S(i)}\alpha D(m,j)\right) \tag{22}$$

where S(i) is the number of surface element adjacent a surface node. The derivation of the extra matrices [C],[D] are well described by Francis D.T.I.[24]. Equation (22) may be reduced in its formulation using superscript ⊕ for convenience;

$$A^{\oplus}\{\Psi\} = +\rho_f\,\omega^2 B^{\oplus}\{a\} - \Psi^{\oplus}_{inc} \tag{23}$$

where

$$([A]-\beta[I]\oplus\alpha[C]) \equiv A^{\oplus}$$
$$([B]\oplus\alpha[D]) \equiv B^{\oplus}$$
$$(\Psi_{inc}\oplus\alpha\frac{\partial\Psi_{inc}}{\partial n_p}) \equiv \Psi^{\oplus}_{inc}$$

Equation (23) can be written as

$$\{\Psi\} = +\rho_f\,\omega^2(A^{\oplus})^{-1}B^{\oplus}\{a\} - (A^{\oplus})^{-1}\Psi^{\oplus}_{inc} \quad (24)$$

2.4. Coupled FE-BEM

The coupled FE-BEM is composed of equation (15) and fluid loading matrices[25]. The pairs of equation (25) are defined as magnetostrictive matrix equations representing flooded magnetostrictive sonar transducer:

$$\begin{aligned}[F]+[I_u] &= [K_{uu}][a]+[K_{u\phi}][\phi]-\omega^2[M][a] \\ &\quad +j\omega[R][a]+[\rho_f\,\omega^2[L](A^{\oplus})^{-1}B^{\oplus}][a] \\ -[I_\phi] &= [K_{\phi u}][a]+[K_{\phi\phi}][\phi] \end{aligned} \quad (25)$$

where

$[L]$	Coupling matrix at the fluid-structure interface
ρ_f	Fluid density
$A^{\oplus}$	Fluid BEM matrix [A]
$B^{\oplus}$	Fluid BEM matrix [B]

2.5. Terfenol-D magnetostrictive rod

Table 1 shows the three-dimensional material properties of the magnetostrictive Terfenol-D which has fully orthotropic properties[26].

Table 1. Magnetostrictive material Properties of (axially polarized properties) Terfenol-D

		Unit			Unit
ρ	9100	Kg/m³	C_{xy}^{xy}	0.1587E+11	N/m²
C_x^x	1.0673E+11	N/m²	e_z^x	-89.8876	T
C_y^x	0.7478E+11	N/m²	e_z^y	-89.8876	T
C_z^x	0.8211E+11	N/m²	e_z^z	166.2921	T
C_y^y	1.0673E+11	N/m²	e_z^{yz}	167.6647	T
C_z^y	0.8211E+11	N/m²	e_z^{zx}	167.6647	T
C_z^z	0.9810E+11	N/m²	μ_x^x	0.3976E-5	Tm/A
C_{yz}^{yz}	0.0599E+11	N/m²	μ_y^y	0.3976E-5	Tm/A
C_{zx}^{zx}	0.0599E+11	N/m²	μ_z^z	0.2865E-5	Tm/A
K_{33}	0.68		K_{15}	0.74	

4. Results and Discussion

The coupled FE-BEM has been used for the numerical analysis of the Terfenol-D structural dynamics in water. Calculation is done in double precision. The diameter and the Z-axial length of the material are 2.5mm and 25mm respectively. The Terfenol-D rod is divided into 80 finite elements (=8 in circumference x 10 in axial sections) (see the cylindrical elements in Fig. 1). Since the Terfenol-D rod is excited by external currents, a thin (0.1mm) helical coil is assumed to wind around the rod. By this solenoid-like coil, external exciting currents are applied to the Terfenol-D rod. Then a magnetic field by the applied current is formed. The magnetic flux should be continuous through the rod and across the water space surrounding the rod (flux conservation). As explained in section 2.2, the $[H^S]$ should be first calculated separately as boundary conditions for the magnetostrictive problem.

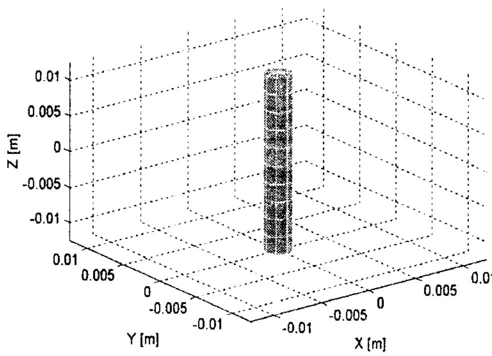


Figure 1. The Terfenol-D rod modelled by finite cylindrical elements.

The outer surface of the surrounding water elements is assumed to have zero magnetic flux condition. It is always desired to have more elements to represent smaller local regions or for higher frequency which requires more time in analysis. Therefore meshing size of elements depends on the upper limit of interest frequency. It is a common criteria to have the size of the largest element to be less than $\lambda/3$. In this paper the interest frequency of the acoustic radiation is less than 96kHz, so that $\lambda/3$ is about 5.2 mm.

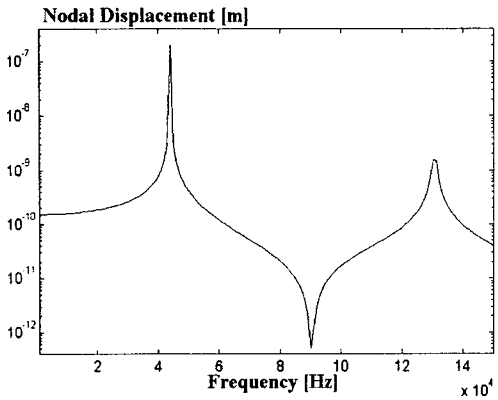


Figure 2. Displacement response of the magnetostrictive rod in water.
Resonant frequency = 44 kHz.

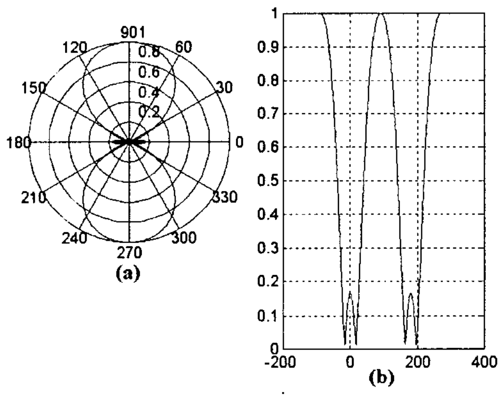
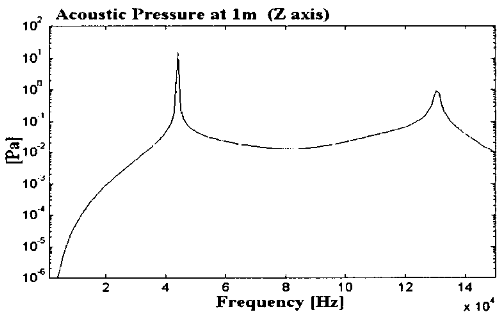
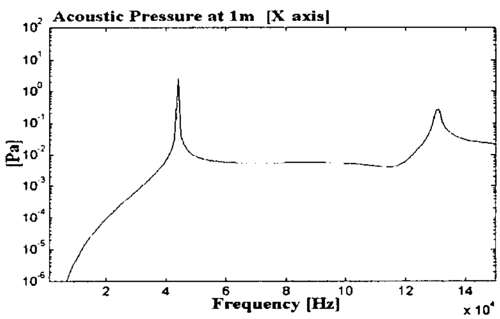


Figure 3. The directivity pattern of the thin and long magnetostrictive rod in water at 44 kHz. (a) in polar form (b) in rectangular form.

Fig. 2 shows the amplitude of the Z-axial displacement as a function of frequency. The resonant frequency of the displacement is about 44 kHz. The second and the third resonant frequencies are rightly appeared at three and five times of the fundamental frequency. Fig. 3 shows the directivity pattern of the rod at the resonant frequency. The main beam is shown in the Z-axis as expected. And Fig. 4 shows the acoustic pressure responses at 1m from the sound source in water as a function of frequency. Fig 4(a) and Fig. 4(b) are for the Z axis and the X axis characteristics respectively. The Z-axis pressure magnitude is always greater than that of X-axix throughout the given frequency range. Finally Fig. 5 shows the real value (Zr: resistance) and the imaginary value (Zi: reactance) of the acoustic radiation impedance as a function of frequency for the radiating magnetostrictive rod in water.



(a)



(b)

Figure 4. The acoustic pressure responses at 1m. (a) Z-axis, (b) X-axis.

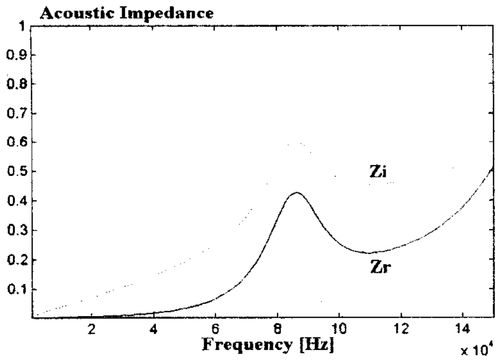


Figure 5. Radiation impedance of a thin magnetostrictive rod in water.
Zr: Resistance Zi: Reactance

4. Conclusion

The coupled FE-BEM has been applied to simulate three-dimensionally a magnetostrictive Terfenol-D transducer. The magnetostrictive rod is linearly simulated in water to transduce external electrical currents in a helical coil around the rod. The steady-state frequency response of the Z-axial displacement indicates the resonant frequency in axial mode. And the directivity pattern and acoustic pressure response are also shown. The theoretical derivation of the magnetostrictive matrix equation such as virtual work principle instead of other variational principle is described in detail.

Acknowledgements

This study was supported by research funds from Chosun University, 1998.

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『센서학회지 제7권 제6호』 논문98-7-6-01 p.1참조

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