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Running Inverse for Implementing Decorrelating Detectors in Synchronous DS/CDMA Systems

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요 약

본 논문에서는 decorrelating 검출기를 사용하는 동기식 직접 시퀀스 부호 분할 다중 접속 시스템에서 동시 사용자의 수가 변하게 될 때 사용자들에게 할당된 확산 부호간의 상호 상관 행렬의 역행렬을 효율적으로 구하는 방법을 제안한다. 시스템의 사용자 수가 n명인 경우 제안된 방법을 사용하여 역행열을 구하게 되면 요구되는 계산량이 $O(n^2)$임을 보인다

Abstract

We propose an efficient method for updating the inverse of the signature waveform cross-correlations (SWC) matrix when the number of users in the synchronous direct-sequence code-division multiple-access (DS/CDMA) system changes. It is shown that the computational complexity of the proposed method is $O(n^2)$ in which n represents the number of active users in the system.

I. Introduction

We consider a synchronous DS/CDMA system where the receiver observes the sum of synchronously transmitted signals from several users in additive noise. Synchronous systems are becoming more of practical interest since quasi-synchronous approach has been proposed for satellite [1] and microcell applications [2]. In this synchronous DS/CDMA system case, the decorrelating detector has been considered as a basic tool for multiuser detectors [2]-[5]. In this work, therefore, we consider the system which takes the decorrelating detector. Whenever the number of active users in this system changes, computation of the inverse of SWC matrix should be needed--this usually requires heavy computation. In an effort to reduce this computational load, we propose an efficient method for updating the inverse based on the symmetry property of the matrix and the current values of the inverse. It is shown that the computational complexity of the proposed method is $O(n^2)$ in which n represents the number of active users in the system.

II. System Model and Decorrelating Detector

We consider BPSK transmission through an AWGN channel shared by K synchronous users employing a direct sequence spread spectrum modulation. K is the available maximum number of users who can use the

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system simultaneously. The kth user's data signal $b_k(t)$ is given by $b_k(t)=\sum_{i=-\infty}^{\infty} b_k^{(i)} p_T(t-iT)$ where $p_T(t)=1$ for $0\le t< T$ and $p_T(t)=0$, otherwise; $b_k^{(i)}\in\{\pm 1\}$ denotes the ith transmitted bit of the k th user. A set of normalized signature waveforms $s_k(t)$, $k=1,\cdots,K$ where each waveform is restricted to a symbol interval of duration T, i.e., $\int_0^T s_i^2(t)\,dt=1$, is considered and these signature waveforms are assumed to be linearly independent. The received signal can be written in the form

$$r(t) = \sum_{i=-\infty}^{\infty}\sum_{k=1}^{K}\sqrt{2P_k}\, b_k^{(i)} s_k(t-iT) + n(t) \quad (1)$$

where P_k is the received power of the kth user's signal; and $n(t)$ is additive white zero-mean Gaussian noise with two sided power spectral density $N_0/2$. The sampled output of the kth matched filter at the end of ith time interval is expressed as $y_k^{(i)}=\int_{iT}^{(i+1)T} r(t)s_k(t-iT)\,dt$ and therefore $\boldsymbol{y}^{(i)}=(y_1^{(i)},\cdots,y_K^{(i)})^T$ are sufficient statistics for demodulating $\boldsymbol{b}^{(i)}=(b_1^{(i)},\cdots,b_K^{(i)})^T$. By using vector notation, the following equation can be obtained:

$$\boldsymbol{y}^{(i)} = \boldsymbol{R}\,\boldsymbol{W}\,\boldsymbol{b}^{(i)} + \boldsymbol{n}^{(i)} \quad (2)$$

where $\boldsymbol{R}$ which is called the signature waveform cross-correlations (SWC) matrix is positive definite and symmetric: $R_{ij}=\int_0^T s_i(t)s_j(t)\,dt$ and $\boldsymbol{W}$ is a diagonal matrix with $W_{ii}=\sqrt{2P_i}$, $i=1,\cdots,K$. $\boldsymbol{n}^{(i)}$ is a zero-mean Gaussian noise vector with the $K\times K$ autocorrelation matrix equal to $(N_0/2)\boldsymbol{R}$. In the decorrelating detector, the decorrelating filter $\boldsymbol{R}^{-1}$ followed by a set of decision devices (sign detection) is applied to $\boldsymbol{y}^{(i)}$ (Fig. 1), i.e., $\hat{\boldsymbol{b}}^{(i)}=sgn(\boldsymbol{R}^{-1}\boldsymbol{y}^{(i)})$ where $\hat{\boldsymbol{b}}^{(i)}$ is the decision vector for $\boldsymbol{b}^{(i)}$.

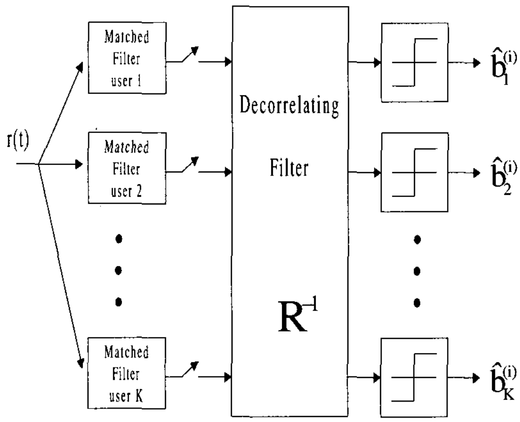

Fig 1. Decorrelating detector for synchronous DS/CDMA systems.

III. Running inverse for implementing decorrelating detector

We assume a set of signature waveforms is fixed and $\boldsymbol{R}$ is precomputed and known to the receiver. Since the number of active users in the system is not fixed to K, however, we must update inverse of the SWC matrix whenever the number of active users changes. We assume that only one new user is admitted to the system at a time. If we compute the inverse directly at each time, computational complexity is proportional to the third power of the number of active users [6]. In this section, therefore, we discuss an efficient method for updating inverse of the SWC matrix based on the fact that the current SWC matrix is symmetric and the inverse of it is known. In the case of addition of a user, specifically, the proposed method is the extension of the formula on the inverse of block matrix [7] to the general case in which a new user is inserted between previous active users for some objective such as reordering. Due to the symmetric property, furthermore, this method has simpler form. We define $\boldsymbol{R}(n)$ as the SWC matrix for n users and this is a sub-matrix of $\boldsymbol{R}$ and symmetric. For the convenience, dimensions of the all matrices used in this section are summarized in Table 1.

Table 1. Dimensions of matrices.

	Case A	Case B
$\boldsymbol{R}_1, \boldsymbol{P}_1$	$(l-1)\times(l-1)$	$(l-1)\times(l-1)$
$\boldsymbol{R}_2, \boldsymbol{P}_2$	$(l-1)\times(n-l)$	$(l-1)\times(n-l+1)$
$\boldsymbol{R}_3, \boldsymbol{P}_3$	$(n-l)\times(n-l)$	$(n-l+1)\times(n-l+1)$
$\boldsymbol{d}_1, \boldsymbol{e}_1$	$(l-1)\times 1$	$(l-1)\times 1$
$\boldsymbol{d}_2, \boldsymbol{e}_2$	$(n-l)\times 1$	$(n-l+1)\times 1$

Case A. Removal of a user

We assume that the number of active users decreases from n to $n-1$; n is a positive integer and $n \leq K$. We assume that the lth user was removed, and then the current SWC matrix, $\boldsymbol{R}(n)$ can be written in the form

$$\boldsymbol{R}(n)=\begin{bmatrix} \boldsymbol{R}_1 & \boldsymbol{d}_1 & \boldsymbol{R}_2 \\ \boldsymbol{d}_1^T & 1 & \boldsymbol{d}_2^T \\ \boldsymbol{R}_2^T & \boldsymbol{d}_2 & \boldsymbol{R}_3 \end{bmatrix} \quad (3)$$

and from this, the updated $(n-1)\times(n-1)$ SWC matrix $\boldsymbol{R}(n-1)$ can be represented as

$$\boldsymbol{R}(n-1)=\begin{bmatrix} \boldsymbol{R}_1 & \boldsymbol{R}_2 \\ \boldsymbol{R}_2^T & \boldsymbol{R}_3 \end{bmatrix} \quad (4)$$

Since $\boldsymbol{R}(n)$ is symmetric, $\boldsymbol{R}(n)^{-1}$ is also symmetric and can be consequently written as

$$\boldsymbol{R}(n)^{-1} = \begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{e}_1 & \boldsymbol{P}_2 \\ \boldsymbol{e}_1^T & c & \boldsymbol{e}_2^T \\ \boldsymbol{P}_2^T & \boldsymbol{e}_2 & \boldsymbol{P}_3 \end{bmatrix} \quad (5)$$

where c is a constant. From $\boldsymbol{R}(n)\,\boldsymbol{R}(n)^{-1}=I$, we obtain

$$\begin{bmatrix} R_1P_1+d_1e_1^T+R_2P_2^T & R_1e_1+d_1c+R_2e_2 & R_1P_2+d_1e_2^T+R_2P_3 \\ d_1^TP_1+e_1^T+d_2^TP_2^T & d_1^Te_1+c+d_2^Te_2 & d_1^TP_2+e_2^T+d_2^TP_3 \\ R_2^TP_1+d_2e_1^T+R_3P_2^T & R_2^Te_1+d_2c+R_3e_2 & R_2^TP_2+d_2e_2^T+R_3P_3 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{bmatrix} \quad (6)$$

By solving the nine equations in (6), we can obtain the following representation of $\boldsymbol{R}(n-1)^{-1}$

$$R(n-1)^{-1}=\begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}+\frac{1}{c}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}\begin{bmatrix} d_1^T & d_2^T \end{bmatrix}\begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}$$

$$R(n-1)^{-1}= \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} + \frac{1}{c}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}\begin{bmatrix} d_1^T & d_2^T \end{bmatrix}\begin{bmatrix} P_1 & P \\ P_2^T & P \end{bmatrix} \quad (7)$$

where $\boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_3, \boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{e}_1, \boldsymbol{e}_2$, and C are known because these are elements of $\boldsymbol{R}(n)$ and $\boldsymbol{R}(n)^{-1}$. Note that the updated inverse can be obtained by multiplying the sub-matrix of $\boldsymbol{R}(n)^{-1}$ which has the elements in the same locations as $\boldsymbol{R}(n-1)$ in $\boldsymbol{R}(n)$ by some additional matrix. We can calculate the number of multiplications and additions which are needed for the computation of $\boldsymbol{R}(n-1)^{-1}$ and the result is shown in Table 2.

Table 2. Computational complexity.

	Multiplication	Addition
Case A	$2n^2-3n+1$	$2n^2-3n+2$
Case B	$3n^2+2n+1$	$3n^2-n$

Case B. Addition of a user

We assume that the number of active users increases from n to $n+1$; n is a positive integer and $n<K$. We assume that a new user is inserted in between the $(l-1)$th and the lth users, and then a new SWC matrix $R(n+1)$ can be written in the form

$$R(n+1)=\begin{bmatrix} R_1 & d_1 & R_2 \\ d_1^T & 1 & d_2^T \\ R_2^T & d_2 & R_3 \end{bmatrix} \tag{8}$$

and

$$R(n)=\begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix}. \tag{9}$$

The inverse matrix $R(n+1)^{-1}$ which is to be computed can be expressed as

$$R(n+1)^{-1}=\begin{bmatrix} P_1 & e_1 & P_2 \\ e_1^T & c & e_2^T \\ P_2^T & e_2 & P_3 \end{bmatrix}. \tag{10}$$

Though we use the same notation, the values of matrices used in this subsection are different from those of subsection A. By solving $R(n+1)\ R(n+1)^{-1}=I$, we obtain

$$c=\frac{1}{1-\begin{bmatrix} d_1^T & d_2^T \end{bmatrix} R(n)^{-1}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}} \tag{11}$$

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=-c\,R(n)^{-1}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}=R(n)^{-1}-R(n)^{-1}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}\begin{bmatrix} e_1^T & e_2^T \end{bmatrix} \tag{13}$$

where d_1, d_2 and $R(n)^{-1}$ are known. From these equations and (10), we can obtain $R(n+1)^{-1}$. It is worthwhile noting that the same results can be obtained by using the formula on the inverse of block matrix [7] in the case a new user is ordered to be last. The number of multiplications and additions which are needed for the computation of $R(n+1)^{-1}$ is also shown in Table 2.

IV. Conclusion

A new method for computation of the inverse of the SWC matrix was proposed and its computational complexity was also calculated. It was found that because of its low complexity, this proposed method is very efficient in case the number of active users in the system is often varied.

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