

Production switching mechanism for an unreliable two-stage production line

고장이 있는 두단계 생산라인의 생산률 변환정책

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Abstract

This paper deals with a production line which consists of two production stages that are separated by a finite storage buffer. The inventory level in the storage buffer controls the production rate of the preceding stage. That is, the production rate becomes high (low) when the buffer inventory is low (high). We analyze the system characteristics utilizing the Markov process theory and then find an optimal control policy which maximizes a given system profit function. Also, a sensitivity analysis is made to examine the effects of various system parameters on the system performances.

1. Introduction

Automatic transfer lines are frequently adopted for mass production systems. In these lines, workpieces pass through successive machines with specific operations being performed at each machine. A major causes of line inefficiency are machine breakdowns and unbalances in processing times. The storage buffers between two adjacent machines are used to reduce the harmful effects of interferences between the machines. In this paper, we consider an automatic transfer line which consists of two unreliable machines with random processing times and a finite buffer storage, as shown in Figure 1. In the system, raw production units come from the outside of the line

and are processed at the first (preceding) machine. Then they move to the buffer, and are processed at the second (succeeding) machine before the units leave the system. Each machine has a random processing time and is subject to failure while processing. Repair is started as soon as it fails. During the time the preceding machine is under repair, the succeeding machine continues to process the units supplied from the buffer storage. With a long repair time, the buffer may become empty. Consequently, the succeeding machine has no units to process and is forced to stop, or 'starved'. Similarly, when the buffer becomes full due to the failure of the succeeding machine, the preceding machine may be forced to be idle, or 'blocked'. Even without machine failures, the blocking and starvation

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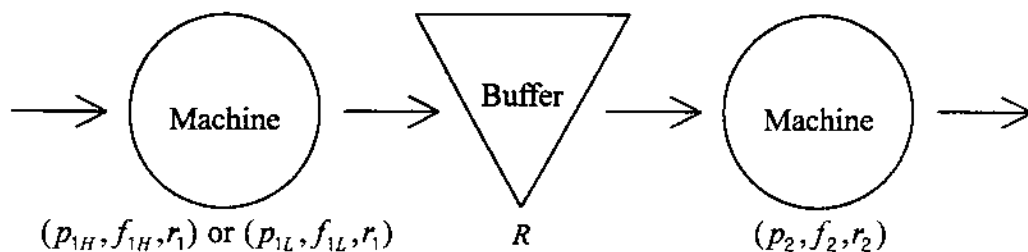


Figure 1. Automatic transfer line with two machines

can occur due to the variability in processing times at the two machines.

Such two-machine production systems have been extensively studied. Buzacott and Hanifin [2] presented a survey paper on related topics. Gershwin and Berman [5] introduced an efficient analytical method for the system with discrete workpieces, finite buffer, random processing times, and random failures of each machine. Meyer et al. [11] discussed a variant of the system in which the succeeding machine does not break down. Buzacott and Kostelski [3] developed algorithms: a matrix-geometric algorithm and a recursive algorithm in a generalized two-machine production model. The objective of all these researches is the determination of an optimum buffer size.

Some researchers discussed the control aspect of the system. Hopp et al. [7] and Hwang and Koh [9] studied switching policies in continuous and discrete flow lines, respectively. Their switching methods are similar to (s, ζ) inventory model.

There exists another type of switching mechanism. Yadin and Naor [13] analyzed a queueing model in which the service rate is variable in accordance with the queue length. This concept was applied to production scheduling problems by Barman and Burch [1], Hwang and Cha [8], and Mellichamp and Love [10]. In their studies, the production levels are restricted to certain finite number of levels (e.g., low, normal and high) and a specific level is determined for each production period on the basis of the current inventory and forecasted demand. This paper

is an extension of Hwang and Koh [9] and utilize the concept of the production switching heuristic. That is, the preceding machine produces items at a high rate to prevent starvation of the succeeding machine when the inventory level in the buffer is low. On the other hand, if the inventory level is high, the production rate of the preceding machine becomes low to reduce the inventory holding cost.

In this study, random machine failures and random processing times are assumed in the both machines. It is also assumed that each machine can be described by three exponentially distributed random variables, i.e., processing time, time to failure, and repair time. The (R, r) policy is adopted to control the production rate of the preceding machine. Under the policy, the preceding machine produces items at a high rate when the buffer inventory level is less than r , and at a low rate when the level is greater than or equal to r . The machine stops its production when the buffer inventory is greater than R (in other words, when the preceding machine has just produced a unit while the inventory level is R).

2. Model development

2.1. Model description and assumption

The system consists of two machines which are separated by a finite buffer as shown in Figure 1. In order to describe the system, we define the system state with three variables as follows:

n = inventory level in the buffer,

$$\alpha = \begin{cases} B, & \text{if the preceding machine is blocked} \\ 1, & \text{if the preceding machine is processing a unit} \\ 0, & \text{if the preceding machine is under repair} \end{cases}$$

$$\beta = \begin{cases} S, & \text{if the succeeding machine is starved} \\ 1, & \text{if the succeeding machine is processing a unit} \\ 0, & \text{if the succeeding machine is under repair} \end{cases}$$

With these variables, the system state is expressed by (α, β, n) . For example, $(1, S, 0)$ implies that the preceding machine is processing a unit, the succeeding machine is starving, and the storage buffer is empty. Also, $(0, 1, n)$ implies that the preceding machine is under repair, the succeeding machine is processing a unit, and inventory level in the storage buffer is n . Therefore, the state space E and the total number of states N are

$$E = \{(0, S, 0), (1, S, 0), (0, 0, n), (0, 1, n), (1, 0, n), (1, 1, n), n = 0, 1, \dots, R, (B, 0, R), (B, 1, R)\} \tag{1}$$

$$N = 2 + 4(R + 1) + 2 = 4R + 8. \tag{2}$$

Let $P(\alpha, \beta, n)$ be the steady state probability that the system is in the corresponding state in the long run average manner. To find these steady state probabilities, the following assumptions are made:

- 1) The blocked or starved machine is not vulnerable to failure.
- 2) The repair facilities are always available to start repair as soon as machine failure occurs.
- 3) When a machine fails, the unit being processed remains on that machine. The machine resumes processing the unit as soon as it is repaired.
- 4) The processing time of the preceding machine is an exponential random variable whose mean is $1/P_{iH}$ if the inventory level is less than r and $1/P_{iL}$, otherwise.

$$(P_{iL} \leq P_{iH})$$

- 5) The time to failure of the preceding machine is an exponential random variable whose mean is $1/f_{iH}$ when the machine produces items at a high rate and $1/f_{iL}$, otherwise. ($f_{iL} \leq f_{iH}$)
- 6) The processing time of the succeeding machine is an exponential random variable with mean $1/P_s$.
- 7) The time to failure of the succeeding machine is an exponential random variable with mean $1/f_s$.
- 8) The repair times of the preceding and succeeding machines are exponential random variables with means $1/r_1$ and $1/r_2$, respectively.
- 9) All the above exponential random variables are mutually independent.

2.2. Markov process model

In this section, $p(\alpha, \beta, n)$ is determined through the discrete Markov Process Theory. First, we find $q(\alpha, \beta, n)$ which is the probability that the state after transition is (α, β, n) . Define steady state probability distribution vector of $q(\alpha, \beta, n)$ as

$$q = (q_s, q_b, q_0, \dots, q_n, \dots, q_R, q_B) \tag{3}$$

where

$$\begin{aligned} q_s &= \{ q(0,S,0), q(1,S,0) \}, \\ q_n &= \{ q(0,0,n), q(0,1,n), q(1,0,n), q(1,1,n) \}, n=0,1,\dots,R. \\ q_B &= \{ q(B,0,R), q(B,1,R) \}. \end{aligned}$$

According to Cinlar [4], there is a unique solution to

$$q = qQ' \text{ and } qe = 1, \tag{4}$$

where Q' is the $N \times N$ transition matrix of the system and $e = (1, 1, \dots, 1)^T$ of size N . The first equation of (4) can be changed into

$$qQ = 0, \tag{5}$$

where $Q = Q' - I$, I is $N \times N$ identity matrix, and $0 = (0, 0, \dots, 0)$ of size N . Using a property of independent exponential random variables, one can derive the matrix Q as follows:

$$\begin{matrix}
 & S & 0 & 1 & 2 & \cdots & r-1 & r & r+1 & \cdots & R-1 & R & B \\
 \begin{matrix} S \\ 0 \\ 1 \\ \dots \\ Q= r-1 \\ r \\ \dots \\ R-1 \\ R \\ B \end{matrix} & \left[\begin{array}{cccccccccccc}
 S_1 & S_2 & & & & & & & & & & & \\
 Q_s & Q_s & Q_s & & & & & & & & & & \\
 Q_1 & Q_2 & Q_3 & & & & & & & & & & \\
 \dots & & & \dots & & & & & & & & & \\
 & & & & Q_1 & Q_2 & Q_3 & & & & & & \\
 & & & & & Q_4 & Q_5 & Q_6 & & & & & \\
 & & & & & & & \dots & & & & & \\
 & & & & & & & & Q_4 & Q_5 & Q_6 & & \\
 & & & & & & & & & Q_4 & Q_5 & Q_6 & \\
 & & & & & & & & & & B_1 & B_2 &
 \end{array} \right]
 \end{matrix} \quad (6)$$

where

$$S_1 = \begin{bmatrix} -1 & 1 \\ \frac{f_{1H}}{P_{1H}+f_{1H}} & -1 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{P_{1H}}{P_{1H}+f_{1H}} \end{bmatrix},$$

$$Q_s = \begin{bmatrix} 0 & 0 \\ \frac{P_2}{r_1+p_2+f_2} & 0 \\ 0 & 0 \\ 0 & \frac{p_2}{P_{1H}+f_{1H}+P_2+f_2} \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{P_2}{r_1+p_2+f_2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{p_2}{P_{1H}+f_{1H}+P_2+f_2} \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} -1 & \frac{r_2}{r_1+r_2} & \frac{r_1}{r_1+r_2} & 0 \\ \frac{f_2}{r_1+p_2+f_2} & -1 & 0 & \frac{r_1}{r_1+p_2+f_2} \\ \frac{f_{1H}}{P_{1H}+f_{1H}+r_2} & 0 & -1 & \frac{r_2}{P_{1H}+f_{1H}+r_2} \\ 0 & \frac{f_{1H}}{P_{1H}+f_{1H}+P_2+f_2} & \frac{f_2}{P_{1H}+f_{1H}+P_2+f_2} & -1 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{P_{1H}}{P_{1H}+f_{1H}+r_2} & 0 \\ 0 & 0 & 0 & \frac{P_{1H}}{P_{1H}+f_{1H}+P_2+f_2} \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{P_2}{r_1+p_2+f_2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{P_2}{P_{1L}+f_{1L}+P_2+f_2} \end{bmatrix},$$

$$Q_5 = \begin{bmatrix} -1 & \frac{r_2}{r_1+r_2} & \frac{r_1}{r_1+r_2} & 0 \\ \frac{f_2}{r_1+p_2+f_2} & -1 & 0 & \frac{r_1}{r_1+p_2+f_2} \\ \frac{f_{1L}}{P_{1L}+f_{1L}+r_2} & 0 & -1 & \frac{r_2}{P_{1L}+f_{1L}+r_2} \\ 0 & \frac{f_{1L}}{P_{1L}+f_{1L}+P_2+f_2} & \frac{f_2}{P_{1L}+f_{1L}+P_2+f_2} & -1 \end{bmatrix}$$

$$Q_6 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{P_{1L}}{P_{1L}+f_{1L}+r_2} & 0 \\ 0 & 0 & 0 & \frac{P_{1L}}{P_{1L}+f_{1L}+P_2+f_2} \end{bmatrix}$$

$$Q_B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{p_{1L}}{p_{1L}+f_{1L}+r_2} & 0 \\ 0 & \frac{p_{1L}}{p_{1L}+f_{1L}+p_2+f_2} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -0 & 0 & 0 & 0 \\ -0 & 0 & 0 & \frac{p_2}{p_2+f_2} \end{bmatrix}$$

and $B_2 = \begin{bmatrix} -1 & 1 \\ \frac{f_2}{p_2+f_2} & -1 \end{bmatrix}$

To solve equation (5), we need to determine the inverse matrix of Q. Several studies [6, 12, 14] proposed efficient algorithms to find a solution of the equation which can be decompose into R+ 3 systems of linear equations as follows:

$$q_1 S_1 + q_0 Q_1 = 0, \tag{7}$$

$$q_1 S_2 + q_0 Q_2 + q_1 Q_1 = 0, \tag{8}$$

$$q_{n-1} Q_2 + q_n Q_2 + q_{n-1} Q_1 = 0, \quad n=1,2,\dots,r-2, \tag{9}$$

$$q_{r-1} Q_2 + q_r Q_2 + q_{r-1} Q_1 = 0, \tag{10}$$

$$q_{r-1} Q_2 + q_r Q_2 + q_{r-1} Q_1 = 0, \tag{11}$$

$$q_{m-1} Q_3 + q_m Q_3 + q_{m-1} Q_1 = 0, \quad m=r+1, r+2,\dots, R-1, \tag{12}$$

$$q_{R-1} Q_3 + q_R Q_3 + q_{R-1} B_1 = 0, \tag{13}$$

$$\text{and } q_R Q_3 + q_R B_2 = 0, \tag{14}$$

From the equations (7), (8), ..., and (13), we have

$$q_s = q_0 A_{1s}, \text{ where } A_{1s} = -Q_s S_1^{-1}, \tag{15}$$

$$q_0 = q_1 A_1, \text{ where } A_1 = -Q_1(A_0 S_1 + Q_2)^{-1}, \tag{16}$$

$$q_n = q_{n+1} A_{n+1}, \text{ where } A_{n+1} = -Q_n(A_n Q_2 + Q_3)^{-1}, \quad n=1,\dots,r-2, \tag{17}$$

$$q_{r-1} = q_r A_r, \text{ where } A_r = -Q_r(A_{r-1} Q_2 + Q_3)^{-1}, \tag{18}$$

$$q_r = q_{r+1} A_{r+1}, \text{ where } A_{r+1} = -Q_r(A_r Q_2 + Q_3)^{-1}, \tag{19}$$

$$q_m = q_{m+1} A_{m+1}, \text{ where } A_{m+1} = -Q_m(A_m Q_2 + Q_3)^{-1}, \tag{20}$$

$$m=r+1,\dots, R-1,$$

$$\text{and } q_R = q_B A_B, \text{ where } A_B = -B_1(A_R Q_3 + Q_3)^{-1}. \tag{21}$$

For the above equations, we can easily determine $A_1, A_2, \dots, A_r, \text{ and } A_B$ by successively inverting 2×2 or 4×4 matrices.

According to Cinlar [4], equation (5) contains one redundant equation. Thus we select the following one from equation (14):

$$q(B,0,R) = q(1,0,R) p_{1L} / (p_{1L} + f_{1L} + r_2) + q(B,1,R) f_2 / (p_2 + f_2). \tag{22}$$

With equations (15), (16), ... (22), and the second equation of (4), the probability distribution vector q can be determined through the following procedure:

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procedure find_q
q(B, 1, R) ← 1, q(B, 0, R) ← 1.
Repeat
    TMP ← q(B, 0, R)
    Compute qk from (21).
    Compute q(B, 0, R) from (22).
Until |TMP - q(B, 0, R)| < ε
Compute qn, qn+1, ..., q0, and q, from (21), (20), ... and
(15), respectively.
Normalize. (Divide each element of the vector by the
sum of all the elements of the vector)
end procedure

Next, let Y(α, β, n) be the duration of time the system
remains at state (α, β, n). Then Y(α, β, n) is an
exponential random variable with a mean of 1/μ(α, β,
n) where
μ(0, S, 0) = r1, μ(1, S, 0) = pm + fm,
μ(0, 0, n) = r1 + r2, μ(0, 1, n) = r1 + p2 + f2,
μ(1, 0, n) = pm + fm + r2, μ(1, 1, n) = pm + fm + p2 + f2,
μ(0, 0, m) = r1 + r2, μ(0, 1, m) = r1 + p2 + f2,
μ(1, 0, m) = p1L + f1L + r2, μ(1, 1, m) = p1L + f1L + p2 + f2,
μ(B, 0, R) = r2, μ(B, 1, R) = p2 + f2,
for n=0, 1, ..., r-1 and m=r, r+1, ..., R. \tag{23}
    
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For special cases such as $r=0$, $r=1$, $r=R$, or $r=R=1$, the above procedures needs a slight modification to find $q(\alpha, \beta, n)$ and $\mu(\alpha, \beta, n)$.

Now consider a time interval(cycle time) between two successive entrances to the state (α, β, n) . It can be shown that the expected cycle time for state (α, β, n) is

$$\lambda(\alpha, \beta, n) = \frac{1}{\mu(\alpha, \beta, n)} + \frac{1}{q(\alpha, \beta, n)} \sum_{(a,b,k) \neq (\alpha, \beta, n)} \frac{q(a,b,k)}{\mu(a,b,k)}$$

$$= \frac{1}{q(\alpha, \beta, n)} \sum_{(a,b,k) \in E} \frac{q(a,b,k)}{\mu(a,b,k)} \quad (24)$$

Finally, we determine steady state probability distribution $p(\alpha, \beta, n)$ from the following equation:

$$p(\alpha, \beta, n) = \mu(\alpha, \beta, n) / \lambda(\alpha, \beta, n)$$

$$= \frac{q(\alpha, \beta, n) \mu(\alpha, \beta, n)}{\sum_{(a,b,k) \in E} q(a,b,k) \mu(a,b,k)} \quad (25)$$

Once the probability of each state is obtained, some performance measures can be determined as follows:

- 1) Utilization ratio of each machine

$$e_1 = p(1, S, 0) + \sum_{n=0}^R \sum_{\beta=0}^1 p(1, \beta, n) \quad (26)$$

$$e_2 = \sum_{n=0}^R \sum_{\alpha=1}^1 p(\alpha, 1, n) + p(B, 1, R) \quad (27)$$

- 2) System productivity

$$P = p_1 e_1 \quad (28)$$

- 3) Average inventory level

$$\bar{I} = \sum_{n=0}^R \sum_{\alpha=1}^1 \sum_{\beta=0}^1 np(\alpha, \beta, n) + (R+1) \sum_{\beta=0}^1 p(B, \beta, R) \quad (29)$$

3. Optimal control policy

R and r are the most important decision variables which characterize the system performances. To select an optimal control policy $(R, r) \in E$ the following objective function is

developed, which expresses a profit index of the system per unit of time:

$$PROF(R, r) = CP - C_H \left\{ p(1, S, 0) + \sum_{n=1}^{r-1} \sum_{\beta=0}^1 p(1, \beta, n) \right\}$$

$$- C_{iL} \sum_{n=r}^R \sum_{\beta=0}^1 p(1, \beta, n) - C_H \bar{I} \quad (30)$$

where C = value of finished product, C_m = operating cost of the preceding machine when its production rate is high, C_{iL} = operating cost of the preceding machine when its production rate is low, and C_h = inventory holding cost. The first term of the equation is system revenue in unit time since P is the number of finished items in unit time. The second term is the operating cost of the preceding machine in unit time when producing in a high rate while the third is the operating cost of the machine in unit time when producing in a low rate. Inventory holding cost of the system in unit time is expressed at the last term.

With the machine parameters $(p_H, p_L, f_H, r, p, f, \text{ and } r_1)$ and cost parameters $(C, C_H, C_{iL}, \text{ and } C_h)$, we have to find an optimal (R, r) to maximize the equation (30). But the equation is too complicated to find an optimal solution with analytic method. So, we propose a numerical search procedure to find a local optimal solution as follows. Although we can not be convinced that the solution from the procedure is a global optimum, in many example problems the procedure found global optimums.

procedure find_optimal

$R \leftarrow 0$.

Repeat

$R \leftarrow R + 1, r \leftarrow -1, PROF(R, r) \leftarrow -\infty$.

Repeat

$r \leftarrow r + 1$.

Compute $p(\alpha, \beta, n)$ for all $(\alpha, \beta, n) \in E$.

Compute $PROF(R, r)$ by equation (30).

Until $r = R$ or $PROF(R, r) < PROF(R, r-1)$

If $r = R$ then

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PROFR ← PROF(R,r), rR* ← r.
else
  PROFR ← PROF(R,r-1), rR* ← r-1.
Until PROFR < PROFR-1.
R* ← R-1, r* ← rR-1.
end procedure
    
```

An example problem with the following machine and cost parameters is solved:

$p_{iH}=10, p_{iL}=7, f_{iH}=1, f_{iL}=0.7, r_i=2, p_2=10, f_2=1, r_2=2,$
 $C=\$ 5/unit, C_{iH}=\$ 10/time, C_{iL}=\$ 5/time, \text{ and } C_h=\$ 1/unit/time.$

Applying the data to the search procedure, the optimal control policy (R^*, r^*) is obtained along with other system measures. They are

$R^* = 6, r^* = 2,$
 $e_1 = 0.6131, e_2 = 0.5075,$
 $P = 5.075, \bar{I} = 2.599,$
 and $PROF(R^*, r^*) = 18.407.$

We apply the objective function and the solution procedure to non-switching system in which all the related parameters of the preceding machine follow the case of high rate production, i.e., $p_{iL}=p_{iH}, f_{iL}=f_{iH},$ and $C_{iL}=C_{iH}.$ And we get the optimal buffer size $R^*=4$ with its profit of \$17,954. It implies that the switching system gains more than non-switching system by \$0.453 per unit of time.

4. Sensitivity analysis

A sensitivity analysis is made to answer the following questions:

- 1) When is it preferable to have switching system rather than non-switching one?
- 2) Under what circumstances do the optimal values of R and r become large (or small)?
- 3) How do the system productivity and average inventory respond to the parameter changes?

For the analysis, we set $C=5, C_{iH}=10, C_{iL}=0.5C_{iH},$ and

$C_h=1$ as the standard cost parameters. For each cost parameter, nine different levels are chosen by multiplying the standard value by 1/5, 1/4, 1/3, 1/2, 1, 2, 3, 4 and 5.

Assuming that $p_{iH}=10, p_{iL}=7, f_{iH}=1, f_{iL}=0.7, r_i=2, p_2=10, f_2=1$ and $r_2=2$ the following variables of interest are determined for each level of the cost parameter with the others being the same as the standard values.

- 1) R^* ,
- 2) r^* ,
- 3) system productivity P ,
- 4) average inventory \bar{I} , and
- 5) Ratio = $PROF(R^*, r^*) / \overline{PROF}$,

where \overline{PROF} is optimal profit of the non-switching system.

The results are depicted in Figure 2, 3, 4, 5, 6, and 7. Based on the graphs, the following observations can be made:

- 1) The productivity is highly related to \bar{I} and they have the same trend. This means that we can get high productivity (or system output) with high inventory since it decreases the probability of starvation.
- 2) The average in-process inventory is closely related to the values of R^* and r^* and shows the same trend as those of R^* and r^* .
- 3) If C (value of the product) increases, we need to increase system productivity for high profit. This results in high inventory, R^* and r^* . Therefore, the switching system becomes more preferable to the non-switching one as C decreases since the preceding machine should produce items at high level for high productivity.
- 4) The result of changes of C_h is reverse of C 's. If C_h increases, we need to decrease inventory level, i.e., low R^* and r^* .
- 5) If C_{iH} increases, (this results in high difference of C_{iH} and C_{iL} since we set $C_{iL}=0.5C_{iH}$) we need to increase the difference R^* and r^* , and then low inventory and low productivity.

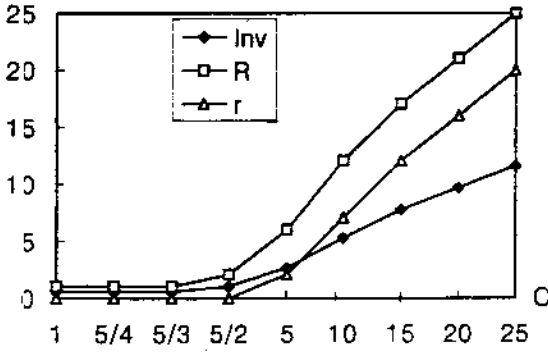


Figure 2. Effect of C on R^* , r^* , and \bar{I}

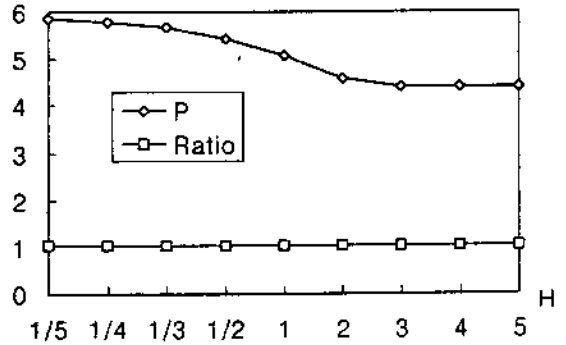


Figure 5. Effect of C_h on P and Ratio

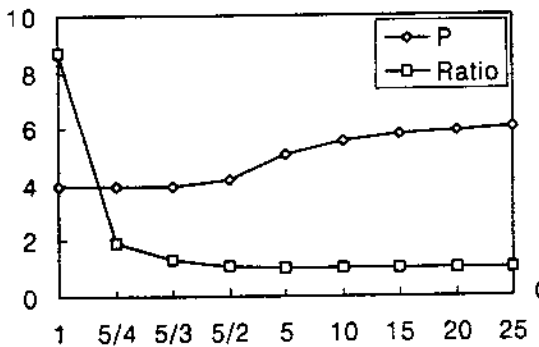


Figure 3. Effect of C on P and Ratio

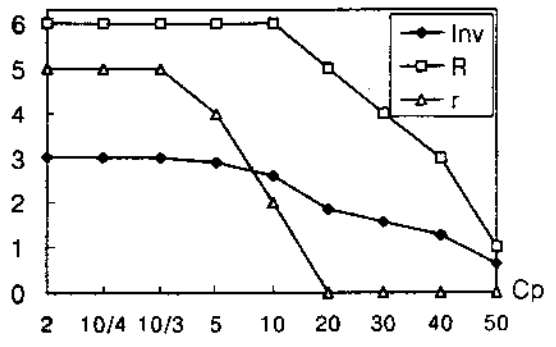


Figure 6. Effect of C_w and $C_{IL}(=0.5C_p)$ on R^* , r^* and \bar{I}

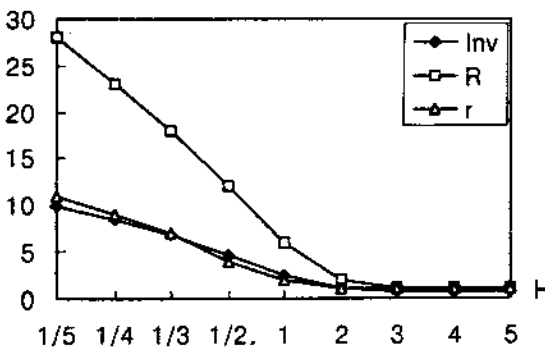


Figure 4. Effect of C_h on R^* , r^* , and \bar{I}

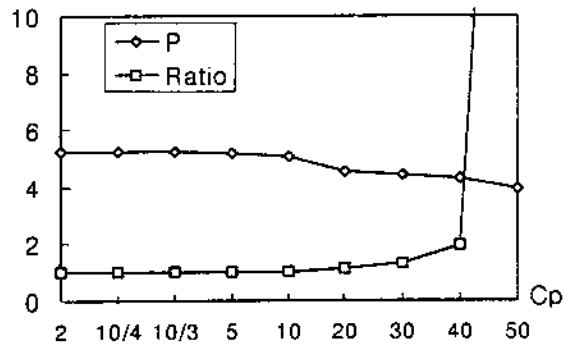


Figure 7. Effect of C_H and $C_{IL}(=0.5C_H)$ on P and Ratio

5. Conclusions

In this paper, an automatic production line is studied which consists of two unreliable machines with random processing times and a finite storage buffer controlled by a production switching mechanism. Using the Markov process model, we determine steady state probability distribution of the system states. Then, the optimal control parameters are determined which maximize the system profit per unit of time. From a sensitivity analysis, it is shown that the production switching system is better than or equal to the non-switching one in terms of the system profit.

It is suggested that future research could extend the use of the current model to other cases including systems of continuous flow.

References

- [1] Barman, S. and Burch, E.E., "The production switching heuristic : A practical revision", *International Journal of Production Research*, Vol.27, No.11, pp. 1863-1875, 1989.
- [2] Buzacott, J.A. and Hanifin, L.E., "Models of automatic transfer lines with inventory banks - A review and comparison", *AIIE Transactions*, Vol.10, No.2, pp. 197-207, 1978.
- [3] Buzacott, J.A. and Kostelski, D., "Matrix-geometric and recursive algorithm solution of a two-stage unreliable flow line", *IIE Transactions*, Vol.19, No. 4, pp.429-438, 1987.
- [4] Cinlar, E., *Introduction to Stochastic Processes*, Prentice-Hall, Englewood Cliffs, NJ, 1975.
- [5] Gershwin, S.B. and Berman, O., "Analysis of transfer lines consisting of two unreliable machines with random processing times and finite storage buffers", *AIIE Transactions*, Vol.13, No.1, pp.2-11, 1981.
- [6] Hong, Y. and Seong, D.H., "The analysis of an unreliable two-machine production line with random processing times", *European Journal of Operational Research*, Vol.68, No.2, pp.228-235, 1993.
- [7] Hopp, W.J., Pati, N., and Jones, P.C., "Optimal Inventory control in a production flow system with failures", *International Journal of Production Research*, Vol.27, No.8, pp.1367-1384, 1989.
- [8] Hwang, H. and Cha, C.N., "An improved version of the production switching heuristic for the aggregate production planning problem", *International Journal of Production Research*, Vol.33, No.9, pp.2567-2577, 1995.
- [9] Hwang, H. and Koh, S.G., "Optimal control of work-in-process with dual limit switches in two-stage transfer line", *European Journal of Operational Research*, Vol.63, No.1, pp.66-75, 1992.
- [10] Mellichamp, J.M. and Love, R.M., "Production switching heuristic for the aggregate planning problem", *Management Science*, Vol.24, No.12, pp. 1242-1251, 1978.
- [11] Meyer, R.R., Rothkopf, M.H., and Smith, S.A., "Reliability and inventory in a production storage system", *Management Science*, Vol.25, No.8, pp. 799-807, 1979.
- [12] Shanthikumar, J.G. and Tien, C.C., "An algorithmic solution to two-stage transfer lines with possible scrapping of units", *Management Science*, Vol.29, No.9, pp.1069-1086, 1983.
- [13] Yadin, M. and Naor, P., "On queueing systems with variable service capacities", *Naval Research Logistics Quarterly*, Vol.14, No.1, pp.43-53, 1967.
- [14] Yeralan, S. and Muth, E.J., "A general model of a production line with intermediate buffer and station breakdown", *IIE Transactions*, Vol.19, No.2, pp. 130-139, 1987.