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Evaluation of Economic Alternatives with Dynamic Measures*

동태적 측도를 이용한 경제성 평가*

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Abstract

This paper presents a new present value measure, the dynamic present value, or DPV. DPV takes into account not only the value of the realized cash flow but also that of potential cash flow. The DPV approach enables the analyst to observe differences in the present value of the alternatives every period over the whole time period of interest. This is the most fundamental advantage of the DPV approach over the traditional present value approach in which the present value of the alternatives is evaluated only one particular point of time. The concepts of the realized and potential cash flows are also developed in this paper. These new concepts are found to be useful elements in evaluating economic alternatives.

1. Introduction

The present value measure is one of the most generally used criteria for economic evaluation of alternatives or projects. We will refer to the process of evaluating economic alternatives with the present value criterion as the present value approach. In this traditional present value approach, the present value of a project is normally calculated only once for *a* given length of a project life.

The traditional present value approach, however, has some limitations as follows: First, the calculation of the present value for one particular period is only a cross-section examination of the project. In practice, the present values of the project at different points of time may also be of interest to analysts. Another limitation of the traditional present value approach is that the length of a project life is determined a priori. It is, however, often of nature of variable to be determined after examination of project's present values over time.

As a way to overcome such limitations of the traditional present value approach, Park [2] proposed an alternative approach to calculating the present value, in which time is a variable rather than a fixed value. In particular, in the alternative approach, the present value of a project is calculated for a series of time points. This series of present value represents the present worths of a project over multiple time periods.

In this paper, we will call this dynamic measure

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showing the present values of a project at various points of time 'the dynamic present value' (DPV) measure. Thus, DPV at time t may be defined as the present value of the total project worth as of time t as a variable for a given discount rate. This concept of the dynamic present value is different from that of the traditional present value approach in two respects: One is that time t is a variable in DPV while it is of a nature of constant in the traditional present value approach. The other is that the traditional present value is based mainly on the cash flows that actually take place except for the final period while DPV takes not only the value of the actual cash but also that of potential ones into consideration every period. Consequently, DPV reveals the total present value of a project every period while the traditional present value does it only once for the final period.

In the previous paper, Park [2] described how to calculate this dynamic measure very briefly only. So more detailed explanations for how to calculate it is necessary to help potential applicants use it at ease. The purpose of this paper is to provide extended methods of analysis and illustration about how to calculate DPV and use it in evaluating economic alternatives under various situations.

2. Determination of Cash Flows and the Dynamic Present Value

2.1 Determination of After-tax Cash Flows

There are many types of cash flows. However, all final decisions must be based on the after-tax cash flows. A general formulation for determination of the after-tax cash flow may be the generalized cash flow by Park and Sharp [1] given as

After-tax (A/T) cash flow from project at time t
 = - (Investment at time t, denoted by I)
 + (after-tax Proceeds from sale of investment at time t, denoted by P)
 + (bank Loan at time t, denoted by L)
 - (loan principal repayment at time t, denoted by Y taken from repayment)
 + A(t)(Depreciation at time t, denoted by D)
 + (1 - A(t)) (Revenue at time t, denoted by R)
 - (1 - A(t)) (Expenses at time t for such items as labor, materials, and interest, denoted by E) (1)

which may be written as

$$CF(t) = -I(t) + P(t) + L(t) - Y(t) + A(t)D(t) + (1 - A(t))R(t) - (1 - A(t))E(t) \qquad (2)$$

where A(t) (taken from tax) represents the tax rate at time t.

2.2 The Realized Cash Flow, the Potential Cash Flow and the Dynamic Present Value

In order to evaluate the 'total' present value of a project, it is necessary to understand the nature of cash flow components in Equation (2). The cash flow components in Equation (2) may be categorized into two types. One type is the cash flows that actually realize at a particular time point of interest, and the other the cash flows that 'could' potentially take place even if they would not actually do. The former may be referred to as the realized cash flow (RCF) and the latter the potential cash flow (PCF). For example, investment, loan, depreciation, revenue and expenses are usually elements of RCFs. On the other hand, proceeds from sale of investment may not take place every period. However, it should be included in evaluation of the total present value of a project. Therefore, proceeds from sale of investment P is often an element of PCF. Loan principal repayment is of the similar nature and is often a typical element of PCF.

The present values of the RCF and the PCF may be called the present value of RCF (PVRCF) and the present value of PCF (PVPCF), respectively. Using the concepts of PVRCF and PVPCF, the DPV of a project at time t for a given discount rate i may be expressed as the sum of PVRCF and PVPCF, or

$$DPV(t|i) = CSPVRCF(t|i) + PVPCF(t|i) \quad (3)$$

where CSPVRCF(t|i) stands for the cumulative sum of the present value of RCF, or

$$CSPVRCF(t|i) = \sum_{T=0}^{t} RCF(T)/(1+i)^T$$

and $PVPCF(t|i) = PCF(t)/(1+i)^t$

3. Computation of the Dynamic Present Value

3.1 The Case with Investment, Revenue and Expense

Alternative A1: Suppose that a graduating senior is considering the alternative of going to graduate school for two years first and then getting a job with a masters's degree. The yearly tuition and fees of 6,000,000 won to be paid at the beginning of the first and second years means the investments of 6,000,000 won at t=0 and t=1, respectively. Living expense for t=1 is 8,000,000 won and it will increase by two percent annually due to increasing standard of living (not due to inflation). Revenue occurs for the first time at t=3 by the amount of 27,000,000 won. It is assumed to increase 15 percent per year.

According to the scenario described in A1 which includes no loan from the bank, no repayment of the loan hence, and no depreciation, L(t)=0, Y(t)=0, and D(t)=0 and it is assumed that A(t)=0. Then, Equation (2) reduces to

$$CF(t) = -I(t) + P(t) + R(t) - E(t) \quad (4)$$

where I(t), R(t) and E(t) are elements of RCF, and P(t) is a PCF.

At t = 0, I(t) = 6,000,000, R(t) = E(t) = 0. Estimation of proceeds from sale of investment P(t) for the case of A1 is not straightforward. To estimate it, note that the value of P(t) can be determined by the amount of refund for paid tuition and fees (investment) at t=0. Following Equation (3), the dynamic present value at t = 0 is the sum of the investment (realized cash outflow) I(0), and P(0) determined by the amount of possible refund (potential cash inflow), or

$$DPV(t=0) = -I(0) + P(0) + R(0) - E(0) \quad (5)$$

Estimation of DPV(0) does not appear to have great bearings in practical decision making because normally the size of DPV at t = 0 would not affect decision making about a project. However, examination of this estimation problems appears to be important for the theoretical completeness of DPV and for understanding calculation procedure in the subsequent periods. DPV(0) can have different values depending on the real situation and the assumption to be made as follows:

(1) DPV(0) = 0: The present value of any project is zero before any cash flow takes place. Once investment is made at t = 0, the value of DPV(0) depends on the assumption about the recovery of the investment. In the case of A1, RCF(0) due to the initial investment is 6,000,000 won. If investment 6,000,000 won would be refundable in its original amount, we may say that proceeds from sale of investment would be P(0) = 6,000,000 and

$$DPV(0) = -I(0) + P(0) = -6{,}000{,}000 + 6{,}000{,}000 = 0.$$

(2) DPV(0) 〈 0: If, in the case of A1, the paid tuition and fees of 6,000,000 won is only partially refundable, then P(0) 〈 6,000,000, and DPV(0) = -I(0) + P(0) 〈 0.

(3) DPV(0) 〉 0: In the case of A1, P(0) due to refund for the paid tuition and fees cannot normally be greater than the paid amount. Consequently, DPV(0) can not be greater than zero.

It should be noted here that the size of DPV in one period would not affect those of DPVs in subsequent periods. Therefore, assumption related to the size of DPV (0) is not critical in evaluation of A1. Calculations of DPVs for t = 1, 2, ..., 10 for i = 0.1 can be done as follows:

At t = 1, are I(1) = 6,000,000, R(1) = 0, E(1) = 8,000,000 and PCF is P(0) = 6,000,000 (under full refund assumption). Then

$$\begin{aligned} DPV(1) &= RCF(0) + RCF(1)/(1+i)^1 + PCF(1)/(1+i)^1 \\ &= RCF(0) + PVRCF(1) + PVPCF(1) \\ &= -6{,}000{,}000 - 12{,}730{,}000 + 5{,}450{,}000 \\ &= -13{,}270{,}000 \end{aligned}$$

At t = 2, are I(2) = 0, R(2) = 0, and E(2) = 8,000,000x1.02 = 8,160,000. There is no potential cash flow PCF(2) = 0, therefore,

$$\begin{aligned} DPV(2) &= RCF(0) + RCF(1)/1.1 + RCF(2)/1.1^2 + PCF(2) \\ &= -6{,}000{,}000 - 14{,}000{,}000/1.1 - 8{,}160{,}000/1.1^2 + 0 = -25{,}470{,}000 \end{aligned}$$

...

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...

$$\begin{aligned} DPV(10) &= RCF(0) + PVRCF(1) + PVRCF(2) + \cdots + PVRCF(9) + PVRCF(10) \\ &= 12{,}613 \end{aligned}$$

The cash flows and DPVs of Alternative A1 are summerized in Table 1.

We may define a measure that indicates the payback period. It should be recalled that the payback period is determined from the equity cash flows. In this regard, we may define the cumulative sum of equity-based RCFs up to time t (CSERCF(t)) as

$$CSERCF(t) = \sum_{T=0}^{t} ERCF(T)) \tag{6}$$

where ERCF(T) represents equity-based RCFs which contain RCFs due to equity and operation of the project under consideration, excluding cash flows due to borrowing, repayment of the loan, and payment of interest. Then, the payback period is the minimum value of t such that CSERCF(t) $\geq$ 0 (Park and Sharp, [1]). We may also define the cumulative sum of future values of ERCFs up to time t at discount rate i (CSFVERCF(t | i)) as

Table 1. Cash Flows and Dynamic Present Values of Alternative A1 (in 10,000 won)

	Period										
	0	1	2	3	4	5	6	7	8	9	10
I (investment)	600	600	0	0	0	0	0	0	0	0	0
P (proceeds)	600	600	0	0	0	0	0	0	0	0	0
R(revenue)	0	0	0	2,700	3,105	3,571	4,106	4,722	5,431	6,245	7,182
E(Expense)	0	800	816	832	849	866	883	901	919	937	956
RCF	-600	-1,400	-816	1,868	2,256	2,705	3,223	3,821	4,512	5,308	6,226
PCF	600	600	0	0	0	0	0	0	0	0	0
PVRCF(i=0.1)	-600	-1,273	-674	1,403	1,541	1,679	1,819	1,961	2,105	2,251	2,400
PVPCF(i=0.1)	600	545	0	0	0	0	0	0	0	0	0
DPV(i=0.1)	0	-1,328	-2,547	-1,144	397	2,076	3,896	5,857	7,962	10,213	12,613
CSERCF	-600	-2,000	-2,816	-948	1,308	4,013	7,236	11,057	15,569	20,877	27,103
CSFVERCF	-600	-2,060	-3,082	-1,523	581	3,344	6,902	11,413	17,066	24,081	32,715

$$CSFVERCF(t|i) = \sum_{T=0}^{t} RCF(T)\ (1+i)^{t-T} \qquad (7)$$

Then, the smallest value of t satisfying CSFVERCF(t|i) ≥ 0 is the time required for the project to break even (Park and Sharp [1], p. 233). The smallest value of t making CSFVERCF ≥ 0 is the discounted payback period.

In calculation of CSERCF(t) and CSFVERCF(t) for Alternative A1, the RCFs are the same as ERCFs because there is no cash flows due to borrowing. Figure 1 represents values of DPV(t), CSERCF(t), and CSFVERCF(t) for A1 for t = 1, 2, ..., 10. The horizontal axis in Figure 1 is time or year and the vertical axis values of various measures. In Figure 1, CSERCF 〉 0 for the first time at t = 4 in Figure 1, which means that the payback period of A1 is four. CSFVERCF 〉 0 for the first time at t = 4 in Figure 1, which means that the discounted payback period is also four.

3.2 The Case with Loan, Depreciation and Proceeds from Sale of Investment

We will consider a more complex situation where investment incurs assets carrying salvage value, and loan. We consider a project which include purchase a machine whose life is twelve years. For the sake of simplicity we assume that the tax rate A = 0. In this case the generalized cash flow in Equation (2) would be reduced to

$$CF(t) = -I(t) + P(t) + L(t) - Y(t) + D(t) + R(t) - E(t) \qquad (8)$$

Alternative 2 (A2): The graduating senior considers a startup of a small business which requires 50,000,000 won at t = 0. Of 50,000,000 won, 40,000,000 won is to be used to purchase a machine, which is to be depreciated over five years with the straight-line method. Salvage value is assumed to be ten percent of the purchase price at the end of fifth period. This results in

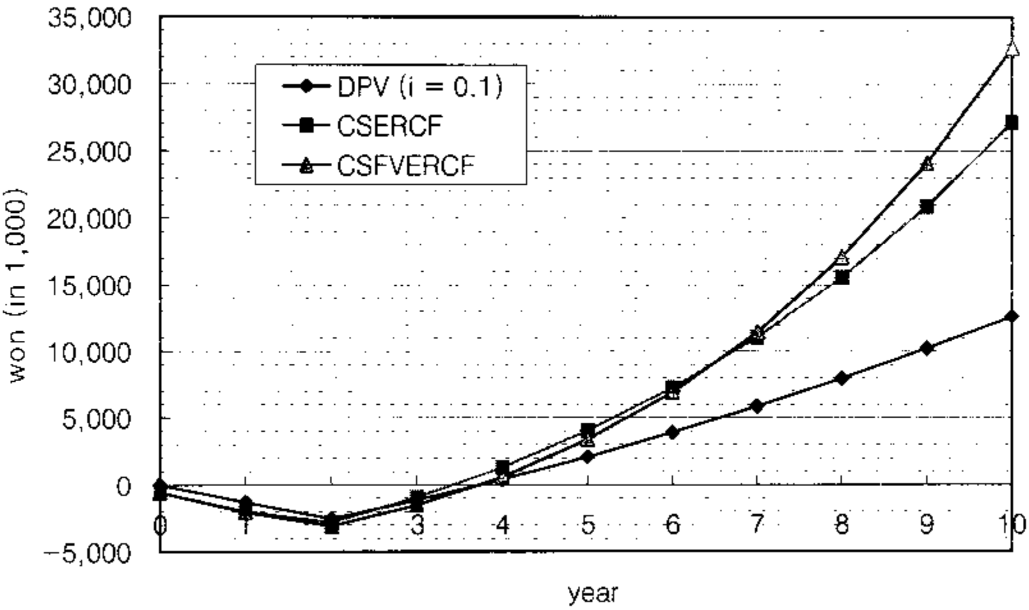

Figure 1. Graphs of DPVs, CSERCFs and CSFVERCFs for Alternative A1

$D(t) = 40,000,000(1 - 0.1)/5 = 7,200,000$ for $t = 1, 2, ..., 5$.

The graduating entrepreneur is going to make a bank loan of 50,000,000 won at $t = 0$ at annual interest rate of 12 percent. Calculation of cash flows and DPVs is done as follows:

At $t = 0$, Investment $I(0) = 50,000,000$. However, 40,000,000 won of the total investment is used to purchase a machine and the remaining 10,000,000 won to buy raw materials which can be resold at 80 percent of its original value at any time while the business is going on. The machine is assumed to maintain the same cash value as its original price at $t = 0$, meaning that P(0) from sale of the machine is 40,000,000. However, proceeds from sale of raw materials would be 80 percent of its original value, meaning that P(0) from sale of raw materials is 8,000,000. Bank loan $L(0) = 50,000,000$, and it can be returned without any cost resulting in $Y(0) = 50,000,000$. In summary,

$$RCF(0) = -I(0) + L(0) = -50,000,000 + 50,000,000 = 0$$

$$PCF(0) = P(0) - Y(0) = 48,000,000 - 50,000,000 = -2,000,000$$

$$DPV(0) = RCF(0) + PCF(0) = 0 - 2,000,000 = -2,000,000$$

At $t = 1$, the young entrepreneur predicts that $R(1) = 50,000,000$. Expense is estimated to be $E(1) = 55,000,000$. Proceeds from sale of investment come from two sources. One is sale of raw material at 80 percent of the original value, i.e., 0.8x10,000,000 and the other the sale of the machine at the book value after depreciation, i.e., $40,000,000 - 7,200,000 = 32,800,000$. The amount of repayment at $t = 1$ is the sum of the principal and 12 percent of interest, or 56,000,000 won. As results,

$$RCF(1) = D(1) + R(1) - E(1) = 7,200,000 + 50,000,000 - 55,000,000 = 2,200,000$$

$$PCF(1) = P(1) - Y(1) = 8,000,000 + 32,800,000 - 56,000,000 = -15,200,000$$

$$DPV(1 | 0.1) = RCF(0) + RCF(1)/1.1 + PCF(1)/1.1 = 0 + 2,200,000/1.1 - 15,200,000/1.1 = -11,820,000$$

At $t = 2$, $I(2) = 0$, $P(2) = 40,000,000 - 2x7,200,000 + 8,000,000 = 33,600,000$, $L(2) = 0$, and $Y(2) = 1.12^2x50,000,000 = 62,720,000$. The revenue is expected to rise to $R(2) = 70,000,000$, and expense $E(2)$ is 70,000,000. Based on these assumptions and estimations, calculations for PCFs and DPVs are as follows:

$$RCF(2) = D(2) + R(2) - E(2) = 7,200,000 + 70,000,000 - 70,000,000 = 7,200,000$$

$$PCF(2) = P(2) - Y(2) = 33,600,000 - 62,720,000 = -29,120,000$$

$$DPV(2 | 0.1) = RCF(0) + RCF(1)/1.1 + RCF(2)/1.1^2 + PCF(2)/1.1^2 = -16,120,000$$

At $t = 3$, $R(3) = 130,000,000$ and expense is assumed to be 70 percent of R(3), or 91,000,000. At $t = 4$, $R(4) = 150,000,000$ and $E(4) = 0.7xR(4) = 105,000,000$. Other terms are calculated in the same way as for $t = 2$. Then,

$$DPV(3) = 9,720,000$$

$$DPV(4) = 37,690,000$$

At $t = 5$, note that Y(t) is not PCF any more. It is RCF because it is to be repaid at $t = 5$. $D(5) = 7,200,000$. R(5) is assumed to rise to 130,000,000 and $E(5) = 910,000,000$. Consequently, $DPV(5) = 59,740,000$. Note here that DPV(5) is not affected whether it is assumed that the principal and the interest is paid or remain due to pay at $t = 5$.

For the periods of $t = 6, 7, 8, 9$, and 10, $R(6) = 90,000,000$, $R(7) = 60,000,000$, $R(8) = 50,000,000$, R(9)

Table 2. Cash Flows and Dynamic Present Values of Alternative A2 (in 10,000 won)

	Period or time t										
	0	1	2	3	4	5	6	7	8	9	10
I	5,000	0	0	0	0	0	0	0	0	0	0
P	4,800	4,080	3,360	2,640	1,920	1,200	1,200	1,200	1,200	1,200	1,200
L	5,000	0	0	0	0	0	0	0	0	0	0
Y	5,000	5,600	6,272	7,025	7,868	8,812	0	0	0	0	0
D	0	720	720	720	720	720	0	0	0	0	0
R	0	5,000	7,000	13,000	15,000	13,000	9,000	6,000	5,000	4,000	3,000
E	0	5,500	7,000	9,100	10,500	9,100	6,300	4,200	3,500	6,000	6,500
RCF	0	220	720	4,620	5,220	-4,192	2,700	1,800	1,500	-2,000	-3,500
PCF	-200	-1,520	-2,912	-4,385	-5,948	1,200	1,200	1,200	1,200	1,200	1,200
PVRCF	0	200	595	3,471	3,565	-2,603	1,524	924	700	-848	-1,349
PVPCF	-200	-1,382	-2,407	-3,294	-4,062	745	677	616	560	509	463
DPV	-200	-1,182	-1,612	972	3,769	5,974	7,430	8,292	8,936	8,037	6,641
CSERCF	-5,000	-4,780	-4,060	560	5,780	10,400	13,100	14,900	16,400	14,400	10,900
CSFVERCF	-5,000	-4,780	-4,530	-372	4,811	9,912	13,603	16,764	19,940	19,934	18,424

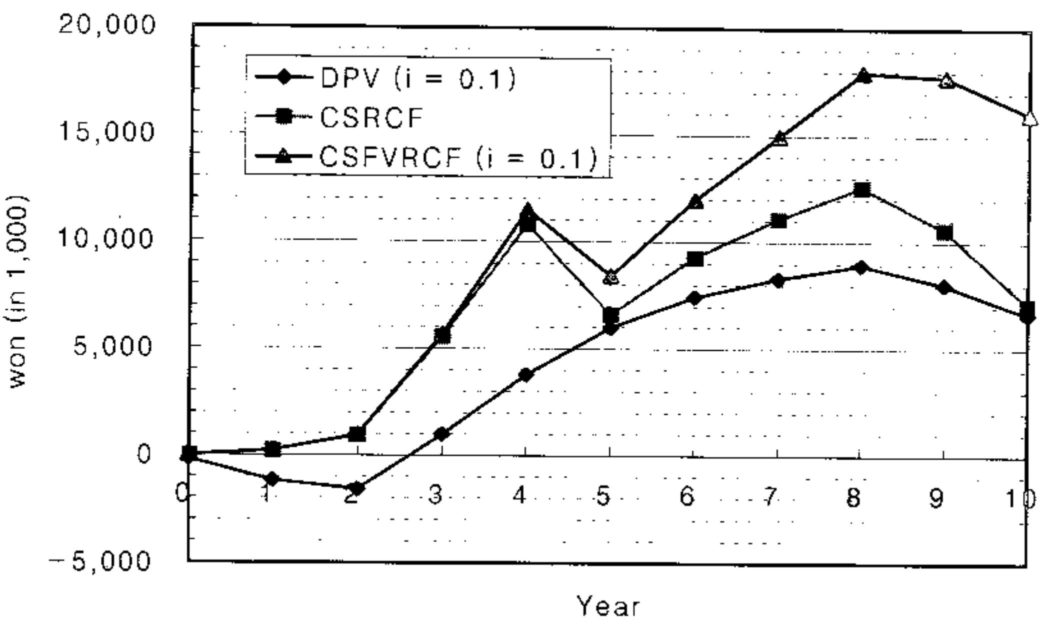

Figure 2. Graphs of DPVs, CSERCFs and CSFVERCFs for Alternative A2

= 40,000,000, R(10) = 30,000,000. Expenses are E(6) = 63,000,000, E(7) = 42,000,000, E(8) = 35,000,000, E(9) = 60,000,000, and E(10) = 65,000,000. D(t) = 0, L(t) = 0, Y(t) = 0. As a result, DPV(6) = 74,300,000, DPV(7) = 82,920,000, DPV(8) = 89,360,000, DPV(9) = 80,370,000, and DPV(10) = 66,410,000, respectively.

The cash flow elements and DPVs for A2 are given in Table 2. In Table 2, CSERCF(t) 〉 0 for the first time at t = 3, which means that the payback period of A2 is three. CSFVERCF(t) 〉 0 for the first time at = 4, which means that the project balance becomes positive for the first time at t = 4. DPVs of A2 is maximum at t = 8, which means that the optimal life of the project A2 is eight years (cf, Park [2]) for the definition of the optimal life of a project). Figure 2 shows changes in values of DPV(t), CSERCF(t), and CSFVERCF(t) for A1 for t = 1, 2, ..., 10.

4. Comparison of Alternatives

The DPVs for Alternatives A1 and A2 are shown in Figure 3. The horizontal axis in Figure 3 is time or year, and the vertical axis quantity of value. For the periods of t = 4, 5, 6, 7, and 8, A2 is superior to A1 because DPVs of A2 during the periods is greater than those of A1. However, For t = 9 and 10, A1 is superior to A2.

As seen in the comparison of A1 and A2 just above. DPV approach enables the analyst to observe differences in present value of the two alternatives every period over the whole time period of interest. This is the most fundamental advantage of the DPV approach over the traditional present value approach.

5. Conclusions

This paper proposes a new present value measure, DPV. DPV takes into account not only the value of the realized cash flow but also that of potential cash flow. The DPV approach enables the analyst to observe differences in the present value of the alternatives every period over the whole time period of interest. This is the most fundamental advantage of the DPV approach over the traditional present value approach in which the present value of the alternatives is evaluated only one particular point of time.

The concepts of the realized and potential cash flows developed in this paper are also found useful in

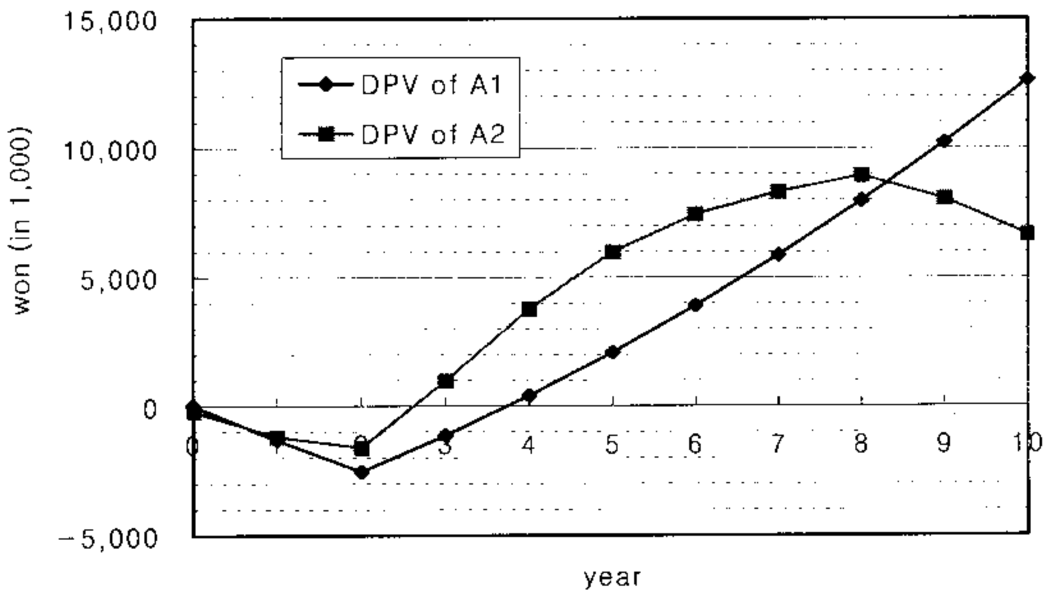

Figure 3. Curves of DPVs for Alternatives of A1 and A2

understanding the nature of project. In particular, the RCF was shown to be useful in defining such measures as the payback period, discounted payback period, and the project balance

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97년 1월 최초 접수, 98년 2월 최종 수정