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An Algorithm to Determine the Spare Inventory Level for Repairable-Item Inventory System with Depot Spares

중앙창 재고를 가진 수리가능시스템의 최적해법

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Abstract

We consider the problem of determining the spare inventory level for a multiechelon repairable-item inventory system. Our model extends the previous results to the system which has an inventory at the central depot as well as at the several bases. We develop an algorithm to find spare inventory levels, which minimize the total expected cost and simultaneously satisfies a specified minimum service level. Comparisons of our algorithm with the simulation show that the algorithm is very accurate and efficient.

1. INTRODUCTION

Repairable items are referred to as components which are expensive, critically important, and subject to infrequent failures such as engines of a fighter plane or a ship. They should be replaced or repaired immediately, if failed, for the system to maintain availability. For example, failed engine of a fighter plane is immediately replaced by a spare engine and is sent to a field repair facility or to a central repair center for repair based on the severity of the failure. For this reason the policy on the inventory or shortage levels is very important and naturally has been studied for a long time by many researchers. There are two main streams of research in this area. METRIC model, developed by Sherbrooke [13] assumes infinite repair capacity. In his model, there are many bases and a central depot. A failed item at a base is dispatched to a repair facility and a spare, if available, is plugged in. Otherwise, it is backordered. A repaired item fills the backorder or is stored at a spare inventory point if there is no backorder. Later, Feeney and Sherbrooke [4], Muckstadt [10, 11] and Muckstadt and Thomas [12] extended this model. However, as Albright [1] has pointed out, models assuming infinite repair capacity always underestimate the amount of congestion in the system and, consequently, result in fewer spares than are really needed to achieve a specified backorder level.

Another stream of study adopts the finite repair capacity, constant-failure-rate assumptions. The models in this stream are more realistic than the comparable METRIC models, and are certainly more difficult to solve due to the huge multidimensional state spaces involved.

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Gross *et al.* [7] considered a two-echelon (two levels of repair, one level of supply) system and presented an implicit enumeration algorithm to calculate the capacities of the base and depot repair facilities as well as the spares level which together guarantee a specified service rate at a minimum cost. Inevitably, the enumeration scheme of the method requires considerable computer running times even for relatively small problems. Gross *et al.* [5, 6] and Albright and Soni [2] analyzed the operating characteristics of a given system with multidimensional Markov process. Albright and Soni [3] applied a similar approach to a two-echelon repairable inventory system. Albright [1] developed an approximation algorithm with a single type of item stocked and repaired by several bases and a central depot. The proposed methods in this stream concentrate on the analysis of the current status of a given system and, consequently, are impractical to apply to optimization problems.

Most of the previous works concerning repairable-item inventory systems use analytic approaches. When it is difficult to construct an analytic model due to the complexity of the system, simulation can be a good tool. Unfortunately, simulation is much more expensive to use and is problem-specific in most cases.

More recently, Kim *et al.* [9] developed an algorithm to determine the optimal inventory level under finite repair capacities. They presented an analytic method to solve a two-echelon (two levels of repair, one level of supply) system with infinite calling population and finite repair facility. In this paper we consider a more general system (two levels of repair, two levels of supply) than the system analyzed in Kim *et al.* [9]. In other words, we consider the system whose central depot also has its inventory as well as the bases. We develop an efficient method to find the amount of spare items at each inventory which minimize the total expected holding plus shortage costs and simultaneously achieve a specified minimum service rate for large real problems.

This article is organized as follows. In Section 2 the model we consider is described and, in Section 3, we introduce the algorithm for the model and present an example to explain the algorithm. Section 4 evaluates the accuracy and the computational efficiency of the algorithm using simulation. Lastly, in Section 5, we summarize the results of the study and identify some areas for future research.

2. MODEL DESCRIPTION

We consider a system with $I (I < \infty)$ bases and a central depot as depicted in Figure 1. The depot has its own spares inventory which enables the depot-repairable item to be replaced immediately with a spare, if available. Time intervals between failures at base i are exponentially distributed with mean $1/\lambda_i$, $i=1,2,\cdots,I$. A failed item at base i is base-repairable with probability α_i and the base-repairable item is replaced by a base spare if one is available. Otherwise, replacement is delayed until a spare becomes available. A failed item, which is depot-repairable with probability $1-\alpha_i$, is sent to the depot for repair. If the depot has spares available, then a spare is immediately sent to the base where the failed item has originated and the failed item, after repair, is stored at the depot inventory. On the other hand, if the depot spares are not available, then the failed item is repaired and is sent to its base after repair. We refer to this case as depot-shortage with respect to a base.

The total number of *failed items* of base i, in the sense that they are currently unavailable for replacement, are the sum of the items at the base repair center, items in depot-shortage with respect to the base i, and items in transit between the depot and base i.

To obtain the steady-state probability distribution of the failed items at each base. Let us denote

$P_i(n)$ = probability distribution that there are n items at the repair center of base i,

$P_d(D)$ = probability distribution that there are D items at the depot repair center,

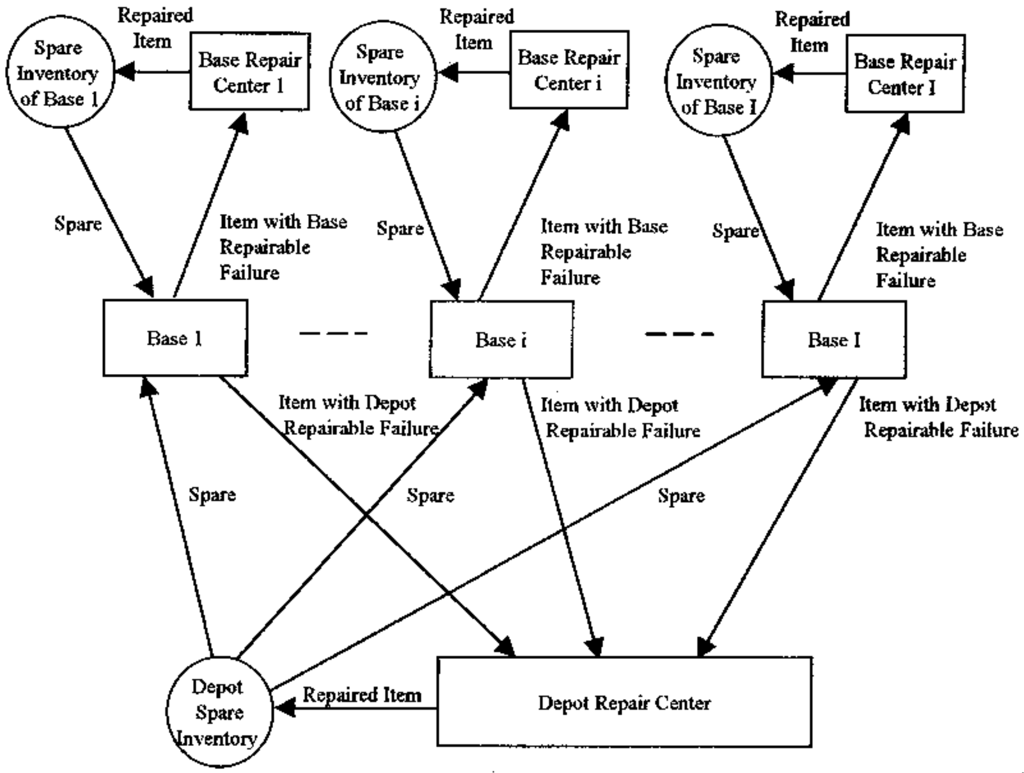

Figure 1. Schematic Representation of the Repairable-item Inventory System

$P_{id}(k_i)$ = probability distribution that there are k_i items at the depot repair center which are depot-shortage with respect to base i,

$P_t(m_i)$ = probability distribution that there are m_i items in transit from or to base i,

$P(z_i)$ = probability distribution that the total number of failed items of base i is z_i

2.1 Probability distribution of items at the base repair center

Let there be c_i service channels at the repair center of base i and the repair times at each channel are assumed to be i.i.d. exponential with mean $1/\mu_i$. Since we assume infinite population, the base repair center can be modeled as an M/M/c_i queueing model, where the arrival (base-repairable failure) rate is $\alpha_i\lambda_i$ and the service rate is μ_i. So the steady-state probability distribution that there are n items at the base repair center i, $P_i(n)$, is given by the following (1) and (2).

$$P_i(n) = \begin{cases} \dfrac{(\alpha_i\lambda_i)^n}{n!\,\mu_i^n}P_i(0) & (1 \le n \le c_i) \\ \dfrac{(\alpha_i\lambda_i)^n}{c_i^{n-c_i}c_i!\,\mu_i^n}P_i(0) & (n \ge c_i) \end{cases} \quad (1)$$

where

$$P_i(0) = \left[\sum_{n=0}^{c_i-1}\frac{(\alpha_i\lambda_i)^n}{n!\,\mu_i^n} + \frac{1}{c_i!}\left(\frac{\alpha_i\lambda_i}{\mu_i}\right)^{c_i} \times \frac{c_i\mu_i}{c_i\mu_i - \alpha_i\lambda_i}\right]^{-1} \quad (2)$$

Note that the above probability distributions do not exist unless the steady-state condition, i.e., $\alpha_i\lambda_i/c_i\mu_i < 1$, is satisfied.

2.2 Probability distribution of depot-shortaged items at the depot repair center

The depot-repairable failures at base i occur according to Poisson process with rate $(1-\alpha_i)\lambda_i$ which are independent of each other. This implies the superposed arrival stream at the depot repair center is Poisson with rate $\sum_{i=1}^{I}(1-\alpha_i)\lambda_i$. As we assume that there are c_d channels at the depot repair center, and also, the repair times are i.i.d. exponential with mean $1/\mu_d$, the probability that there are D items at the depot repair center, $P_d(D)$, is derived from the equations of an M/M/c_d queueing model as follows:

$$P_d(D) = \begin{cases} \dfrac{\left[\sum(1-\alpha_i)\lambda_i\right]^D}{D!\,\mu_d^D} P_d(0) & (D \le c_d) \\ \dfrac{\left[\sum(1-\alpha_i)\lambda_i\right]^D}{c_d^{D-c_d} c_d!\,\mu_d^D} P_d(0) & (D \ge c_d) \end{cases} \tag{3}$$

where

$$P_d(0) = \left[\sum_{D=0}^{c_d-1} \frac{\left[\sum(1-\alpha_i)\lambda_i\right]^D}{D!\,\mu_d^D} + \frac{1}{c_d!}\left[\frac{\sum(1-\alpha_i)\lambda_i}{\mu_d}\right]^{c_d} \times \frac{c_d\mu_d}{c_d\mu_d - \sum(1-\alpha_i)\lambda_i} \right]^{-1} \tag{4}$$

Steady-state condition is $\sum_{i=1}^{I}(1-\alpha_i)\lambda_i / c_d\mu_d < 1$.

Now let us find the probability distribution of k_i, the number of items at the depot repair center which is supposed to be returned to base i, i.e., depot-shortaged items of base i. When we denote actual fill rate of depot as F_d, the arrival rate of total depot-shortaged items is $\sum_{i=1}^{I}(1-\alpha_i)\lambda_i(1-F_d)$ and the rate of depot-shortaged items of base i is $(1-\alpha_i)\lambda_i(1-F_d)$. Since actual fill rate means the percentage of arriving items that are replaced immediately by a depot spare, the ratio of total depot-shortaged items to those of base i, θ_i, is expressed as in equation (5).

$$\theta_i \equiv (1-\alpha_i)\lambda_i(1-F_d) / \sum_{i=1}^{I}(1-\alpha_i)\lambda_i(1-F_d) = (1-\alpha_i)\lambda_i / \sum_{i=1}^{I}(1-\alpha_i)\lambda_i \tag{5}$$

Now, considering there are s_d spare items at the depot, the conditional probability that there are k_i depot-shortaged items with respect to base i given that D items are depot shortaged is given by

$$P_{id}(k_{id}|D) = \begin{cases} 1 & \text{if } k_i = 0, D \le s_d \\ 0 & \text{if } k_i \ne 0, D \le s_d \\ \binom{D-s_d}{k_i}\theta_i^{k}(1-\theta_i)^{D-s_d-k} & \text{if } D > s_d \end{cases} \tag{6}$$

Equation (6) means that, if there still remain spares in the depot, then depot-shortaged item to base i can not exist. On the other hand, if $D > s_d$, i.e., all spares in the depot have been sent to bases, then depot-shortage accumulates and, among them, depot-shortaged item to base i has binomial distribution.

By unconditioning on D in $P_{id}(k_i/D)$, we can obtain $P_{id}(k_i)$ as follows:

$$P_{id}(k_i) = \sum_D P_{id}(k_i/D) \cdot P_d(D) \tag{7}$$

2.3 Probability distribution of items in transit

It is well known that the probability distribution of the number of items in transit from the depot repair center to base i is equal to that of the depot repairable failures at the base in the steady state(see, for example, p. 710 of Hillier and Lieberman [8]). Therefore, the probability distribution of the number of items in transit from the depot to the base is Possion. Since the number in transit from base i to the depot repair center is also Poisson and sum of independent Possion's is Possion, the total number of items in transit is also Possion. When we denote the transit time between base i and the depot by t_i, the probability distribution of the number of items in transit is Poisson as given in equation (8).

$$P_i(m_i) = \frac{\left[2(1-\alpha_i)\lambda_i t_i\right]^{m_i} \times \exp(-2(1-\alpha_i)\lambda_i t_i)}{m_i!} \tag{8}$$

2.4 Probability distribution of total failed items

In the steady state, the total failed items of a base are the sum of the items currently at the base repair center, at the depot repair center and in transit. So we can obtain the probability distribution of the total failed items of base i, $P(z_i)$, by convolution of the previously derived probability distributions as in equation (9).

$$P(z_i) = \sum_{m_i=0}^{\infty}\sum_{k_i=0}^{\infty} P_t(m_i) \cdot P_{id}(k_i) \cdot P_i(z_i-k_i-m_i) \quad (9)$$

2.5 Expected cost and minimum fill rate

If the total failed items of base i, z_i, is greater than the spare inventory level s_i, then the shortage cost b_i is incurred for each backorder. On the other hand, the holding cost h_i is charged on the total number of spares in the system, which is equal to $h_i s_i$ since it is reasonable to consider that the holding cost is charged on the whole spare items regardless of their current locations. When we assume a quadratic shortage cost, the total expected cost of base i, which is the sum of the expected shortage and holding costs, can be obtained by:

$$TC(s_i) = h_i s_i + b_i \sum_{z_i=s_i+1}^{\infty} (z_i-s_i)^2 P(z_i) \quad (10)$$

Similarly, when we denote the depot shortage and holding costs as b_d and h_d, respectively, the total expected cost of the depot is

$$TC(s_d) = h_d s_d + b_d \sum_{D=s_d+1}^{\infty} (D-s_d)^2 P(D) \quad (11)$$

Now in order to find the minimum cost spare level at the base i, we need the following two theorems, which has been proved in Kim *et al.* [9].

Theorem 1. *The expected total cost functions, $TC(s_i)$ and $TC(s_d)$ are unimodal on the interval $[0,\infty]$.*

Proof. See Kim *et al.* [9].

The actual fill rate F_i, which is the probability that the base has more spares than the total failed items, has to be larger than or equal to the minimum required fill rate f_i:

$$F_i = \Pr\{ z_i < s_i \} = \sum_{z_i=0}^{s_i-1} P(z_i) \geq f_i \quad (12)$$

Theorem 2. *Let s_i^* be the inventory level with the minimum total expected cost and $\bar{s}_i$ be the minimum of s_i values satisfying equation (12). Then the spare inventory level to achieve the minimum fill rate at minimum cost is* $\max\{ s_i^*, \bar{s}_i \}$.

Proof. See Kim *et al.* [9].

3. THE ALGORITHM

We now formally present an algorithm to determine the spare inventory level to achieve a predetermined minimum fill rate at a minimum cost

Step 0. Verify that the following steady-state conditions are satisfied.

$$\sum_{i=1}^{I}(1-\alpha_i)\lambda_i/c_d\mu_d < 1 \text{ and } \alpha_i\lambda_i/c_i\mu_i < 1 \text{ for } i=1,2,\cdots,I.$$

If the conditions are met, go to Step 1. Otherwise, stop since the system we are considering can not reach steady-state.

Step 1. Calculate $P_d(D)$ for $D=0,1,\cdots$ until the probability becomes less than $\varepsilon=10^{-4}$.

Step 2. Calculate minimum cost spare level of the depot by calculating $TC(s_d)$ until it starts increasing. The largest spare level before we observe increase of $TC(s_d)$ is the minimum cost spare level.

Step 3. For $i=1,2,\cdots,I$, perform the following 3.1-3.4 Steps.

Step 3.1. Calculate $P_i(n)$, $P_{id}(k_i)$, $P_t(m_i)$ and $P(z_i)$ until the probability becomes less than $\varepsilon=10^{-4}$.

Step 3.2. Calculate $TC(s_i)$ until it starts increasing.

The largest value before the increase is the minimum point of the cost function, i.e., s_i^*.

Step 3.3. Calculate the minimum inventory level satisfying the minimum fill rate $\bar{s}_i$.

Step 3.4. Choose the maximum of s_i^* and $\bar{s}_i$ as desired spare inventory level for the current base i.

Step 1 and 3.1 are to calculate the probability distributions previously introduced. In Step 3.2 and 3.3, we find the minimum point of the cost function and calculate the minimum inventory level satisfying the specified minimum fill rate. Using the results of Steps 3.2 and 3.3, we are able to find the desired spare level in Step 3.4.

3.1 Example

We illustrate the algorithm by the following example. Consider a multiechelon inventory system with two bases and a depot. The shortage costs for each base and a depot are 107.5, and the holding costs are set to 19.6. Other relevant data is described in Table 1. Table 2 shows the results considering the total cost only. For base 1, when we neglect the minimum fill rate, the desirable inventory level is 26 at a minimum total expected cost of 539.468.

Table 1. Data for the example

Base/Depot	Parameters						
	λ_i	α_i	c_i	μ_i	t_i	h_i	b_i
Base 1	20.0	0.623	2	18.0	1.130	19.6	107.5
Base 2	10.0	0.743	1	15.0	1.502	19.6	107.5
Depot	-	-	5	3.0	-	19.6	107.5

Table 2. Minimum cost inventory level

Base/Depot	Inventory level	Minimum cost
Base 1	26	539.468
Base 2	14	308.617
Depot	10	249.697

Table 3. Output of the algorithm for the example

Minimum fill rate	Actual fill rate	Optimal spare level	Optimal cost
0.99	0.994(0.993)*	30(18)	591.428(355.569)
0.95	0.956(0.958)	26(15)	539.468(312.476)
0.90	0.956(0.929)	26(14)	539.468(308.617)
0.85	0.956(0.929)	26(14)	539.468(308.617)
0.80	0.956(0.929)	26(14)	539.468(308.617)
0.75	0.956(0.929)	26(14)	539.468(308.617)
0.70	0.956(0.929)	26(14)	539.468(308.617)
0.65	0.956(0.929)	26(14)	539.468(308.617)
0.6	0.956(0.929)	26(14)	539.468(308.617)

* Entry in parenthesis is for the base 2.

For base 2, it is 14 items at a cost of 308.617. For the depot, it is 10 items at a cost of 249.697. The solution of the example, inventory levels satisfying the minimum fill rate at minimum cost, is summarized in the third column of Table 3. The spares level and the cost are decreased as the minimum fill rate is decreased until the minimum point is reached. But if the inventory level arrives at the minimum point, it remains there despite a further decrease in the minimum fill rate.

4. COMPUTATIONAL EXPERIMENTS

We perform extensive computational experiments for the proposed algorithm. The principal objective of computational experiments is to test the accuracy of the proposed algorithm. For this purpose we compare the minimum total expected cost and actual fill rate calculated by the algorithm with those obtained from a simulation. A secondary objective is to get an idea of how fast the algorithm is for real problems. The proposed algorithm is written in C and the simulation model is programmed in the SLAMSYSTEM (Version 4.0) with an interface program in FORTRAN. The experiments are performed on a Pentium(166MHz CPU) based IBM compatible PC system. The data for each experiment are generated from the following distributions:

Distributions for Constant Parameters

Holding cost ~ N(25, 25),

Shortage cost ~ N(100, 100),

Probability that a failure is a base repairable ~ U(0.4, 0.8),

Number of repair channels at the depot ~ integer nearest to a random number from U(3, 6),

Number of repair channels at base ~ integer nearest to a random number from U(2, 5),

Minimum fill rate ~ U(0.55, 0.99),

Transit time ~ U(1.0, 2.0)

Distributions for Failure and Repair Rate

Failure rate ~ Poisson(10),

Repair rate of a channel at the depot ~ Poisson(20),

Repair rate of a channel at base ~ Poisson(7)

For a given number of bases, we generate a set of data satisfying the steady-state conditions from the distributions for constant parameters. For each set of data, we use the algorithm to calculate the total cost and fill rates of a system based on the failure and repair rates. With the same constant parameters data set, ten independent simulation runs are replicated using random numbers generated from the failure and repair rate distributions to obtain an averaged result. Each simulation is run for

Table 4. Input data for base 5

Base /Depot	Parameters							
	λ_i	α_i	c_i	μ_i	f_i	t_i	h_i	b_i
Base 1	10.0	0.623	4	2.0	0.774	1.630	19.61	107.5
Base 2	5.0	0.743	1	5.0	0.941	1.502	19.61	107.5
Base 3	18.0	0.651	4	5.0	0.721	1.050	19.61	107.5
Base 4	8.0	0.332	2	8.0	0.802	1.324	19.61	107.5
Base 5	11.0	0.649	2	10.0	0.945	1.520	19.61	107.5
Depot	-	-	2	15.0	-	-	19.61	107.5

Table 5. Experimental result for base 5

Base /Depot	Cost		Fill rate	
	Algorithm	Simulation	Algorithm	Simulation
Base 1	509.797	511.445	0.802	0.803
Base 2	308.050	305.904	0.955	0.953
Base 3	440.966	442.208	0.799	0.798
Base 4	402.960	402.883	0.843	0.841
Base 5	397.171	397.198	0.964	0.964
Depot	139.370	140.071	0.739	0.740

Table 6. Input data for base 10

Base /Depot	Parameters							
	λ_i	α_i	c_i	μ_i	f_i	t_i	h_i	b_i
Base 1	12.0	0.539	2	4.0	0.655	1.802	29.96	101.6
Base 2	9.0	0.681	4	2.0	0.643	1.559	29.96	101.6
Base 3	10.0	0.483	2	6.0	0.805	1.918	29.96	101.6
Base 4	9.0	0.423	1	9.0	0.721	1.703	29.96	101.6
Base 5	12.0	0.610	3	5.0	0.928	1.822	29.96	101.6
Base 6	7.0	0.647	1	7.0	0.751	1.748	29.96	101.6
Base 7	12.0	0.704	3	5.0	0.972	1.058	29.96	101.6
Base 8	9.0	0.648	2	9.0	0.627	1.289	29.96	101.6
Base 9	8.0	0.723	2	7.0	0.830	1.382	29.96	101.6
Base 10	11.0	0.522	2	7.0	0.557	1.590	29.96	101.6
Depot	-	-	5	9.0	-	-	29.96	101.6

Table 7. Experimental result for base 10

Base /Depot	Cost		Fill rate	
	Algorithm	Simulation	Algorithm	Simulation
Base 1	723.405	724.759	0.663	0.665
Base 2	595.515	595.382	0.678	0.679
Base 3	820.040	820.357	0.827	0.828
Base 4	724.308	723.738	0.742	0.744
Base 5	819.018	819.243	0.945	0.944
Base 6	448.263	452.453	0.758	0.750
Base 7	542.431	542.606	0.979	0.978
Base 8	383.355	384.441	0.694	0.692
Base 9	345.301	345.320	0.876	0.876
Base 10	696.363	696.618	0.647	0.646
Depot	613.243	613.605	0.683	0.679

Table 8. Input data for base 15

Base /Depot	λ_i	a_i	c_i	μ_i	f_i	t_i	h_i	b_i
Base 1	16.0	0.173	3	6.0	0.831	1.464	29.36	108.4
Base 2	12.0	0.672	3	6.0	0.767	1.314	29.36	108.4
Base 3	11.0	0.757	9	2.0	0.991	1.628	29.36	108.4
Base 4	7.0	0.770	2	7.0	0.771	1.075	29.36	108.4
Base 5	15.0	0.622	2	12.0	0.807	1.115	29.36	108.4
Base 6	14.0	0.591	2	10.0	0.972	1.678	29.36	108.4
Base 7	6.0	0.467	2	3.0	0.758	1.365	29.36	108.4
Base 8	5.0	0.744	1	12.0	0.585	1.576	29.36	108.4
Base 9	9.0	0.685	4	3.0	0.858	1.311	29.36	108.4
Base 10	16.0	0.590	3	5.0	0.976	1.047	29.36	108.4
Base 11	11.0	0.571	2	7.0	0.885	1.766	29.36	108.4
Base 12	11.0	0.525	2	9.0	0.797	1.203	29.36	108.4
Base 13	13.0	0.703	4	3.0	0.899	1.908	29.36	108.4
Base 14	18.0	0.789	3	6.0	0.705	1.105	29.36	108.4
Base 15	5.0	0.607	2	3.0	0.562	1.784	29.36	108.4
Depot	-	-	5	21.0	-	-	29.36	108.4

Table 9. Experimental result for base 15

Base /Depot	Cost		Fill rate	
	Algorithm	Simulation	Algorithm	Simulation
Base 1	653.103	652.464	0.851	0.852
Base 2	483.185	483.201	0.782	0.784
Base 3	676.233	676.236	0.992	0.992
Base 4	222.465	221.957	0.842	0.845
Base 5	556.238	556.667	0.852	0.851
Base 6	885.364	885.214	0.973	0.973
Base 7	419.147	418.411	0.792	0.793
Base 8	212.816	212.011	0.703	0.796
Base 9	430.904	430.805	0.878	0.878
Base 10	796.175	796.207	0.983	0.983
Base 11	724.017	724.631	0.903	0.902
Base 12	536.029	536.501	0.810	0.811
Base 13	827.760	825.781	0.902	0.904
Base 14	580.409	578.346	0.734	0.735
Base 15	289.054	289.083	0.685	0.684
Depot	160.151	160.605	0.676	0.663

50,000 time units and the initial transient period was discarded when accumulating the statistics. The following Tables, 4-9, show the input data and experimental results

Table 10. Analysis for algorithm vs. simulation

No. of bases		Mean percent difference	Maximum of percent difference	Minimum of percent difference	Variance of percent difference
5	Cost	0.3060	0.6965	0.0068	0.0006
	Fill rate	0.3067	0.8683	0.0062	0.0008
10	Cost	0.1551	0.9348	0.0055	0.0007
	Fill rate	0.2596	0.9830	0.0023	0.0008
15	Cost	0.1155	0.3780	0.0004	0.0001
	Fill rate	0.2867	0.9404	0.0060	0.0007

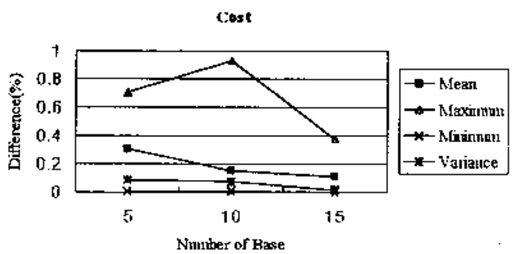

Figure 2. Cost Difference between the Proposed Algorithm and Simulation

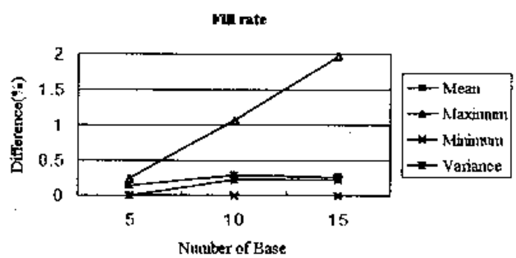

Figure 3. Fill Rate Difference between the Proposed Algorithm and Simulation

for the cases where there are 5, 10 and 15 bases. Table 10 summarizes the results of the experiments using the percent differences of the total cost and fill rate as performance measures. Here, the percent difference means (100×|output of the proposed algorithm - output of simulation|/output of simulation). Figure 2 and Figure 3 graphically show the contents of Table 10.

We conclude from our set of runs:

1. The proposed algorithm finds the optimal spares level estimated from the simulation for all cases.
2. Percent differences in the cost and fill rates are within 2% for all replications.
3. Mean and variance of percent differences do not change significantly as the problem size increases.
4. All results of the proposed algorithm are very close to those obtained from the simulations.
5. Running time of the algorithm is less than 10 seconds in CPU time for all cases.

Even though our experiments are based on rather a small test set of problems due to the long simulation running time (for example, more than 3 hours in CPU time for the 15-base problem), we see the proposed algorithm appears to be quite promising.

5. CONCLUDING REMARKS

In this paper we develop a method to calculate an approximate spare inventory level which satisfies a predetermined minimum service rate at minimum cost. With this approach we are able to solve large problems quickly and accurately. For further study, one can relax the assumption of infinite number of items operating at each base, which has enabled us to make use of the formulas from M/M/s model. In addition, this model can be extended to the more general case where the spares in a base can be transferred to another if it has no spare to replace a failed item. We are currently exploring these problems, and the results to date are very encouraging.

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