Journal of the Korean Institute
of Industrial Engineers
Vol.24, NO.3, September, 1998

이치형 성능변수를 이용한 전수검사 하에서의 공정감시절차의 경제적 설계*

Economic Design of a Process Monitoring Procedure for Dichotomous Performance Variable under 100% Inspection*

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Abstract

An economic process monitoring procedure is presented when the major quality characteristic of the item is dichotomous. Every item is inspected and decided whether it is conforming or not. If an item is found to be nonconforming, the previous number of the successive conforming items is compared with a predetermined number r to check the process for existence of any assignable cause of variation. A cost model is constructed on the basis of costs of inspection, illegal signal, undetected out-of-control state and corrective action. By minimizing the expected total cost per unit time, the optimal value of r is obtained. The effects of cost coefficients are studied numerically.

1. Introduction

Due to the recent advances in inspection systems and the increasing requirements of the marketplace, 100% inspection (screening) becomes very popular at one or more stages of a manufacturing process. For example, in the manufacturing process of a nozzle of the fuel injection equipment, every component (needle and body) is inspected before it is assembled. For detailed literature review on the screening procedure, see Tang and Tang (1994). For more recent studies, see Bai and Kwon(1995), Bai et al.(1995), Boys et al.(1996), Bai and Kwon(1997), and Hong et al.(1997). In many manufacturing processes, it is also important to be able to detect moderate shifts in the performance of the process. For attainment of a state of statistical stability of a process, Shewhart(1926) originates the process control chart. A wide variety of Shewhart control charts and their modifications have been developed for different types of problems. Economic design of control charts has been also studied by many authors. For detailed review, see Montgomery(1980). In most of the control charts, samples are taken over a fixed or variable time interval. When every item is subject to inspection, the usual procedure is to take some convenient

* 이 논문은 1996년 8월 - 1997년 7월 부경대학교 기성회 해외연구 지원으로 연구되었음.

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and meaningful segment of production. Apart from the control chart method, Hui(1991) studied a complete inspection plan with feedback control when the performance variable is continuous. Control charts may be based on attribute data or on measurement data. For detecting shifts in the process, the measurement-data chart is more efficient than the attribute-data chart. However, the attribute-data chart has its own operational advantages; i) the inspection process is often expedited through use of the go-no go type gauge, ii) many separate inspection tests, including some quantitative and some qualitative, may be carried out on each item which is eventually classified into conforming or nonconforming group, iii) no distributional assumptions are needed for attribute data, and so on. When the attribute data is used to monitor changes in fraction nonconforming, p-chart is usually adopted. As an alternative, under 100% inspection, Bourke (1991) suggested to use run-length control chart to detect a shift in fraction nonconforming.

When p-chart is used under 100% inspection, even the data taken from the same segment may not be homogeneous since the process may change during any segment and it is difficult to form a rational subgroup. The run length chart suggested by Bourke(1991) has a similar drawback because the data of the same run may be heterogeneous. Considering the time point of process shift, this paper presents an economic process monitoring procedure when the performance variable is dichotomous. In the suggested procedure, if an item is found to be nonconforming during inspection, the previous number of the successive conforming items is compared with a predetermined number r to check if the process is in control or out of control. In section 2, a cost model is constructed on the basis of the costs of inspection, illegal signal, undetected out-of-control state and corrective action. Section 3 provides a solution procedure to obtain the optimal value of r. In section 4, a numerical example is given and the effects of cost coefficients are studied.

2. The Model

2.1 The Procedure

Let Y be a binary performance variable taking 0 if the corresponding item is conforming and 1 otherwise. Let p_i, i=0,1 be the proportions of defective items under in-control and out-of-control states of the process, respectively. Every item is subject to inspection and the process is monitored as to the following procedure:

i) *For each incoming item, obtain the observed value y of Y. If y = 0, accept the item and repeat inspecting subsequent items. Otherwise, reject the item and go to the next step.*

ii) *Compare the previous number of successive conforming items (the length of the preceding run of conforming items) with the predetermined number r. If it is greater than or equal to r, go to step i). Otherwise, go to the next step.*

iii) *Determine whether the process is in control or out of control. If in control, go to step i). Otherwise, go to the next step.*

iv) *Stop the process, investigate the process for some change, identify the assignable cause of variation and eliminate it. Repeat the procedure from step i).*

Here, the number r is the decision variable to be determined.

2.2 The Cost Model

The procedure described in the previous section will generates three periods; i) in-control period, ii) out-of-control period until detection of the change in the process, and iii) period of identification and correction of the assignable cause, during which the process is stopped. Here, it is assumed that it takes a negligible length of time to know whether the process is in control or out of control. Figure 1 illustrates the three periods of a cycle.

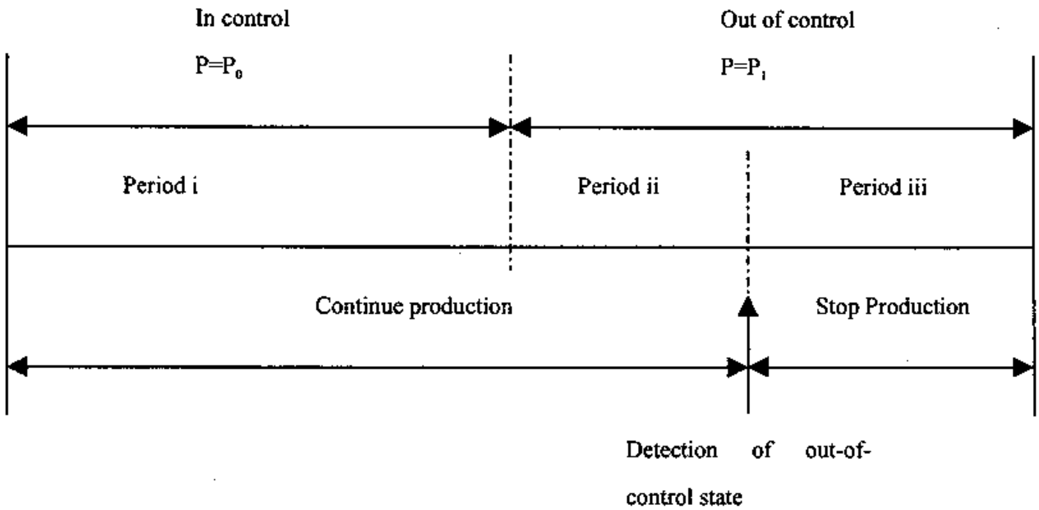

Figure 1. Cycle of the Process

In each of these period, some kind of costs are incurred, that is, i) the cost of inspection, ii) the cost of illegal signal, iii) the cost of undetected out-of-control state, and iv) the cost of correction. Each of these factors is associated with at least one of the three periods. The cost of inspection will be incurred during period i and ii of Figure 1. If we denote the unit cost of inspection by c_s, the cost of inspection per cycle will be

$$c_s(N+M), \tag{1}$$

where N and M are the numbers of items produced under the in-control and out-of-control state per cycle, respectively. Let c_i be the unit cost incurred by an illegal signal and D be the number of illegal signals issued during the in-control-state period. Then the cost due to the illegal signals per cycle is

$$c_i D \tag{2}$$

since the illegal signal can be issued during only the in-control-state period. During period ii of Figure 1, the process is considered as in-control state and keeps producing items even though it is out of control. Let c_u denote the cost of the undetected out-of-control state per unit time. If we take the unit time as the length between the completion time points of two successive items, the cost of the undetected out-of-control state per cycle will be

$$c_u M. \tag{3}$$

The cost of correction includes the cost of corrective action c_a and the opportunity cost incurred by stopping the process. Let c_o be the unit opportunity cost and A be the number of items that might have been produced during the correction period if the process were not stopped. Then the cost of correction per cycle will be

$$c_a + c_o A. \tag{4}$$

By combining (1), (2), (3), and (4), the total cost per cycle is obtained by

$$TC_{cycle} = c_s N + (c_s + c_u)M + c_i D + c_o A + c_a. \tag{5}$$

The length of a cycle is $N + M + A$ and thus, the total cost per unit time is given by

$$TC = \frac{c_s N + (c_s + c_u)M + c_l D + c_o A + c_a}{N + M + A}. \tag{6}$$

Since N, M, and D are random variables, the optimum value r^* of r is determined so that the expected value of TC is minimized.

3. The Optimal Solution

To obtain the expected total cost per unit time ETC, $E(N)$, $E(M)$, and $E(D)$ must be first obtained. Assume λ to be the probability that the process shifts from the in-control state to the out-of-control state at any production point. Then the probability function of N is

$$f_N(n) = (1-\lambda)^n \lambda, \quad n=0,1,2,\cdots \tag{7}$$

and $E(N)$ is given by

$$E(N) = \frac{1-\lambda}{\lambda}. \tag{8}$$

The probability function of D can be obtained as

$$f_D(d) = \frac{\lambda}{\lambda + (1-\lambda)p_0[1-(1-p_0)^r]} \left[\frac{(1-\lambda)p_0[1-(1-p_0)^r]}{\lambda + (1-\lambda)p_0[1-(1-p_0)^r]} \right]^d, \quad d = 0,1,2,\cdots. \tag{9}$$

See Appendix for detailed derivation. Thus, the expected value of D is

$$E(D) = \left(\frac{1-\lambda}{\lambda} \right) p_0[1-(1-p_0)^r]. \tag{10}$$

The probability function of M is difficult to be obtained in a simple form. In Appendix, we obtain its expected value without deriving its probability function as

$$E(M) = \frac{1}{p_1} \left[1 + \frac{p_1(1-p_0)^{r+1} - p_0(1-p_1)^{r+1}}{(p_1 - p_0)[1-(1-p_1)^r]} \right]. \tag{11}$$

The expected value of the ratio of two random variables is generally not equal to the ratio of their expectations. However, the sequence of production monitoring adjustment, with accumulation of costs over the cycle, can be viewed as a particular type of stochastic process called renewal reward process. Thus, the total expected cost per unit time can be obtained by dividing the total expected cost per cycle with the expected length of a cycle as follows:

$$ETC = \frac{c_s E(N) + (c_s + c_u)E(M) + c_l E(D) + c_o A + c_a}{E(N) + E(M) + A}. \tag{12}$$

Since $E(D)$ is an increasing function and $E(M)$ is a decreasing function of r for given p_0 and p_1, the optimum value r^* exists for some cost coefficients and can be obtained numerically using a simple computer program.

4. Numerical Example

This section provides a numerical example and the effects of cost coefficients are studied based on this example.

Example. In a certain factory, synthetic and natural gut casings are produced for a process meat packer. Natural gut materials are visually inspected upon receipt, graded, and sent to processing. After processing, all finished casings are tested under pressure on a special device to ensure a specified strength before shipping to the meat packer. Suppose the proportion of casings that burst during test is p_0=1% if the process is in control state. When the proportion is p_1=5%, the process is considered as out of control and the out-of-control state should be detected as soon as possible. Suppose that λ=0.0001, c_s=\$0.01, c_l=\$ 0.5, c_u=\$1.0, c_a=\$10.0, c_o=\$5.0, and A=5. Using a computer program, r^*=36 and ETC^*=\$0.01895.

Effects of Cost Coefficients on r^*

Among the cost coefficients, the effects c_u and c_i are studied here. Figure 2 shows that r^* tends to increase as c_u increases. And r^* takes smaller values as c_i becomes larger. The other cost coefficients do not seem to affect r^* much.

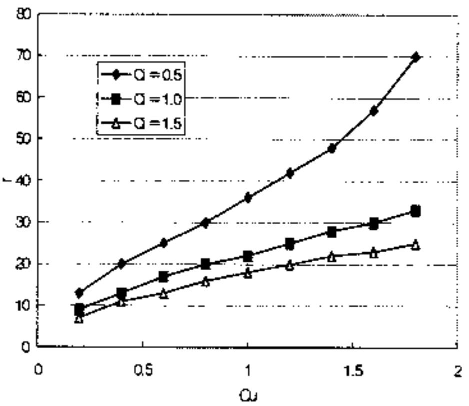

Figure 2. Effects of c_u and c_i on r^*

Use of Incorrect Cost Coefficient

Sometimes it may be difficult to estimate the cost coefficients correctly. When incorrect values of the cost coefficients are used, the obtained r^* is not truly the optimal value of r and additional cost will be unduly incurred. Among the cost coefficients, the effects of using incorrect values for c_u are studied for different values of c_i. Figure 3 shows thepercentage increase in *ETC* when

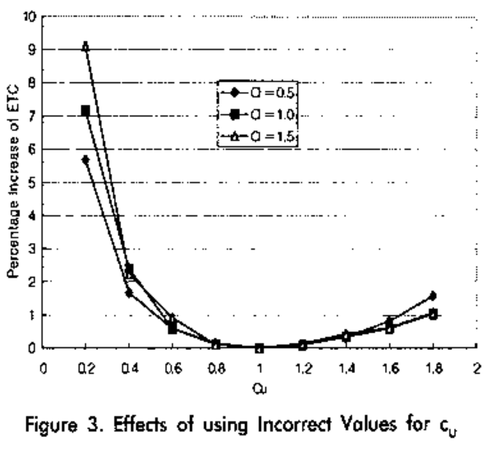

Figure 3. Effects of using Incorrect Values for c_u

incorrect values of c_u are used given its true value c_u= 1.0. The graph shows that *ETC* is more sensitive when c_u is underestimated than when it is overestimated. Thus, if the true value of c_u is unknown, it will be better to use an overestimated value.

Comparison with Run Length Control Chart

If we use a run length control chart with only a lower control limit, the lower limit L will be determined by

$$P\left[\text{Run Length} < L \,|\, \text{Fraction Nonconforming}=p_0\right] = 1 - (1-p_0)^L \leq \alpha. \tag{13}$$

When p_0=1% and α=0.05, (13) will give L=5. This means that the process is hardly checked if it is not severely changed. The process with fraction nonconforming p_1=5% will be checked with probability $1-(1-0.05)^5=0.2262$ only, i.e., the change in the process will be undetected with probability 0.7738. If this is not a problem from the economic point of view, it does not matter. If it is, however, the process control scheme will be better designed economically.

Moreover, when plotting points the run length control chart under 100% inspection, there can be a point which is based on heterogeneous data. That is, a run may consist of mixed data from in-control state and out-of-control state process. some of the items of a run may be from the in-control state while the others of the same run may be from the out-of-control state. Thus, some point does not represent exclusively for in-control process or out-of-control process. In the suggested model, this trouble is taken into consideration by treating the numbers of items produced under in-control and out-of-control states separately.

5. Conclusion

An economic process monitoring procedure is presented when the major quality characteristic of the item is dichotomous and every item is subject to inspection. Under an assumed cost model, the optimum number of successive conforming items is provided, by which the process is decided to be checked or not.

The solution is not given in a closed form but it can be numerically calculated using a simple computer program. The effects of cost coefficients on the optimal solution and the effect of using incorrect cost coefficients on the expected total cost are studied on the basis of a numerical example. The differences between the suggested model and run length control chart are also discussed briefly.

The model may be extended to the case where both the lower and upper limits of the number of successive conforming items are to be decided. The case where inspection is based on a surrogate variable may be studied. Further studies are also expected for the case where the observed data are autocorrelated.

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Appendices

1. Derivation of $f_D(d)$

Let L be the number of rejected items during the in-control period. Then

$$P(L=l|N=n)=\binom{n}{l}(1-p_0)^{n-l}p_0^l,\quad l=0,1,2,\cdots,n. \tag{A1}$$

Using (7) and (A1), we have

$$P(L=l)=\left[\frac{\lambda}{\lambda+(1-\lambda)p_0}\right]\left[\frac{(1-\lambda)p_0}{\lambda+(1-\lambda)p_0}\right]^l,\quad l=0,1,2,\cdots. \tag{A2}$$

Note that we have an illegal signal when the number of successive conforming items is smaller than r under the in-control process. So, the probability to have an illegal signal is $1-(1-p_0)^r$. Thus,

$$P(D=d|L=l)=\binom{l}{d}[1-(1-p_0)^r]^d[(1-p_0^r)]^{l-d},\quad d=0,1,2,\cdots,l \tag{A3}$$

From (A2) and (A3), $f_D(d)$ of (9) is obtained.

2. Derivation of $E(M)$

Let K be the number of rejected items until the out-of-control state is detected. It can be shown that

$$P(K=k)=\begin{cases} 1-\dfrac{p_1(1-p_0)^{r+1}-p_0(1-p_1)^{r+1}}{p_1-p_0} & \text{if } k=1 \\ \left[\dfrac{p_1(1-p_0)^{r+1}-p_0(1-p_1)^{r+1}}{p_1-p_0}\right][1-(1-p_1)^r][(1-p_1)^r]^{k-2}, & \text{if } k=2,3,\cdots \end{cases} \tag{A4}$$

Let X_1 be the number of items inspected after the process is changed until the first nonconforming item is found and X_i, $i=2,3,4,\cdots$ be the number of items inspected between $(i-1)^{th}$ and i^{th} nonconforming items, including the i^{th} item under the out-of-control state. Then, we have $E(M|K=k)=E(X_1)+\cdots+E(X_k)$ and obtain $E(M)$ of (11) using the identity $E(M)=E[E(M|K)]$.