Journal of the Korean Institute
of Industrial Engineers
Vol.24, N0.4, December, 1998

An Approach to Determining Storage Capacity of an Automated Storage/Retrieval System under Full Turnover-Based Policy

물품회전율을 기준으로 한 저장정책하에서 자동창고의 저장규모 결정방법

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Abstract

Full turnover-based storage policy (FULL) is often used to minimize the travel time needed to perform storage/retrieval operations in automated storage/retrieval systems (AS/RSs). This paper presents an approach for determining the required storage capacity for a unit load AS/RS under the FULL. An analytic model is formulated such that the total cost related to storage space and space shortage is minimized while satisfying a desired service level. To solve the model, some analytic properties are derived and based on them, an iterative search algorithm which always generates optimal solutions is developed. To illustrate the validity of the approach, an application is provided when the standard economic-order-quantity inventory model is used.

1. Introduction

Automated storage and retrieval systems (AS/RS) have been widely adopted in warehousing applications to handle various materials in an effective way. A typical system mainly consists of storage racks, storage/retrieval (S/R) machines, conveying devices which link the system with outside areas, and a controller. The systems are usually regarded as highly specialized material handling systems which require extensive initial investments. In addition, once constructed, it is difficult to modify the system structure such that new conditions are met. Therefore, issues of design in AS/RS are of considerable importance.

One of the important design issues, determining the storage capacity of the system. is being addressed in this paper. The required storage capacity is defined as the amount of storage space needed to accommodate the materials to be stored in order to meet a desired service level. The major factor that influences storage sizing is the storage assignment policy used in the system. Three kinds of storage assignment policies are usually addressed in the literature. They are randomized storage assignment (RAN), full turnover-based storage (FULL), and N-class turnover-based storage (CN).

Assuming an identical storage size for the three policies, Hausman et al.[2,4] derived analytic expressions for the system throughput, and show that significant improvements in throughput are obtainable when FULL and CN

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are used. Later, Ko and Hwang[7] extended Hausman et al.'s study by taking into account the storage space required for each policy. However, in their study, the storage space were determined without any consideration of costs involved. Assuming dedicated storage assignment, Kim[6] studied the problem of simultaneously determining storage locations and space requirements for correlated items in a mini-load AS/RS. Optimization of multi-class dedicated storage layout was addressed by Yang[12]. The objective of the study was to minimize the expected single-command travel time. Rosenblatt and Roll[10] presented a search procedure for finding the optimal storage design which minimizes capital investment, space shortage cost, and costs associated with storage policies. Later, for a warehouse in a stochastic environment, the major elements that affect the required storage capacity were examined using a simulation model by the same authors[11]. Rosenblatt[9] studied four issues related to the design and control of AS/RSs. One of the issues is the determination of the items and the corresponding inventory levels to be allocated to an AS/RS with limited available space. Based on the probability of space shortage, Francis et al.[3] presented mathematical models of determining the storage capacities under different storage policies. Other related research on storage sizing includes those done by Mullen[8] and Bafna[1] where general procedures that can be used in practice are suggested.

A perusal of the literature shows that no research has investigated as to how the storage capacity is determined economically when space shortage is allowed meeting a given service level. In this paper, we present a model for determining the storage capacity such that the total cost related to storage space and space shortage is minimized while satisfying a desired service level for the FULL storage policy. An application of the model to the economic order quantity (EOQ) inventory system is presented with the investigation of the effects of demand distribution and ratio of storage space cost to space shortage cost on the storage capacity.

2. A Mathematical Model for Determining Storage Capacity

In order to determine the required storage capacity for the storage system, Francis et al.[3] present two different approaches, a service-level approach and a cost-based approach. In the former approach, the total amount of storage space is minimized without exceeding a given probability, α $(0<\alpha<1)$, of a space shortage occurring (hereafter, we call it the shortage probability). If the storage requirement is greater than the storage capacity, a space shortage occurs. Under such conditions, the excess space requirement is assumed to be met via leased storage space. In the cost-based approach, the storage capacity is determined to minimize the sum of the costs of owning space and contracting space incurred by space shortage without any consideration of the service level. In this paper, in order to determine the storage capacity we present a hybrid one in which the total cost is minimized subject to satisfying the service level required. The approach may reflect real-world situations much better than the previous approaches.

Let X and $S(\alpha)$ be the random variables denoting the aggregate inventory level of the system and the storage capacity at the $1-\alpha$ service level, respectively. Then, the following holds :

$$\Pr(X \leq S(\alpha)) \geq 1 - \alpha$$

Let X_i, i=1, ..., n, be a random variable which represents the inventory level of item i. In actual warehousing systems, the inventory level depends on the inventory model which specifies the reorder point and the ordering quantity. In this paper, we consider the case where every X_i follows a uniform distribution as follows :

$$X_i \sim U(a_i, b_i), \qquad i = 1,\ldots,n.$$

One example of such a case is the system in which the standard EOQ model with a_i being zero is applied to all items stored.

Throughout this paper, unless otherwise stated, items are numbered in a decreasing order of b_i values. X is then expressed as a function of X_is and its storage level, which is certainly influenced by the storage assignment policy used.

We now present a model for determining the economic storage capacity which is large enough to accommodate the incoming full pallet loads of items with a probability not less than $(1-\alpha)$. FULL is a kind of dedicated storage policy in which items are assigned dedicatedly to storage locations in an attempt to minimize the time required to perform storage/retrieval operations. Therefore, in this case, items with the larger ratio of their activity levels to storage space are allocated preferably to the locations closer to the I/O point. In the unit load AS/RS concerned in this paper, the ratio is referred to as 'turnover frequency' and is given by

$$TF_i = d_i \;/\; s_i \; q_i$$

where d_i, s_i, and q_i are the demand rate, the space needed to store a unit quantity, and the maximum quantity that can be stored for item i, respectively. If we assume that q_i is measured in a full pallet load of uniform size, then $s_i=1$ and therefore TF_i simply reduces to d_i/q_i.

Let β_i be the shortage probability of item i. Since X_i is assumed to be uniformly distributed with the probability density function $h(x_i)=1/(b_i-a_i)$ for $a_i \le x_i \le b_i$, the shortage probability can be expressed as

$$\beta_i = \Pr(X_i > q_i) = \Pr(X_i > a_i + (1-\beta_i)(b_i - a_i)).$$

Then, because of the statistical independence among the distributions, the probability of no shortage occurring for any item is given by

$$\Pr(\text{no shortage in the system}) = \prod_{i=1}^{n} (1-\beta_i). \qquad (1)$$

The probability is the overall service level of the system. Of course, the service level can be defined in different ways. One example is to achieve $100(1-\alpha)\%$ service for each item. However, in order to examine the performances of storage policies on the same basis, the service level should reflect the overall system as is given by (1).

Now, we want to determine the minimum amount of storage space required to provide the desired level of assurance so that the resulting shortage probability will not be greater than α. Let $w_i=b_i-a_i$ for all i. Here, for convenience number the items such that $w_i \ge w_{i+1}$, i=1,..., n-1. Then, the storage capacity becomes

$$S(\alpha) = \sum_{i=1}^{n} (a_i + w_i(1-\beta_i)).$$

In this case, the expected amount of space shortage per unit time will be

$$E(\alpha) = \sum_{i=1}^{n} \int_{a_i+w_i(1-\beta_i)}^{b_i} [x_i-(a_i+w(1-\beta_i))] \cdot h(x_i)dx_i$$
$$= \sum_{i=1}^{n} w_i\beta_i^2/2.$$

where the probability density function, $h(x_i)=1/w_i$. Therefore, the total cost is given by

$$TC(\alpha) = \lambda_1 S(\alpha) + \lambda_2 E(\alpha)$$
$$= \lambda_1 \sum_{i=1}^{n} (a_i + w_i(1-\beta_i)) + \lambda_2 \sum_{i=1}^{n} w_i\beta_i^2/2$$
$$= \lambda_1 \sum_{i=1}^{n} (a_i + w_i) - (\lambda_1^2/2\lambda_2) \sum_{i=1}^{n} w_i$$
$$+ (\lambda_2/2) \sum_{i=1}^{n} w_i(\lambda - \beta_i)^2$$

where λ_1=discount present worth cost per unit storage space owned and operated for unit time,

λ_2=discount present worth cost per unit leased space or per unit of space shortage for a finite planning period,

and

$\lambda=\lambda_1/\lambda_2$.

If we consider only the variable part, the third term of the total-cost function, the problem reduces to

$$\text{(P1)}\quad \text{Mininimize} \quad \sum_{i=1}^{n} w_i(\lambda-\beta_i)^2$$

$$\text{subject to} \quad \prod_{i=1}^{n}(1-\beta_i)\geq 1-\alpha$$

$$0\leq\beta_i\leq u_s \qquad \forall i$$

where u_s is a common upper bound of shortage probability which holds the inequality, $(1-u_s)^n\geq 1-\alpha$. This model seems to be a complex nonlinear program whose optimum cannot be easily determined

Substituting $\gamma_i=-\ln(1-\beta_i)$ into (P1) yields the equivalent program:

$$\text{(P2)}\quad \text{Minimize} \quad \sum_{i=1}^{n} w_i(\lambda-1+e^{-\gamma_i})^2$$

$$\text{subject to} \quad \sum_{i=1}^{n}\gamma_i\leq -\ln(1-\alpha)$$

$$0\leq\gamma_i\leq u_t \qquad \forall i \qquad (2)$$

where $u_t=-\ln(1-u_s)$.

Theorem 1. Let $g(\gamma_i) = (\lambda-1+e^{-\gamma_i})^2$. Then, $g(\gamma_i)$ is convex over $\gamma_i\geq 0$ for $\lambda\geq 1$ and $0\leq\gamma_i\leq -\ln((1-\lambda)/2)$ for $0<\lambda<1$.

Proof. If we take the first and second derivatives of $g(\gamma_i)$ with respect to γ_i, we obtain

$$g'(\gamma_i) = -2(\lambda-1+e^{-\gamma_i})e^{-\gamma_i}$$

and

$$g''(\gamma_i) = 2(\lambda-1+2e^{-\gamma_i})e^{-\gamma_i}.$$

If $\lambda\geq 1$, it is obvious that for any $\gamma_i\geq 0$, $g''(\gamma_i)\geq 0$ which is the necessary and sufficient condition for the convexity of $g(\gamma_i)$. If $0<\lambda<1$, the condition is equivalent to the following inequality

$$\lambda-1+2e^{-\gamma_i}\geq 0.$$

Solving for γ_i with (2) gives

$$0\leq\gamma_i\leq -\ln((1-\lambda)/2).$$

This completes the proof.

Note that the minimum value of $-\ln((1-\lambda)/2)$ is $-\ln(0.5)$ at $\lambda= 0$. Since $u_t\leq -\ln(0.5)$ assuming $u_s\leq 0.5$ (this assumption may be justified for most real-life situations), the convex region defined in Theorem 2 meets the constraints (2). In addition, it should be noted that the minimum point of $g(\gamma_i)$ lies at $\gamma_i=-\ln(1-\lambda)$ which is certainly less than $-\ln((1-\lambda)/2)$ for all $0<\lambda<1$.

Theorem 2. $g'(\gamma_i)$ is nonpositive and concave over $\gamma_i\geq 0$ for $\lambda\geq 1$ and $0\leq\gamma_i\leq -\ln(1-\lambda)$ for $0<\lambda<1$.

Proof. The third derivative of $g(\gamma_i)$ with respect to γ_i, will be

$$g'''(\gamma_i) = -2(\lambda-1+4e^{-\gamma_i})e^{-\gamma_i}.$$

If $\lambda\geq 1$, then $g'(\gamma_i)\leq 0$ and $g'''(\gamma_i)\leq 0$ for any $\gamma_i\geq 0$. Thus, $g'(\gamma_i)$ is nonpositive and concave over $\gamma_i\geq 0$ for $\lambda_i\geq 1$. If $0<\lambda<1$, the condition that $g'''(\gamma_i)$ is nonnegative should be

$$\lambda-1+4e^{-\gamma_i}\geq 0.$$

Solving for γ_i with (6) gives

$$0\leq\gamma_i\leq -\ln((1-\lambda)/4).$$

It follows that $g'(\gamma_i)$ is concave over $0\leq\gamma_i\leq -\ln((1-\lambda)/4)$ for $0<\lambda<1$.

Finally, since $\lambda-1+e^{-\gamma_i}\geq 0$ for $0\leq\gamma_i\leq -\ln((1-\lambda))$, and $-\ln((1-\lambda)\leq -\ln((1-\lambda)/4)$ for $0<\lambda<1$, $g'(\gamma_i)$ is nonpositive and concave over $0\leq\gamma_i\leq -\ln(1-\lambda)$ for $0<\lambda<1$.

3. A Constrained Convex Problem

Now, in order to deal with a class of similar problems, we consider a constrained convex problem formulated in a general way as follows:

$$\text{(P3)} \quad \text{Minimize} \quad Z = \sum_{i=1}^{n} c_i f(x_i) \tag{3}$$

$$\text{subject to} \quad \sum_{i=1}^{n} x_i \leq r_1 \tag{4}$$

$$0 \leq x_i \leq r_2 \quad \forall i \tag{5}$$

where $c_i(c_i \geq c_j,\ i < j)$, r_1 and r_2 $(r_1 \geq b_2)$ are positive constants and $f(x_i)$ is a nonnegative convex function of x_i over $0 \leq x_i \leq \max(x_0, r_2)$ whose minimum lies at $x_i = x_0 > 0 \quad \forall i$. We also assume that $f'(x_i)$ is nonpositive and concave for $0 \leq x_i \leq x_0$.

Since equation (3) and the region defined by constraints (4) and (5) are both convex, (P3) is a separable convex program. Then, excluding the feasibility conditions, (4) and (5), the Kuhn-Tucker necessary conditions [14] for the optimal solution of the reduced problem will be

$$-c_i f'(x_i) + \mu + \nu_i - \eta_i = 0 \qquad \forall i \tag{6}$$

$$\mu \left(r_1 - \sum_{i=1}^{n} x_i \right) = 0 \tag{7}$$

$$\nu_i (r_2 - x_i) = 0 \tag{8}$$

$$\eta_i x_i = 0 \tag{9}$$

$$\mu \leq 0, \quad \nu_i \leq 0, \quad \eta_i \leq 0 \qquad \forall i \tag{10}$$

where $f'(x_i)$ = the first derivative of $f(x_i)$ and μ, ν_i and η_i = Lagrange multipliers. Notice that since the problem is convex, the Kuhn-Tucker conditions are also sufficient for the optimality. The following Theorems (3) and (4) pave the way to solving the equations (6) to (10), efficiently.

Theorem 3. Let, x_i^*, $i = 1, \ldots, n$ be the optimal solution of (P3). Then, x_i^*s are nonincreasing, as is given by

$$0 = x_n^* = x_{n-1}^* = \ldots = x_l^* \leq x_{l-1}^* \leq \ldots \leq x_k^* = x_{k-1}^* = \ldots = x_1^* = \min(x_0, r_2) \tag{11}$$

$k = 0, 1, \ldots, n$ and $l = 1, 2, \ldots, n+1$

where x_0^* and x_{n+1}^* are dummy variables.

Proof. Let μ^*, ν_i^*, and η_i^* for all i be the solutions of equations (6), (7), ..., (10).

Case 1: $(r_2 \leq \min(x_0, r_1/n))$

In this case, every $c_i f(x_i)$ is strictly decreasing and thus $\sum_{i=1}^{n} x_i$ will be always equal to r_1 for $0 \leq x_i \leq r_2$. Hence, $\mu^* = 0$ and to minimize Z each x_i has to be increased up to r_2. Then the optimal solution will be $x_i^* = r_2$, $\nu_i^* = c_i f'(r_2)$, and $\eta_i^* = 0 \quad \forall i$. Note that this is a special case of (11) with $k = n$.

Case 2 : $(x_0 \leq r_1/n$ and $r_2)$

Since $x_0 \leq r_1/n$ and r_2, $x_i = x_0$ is feasible for all i. Thus, the optimal solution is certainly $x_i^* = x_0$ and $\mu^* = \nu_i^* = \eta_i^* = 0 \quad \forall i$, which is also a special case of (11) with $k = n$.

Case 3: $(r_1/n < x_0 < r_2)$

Since $r_1/n < x_0$ and $r_2 > x_0$, we get $x_i^* \leq x_0 \quad \forall i$ and the constraint (4) should be bounded, which implies $\mu^* < 0$. In addition, it follows from (8) and $x_i^* \leq x_0 < r_2 \quad \forall i$, $\nu_i^* = 0 \quad \forall i$. Therefore, the optimal solutions x_i^* and μ^* can be obtained by solving equations (6), (7), and (9), simultaneously, i.e.,

$$\mu^* = c_i f'(x_i^*) + \eta_i^* \qquad \forall i \tag{12}$$

$$r_1 - \sum_{i=1}^{n} x_i^* = 0 \quad (13)$$

$$\eta_i^* x_i^* = 0. \quad (14)$$

If $\eta_i^* < 0$, then from (14) $x_i^* = 0$. Similarly, if $x_i^* > 0$, $\eta_i^* = 0$. It follows that for a given μ^*, if there exists a value x_i^* such that $\mu^* = c_i f'(x_i^*)$ for $0 < x_i^* < x_0$, then $\eta_I^* = 0$ and thus, $x_i^* > 0$; Otherwise, $\mu^* < c_i f'(x_i^*)$ and therefore $\eta_i^* < 0$ which results in $x_i^* = 0$. Solving equation (12) for x_i^* gives

$$x_i^* = f' - 1((\mu^* - \eta_i^*)/c_i).$$

Here, for any $1 \le i < j \le n$, $c_i \ge c_j$ and $f'(x_i)$ is nonpositive and concave for $0 \le x_i \le x_0$, we have

$$0 \le x_i^* \le x_j^* < x_0 \quad (15)$$

for an optimal value $\mu^* < 0$ and $\eta_i^* = 0$. Finally since from (12) it is obvious that η_i^* is decreasing as i increases, the optimal soultion satisfying (13) will be

$$0 = x_n^* = x_{n-1}^* = ... = x_l^* \le x_{l-1}^* \le ... \le x_1^* \le x_0,$$
$$l = 1, 2, ..., n+1$$

which is a special case of (11) with $k = 0$.

Case 4: $(r_1/n < r_2 < x_0)$

Since $r_1/n < r_2$ and $r_2 < x_0$, in order to minimize Z the constraint (4) should be bounded. Also, μ has to be less than zero assuming $r_2 < x_0$. Now, for a given value $\mu^* < 0$, let t_1 and t_2 be the indices such that for $0 < x_i^* \le r_2$

$$c_i f'(x_i^*) < \mu^* \quad \text{if } 1 \le i \le t_1$$

$$c_i f'(x_i^*) > \mu^* \quad \text{if } t_2 \le i \le n.$$

Then, for $1 \le i \le t_1$, from (6) the following should hold

$$\nu_i^* - \eta_i^* < 0. \quad (16)$$

If $x_i^* = r_2$, then $\eta_i^* = 0$ from (9) and hence equation (16) always holds. On the other hand if $0 < x_i^* < r_2$, from (8) and (9) $\nu_i^* = \eta_i^* = 0$, which contradicts equation (6). A similar argument can be made for the case of $t_2 \le i \le n$ where the following condition needs to be satisfied

$$\nu_i^* - \eta_i^* > 0 \quad (17)$$

In this case, if $0 < x_i^* \le r_2$, then equation (17) is obviously not satisfied as derived above. However, if $x_i^* = 0$, $\nu_i^* = 0$, from (8) and thus equation (17) always true. Finally, when $c_i f'(x_i^*) = \mu^*$, $0 < x_i^* < r_2$ with $\nu_i^* = \eta_i^* = 0$. Note that case 4 is a typical one of (11) with $t_1 = k$, $t_2 = l$, and the right hand side term being r_2. This completes the proof.

It follows from Theorem 3 that the optimal solutions are of the form, either the trivial one such as $x_i^* = r_2(\text{or} x_0)$ or the general one, $0 = x_n^* = x_{n-1}^* = ... = x_l^* \le x_{l-1}^* \le ... \le x_k^* = x_{k-1}^* = ... = x_1^* = r_2$. Therefore, to find the optimal solutions for the trivial cases we need only to check the relationship between the known constants, r_1, r_2, and x_0. However, for the general case, we have to first find the optimal value of the Lagrange variable, μ^* and then the corresponding optimal solution is obtained using it as is given by

$$x_i^* = 0, \text{ if } c_i f'(x_i^*) > \mu^*,$$
$$x_i^* = r_2, \text{ if } c_i f'(x_i^*) < \mu^*$$
$$x_i^* = f'^{-1}(\mu^*/c_i), \text{ otherwise.}$$

Fig.1 show a typical example of this case.

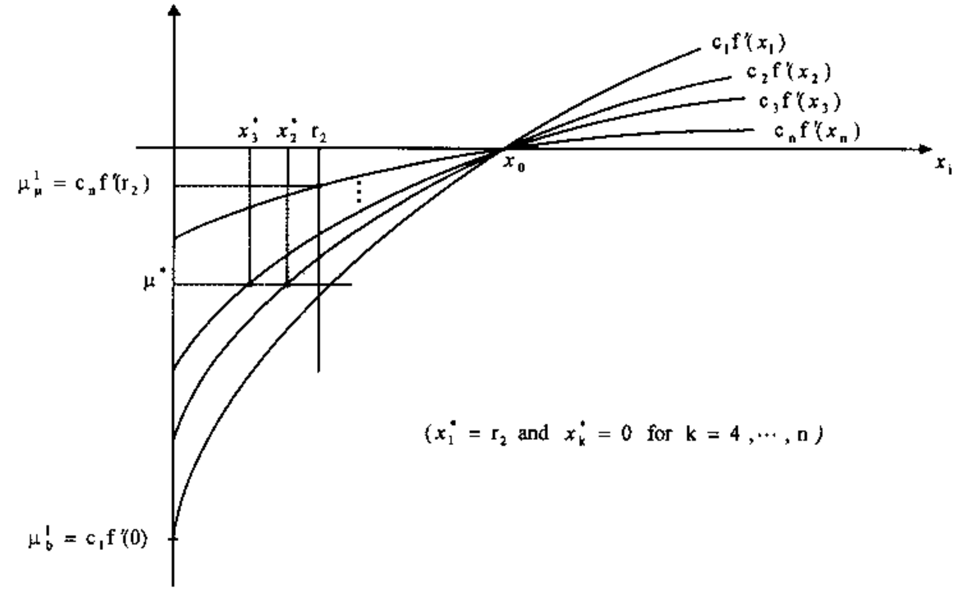

Fig. 1 A typical example of case 4.

Theorem 4. Let x_i be the one that satisfies equation (6). Then, $\sum_{i=1}^{n} x_i$ is nondecreasing over $\mu \leq 0$.

Proof. Since $f'(x_i)$ is assumed to be nonpositive and concave for $0 \leq x_i \leq x_0, x_i = f'^{-1}((\mu + v_i - \eta_i)/c_i)$ is nondecreasing as μ increases to 0 for each i,. Therefore, the sum of x_is, $\sum_{i=1}^{n} x_i$ is also nondecreasing over $\mu \leq 0$.

From the foregoing analysis, we know that determination of μ^* satisfying the constraints (6) will automatically determine every x_j^* in turn for the general case. This can be easily done by using any search procedure such as the binary section method or the Fibonacci search method[5]. Here, we propose an efficient search procedure which is mainly based on the latter one.

Step 0: ⟨Problem Type Check ⟩

If $(r_2 \leq \min(x_0, r_1/n))$, then $x_i^* = r_2 \ \forall i$ and stop.

If $(x_0 \leq r_1/n$ and $x_0 \leq r_2)$, then $x_i^* = x_0 \ \forall i$ and stop. Otherwise, proceed to step 1.

Step 1: ⟨Initialization⟩

Let F_j be the j-th Fibonnacci number which is given by

$$F_j = [((1+\sqrt{5})/2)^{j+1} - ((1-\sqrt{5})/2)^{j+1}],\ j = 0, 1, 2, \ldots$$

where [x] means the largest integer which is not greater than x. Denote μ_u^1 and μ_b^1 the initial upper and lower bounds of μ which are respectively defined as

$$\mu_b^1 = c_1 f'(0),$$

$$\mu_u^1 = c_n f'(r_2) \text{ if } r_2 \leq r_1;$$

$$= 0, \text{ otherwise.}$$

Let J be the smallest integer satisfying $F_J \geq (\mu_u^1 - \mu_b^1)/\varepsilon$ where ε is a required accuracy of the solution.

Step 2: ⟨Initial Solution ⟩

Find the following interior points

$\mu_1^1 = (F_{J-2}/F_J)(\mu_u^1 - \mu_b^1) + \mu_b^1$ and

$\mu_2^1 = (F_{J-1}/F_J)(\mu_u^1 - \mu_b^1) + \mu_b^1$.

Determine the row vector

$\gamma_p^1 = (\gamma_{p1}^1(\mu_p^1), ..., \gamma_{pn}^1(\mu_p^1)),\ \ p = 1, 2$ where

$$\gamma_{pi}^1(\mu_p^1) = \min[\max(0, f'^{-1}(\mu_p^1/c_i)), r_2],\quad i = 1, ..., n \tag{18}$$

Set the iteration index, j = 1 and go to step 4.

Step 3: ⟨Iteration j⟩

Set

$\mu_1^j = (F_{J-1-j} / F_{J+1-j})(\mu_u^j - \mu_b^j) + \mu_b^j$

and (for Case 2 and Case 3) / or (for Case 1)

$\mu_2^j = (F_{J-j} / F_{J+1-j})(\mu_u^j - \mu_b^j) + \mu_b^j$.

The definition of each case is given in Step 4. Using equation (18), find γ_1^j and/or γ_2^j with $\mu = \mu_p^j$, p=1,2, respectively.

Step 4: ⟨Evaluation⟩

Case 1) If $\sum_{i=1}^{n} \gamma_{1i}^j(\mu_1^j) < r_1$ and $\sum_{i=1}^{n} \gamma_{2i}^j(\mu_2^j) > r_1$,

then set $\mu_u^{j+1} = \mu_2^j,\ \mu_2^{j+1} = \mu_1^j,\ \mu_b^{j+1} = \mu_b^j$

or $\mu_u^{j+1} = \mu_u^j,\ \mu_1^{j+1} = \mu_2^j,\ \mu_b^{j+1} = \mu_1^j$.

Case 2) If $\sum_{i=1}^{n} \gamma_{1i}^j(\mu_1^j) > r_1$ and $\sum_{i=1}^{n} \gamma_{2i}^j(\mu_2^j) > r_1$,

then set $\mu_u^{j+1} = \mu_1^j$ and $\mu_b^{j+1} = \mu_b^j$.

Case 3) If $\sum_{i=1}^{n} \gamma_{1i}^j(\mu_1^j) < r_1$ and $\sum_{i=1}^{n} \gamma_{2i}^j(\mu_2^j) < r_1$,

then set $\mu_u^{j+1} = \mu_u^j$ and $\mu_b^{j+1} = \mu_2^j$.

Step 5: ⟨Termination Test⟩

If $\mu_u^{j+1} - \mu_b^{j+1} \le \varepsilon$, then set $\mu^* = (\mu_u^{j+1} - \mu_b^{j+1})/2$, compute the optimum $\gamma^*(\mu^*)$, and stop; Otherwise, set j = j+1 and go to step 3.

Note that the above general procedure guarantees an exact optimum since there must exist a unique μ^* when $r_1/n < x_0$ and r_2.

4. Optimal Solution of the Storage Capacity Problem

Our problem (P2) can be solved by using the above solution procedure with the following substitutions:

$c_i = w_i$,

$x_i = \gamma_i$,

$f(x_i) = (\lambda - 1 + e^{-\gamma_i})^2$,

$r_1 = -\ln(1-\alpha)$,

$r_2 = u_\gamma$,

and

$x_0 = -\ln(1-\lambda)$.

The only thing left is to determine

$x_i = f'^{-1}(\mu/c_i) \qquad \forall i$

for a given μ. Substituting the relating variables into equation (18) and rearranging it, we obtain

$e^{-2\gamma_i} + (\lambda-1)e^{-\gamma_i} + \mu/2w_i = 0.$

Solving for γ, yields

$\gamma_i = -\ln((1-\lambda \pm \sqrt{(1-\lambda)^2 - 2\mu/w_i})/2)$.

Since $\mu \le 0$ and $1-\lambda < \sqrt{(1-\lambda)^2 - 2\mu/w_i}$ for any λ, the only feasible solution is

$\gamma_i = -\ln((1-\lambda + \sqrt{(1-\lambda)^2 - 2\mu/w_i})/2)$.

Considering the constraint (5), the feasible solution for a given μ will be

$\gamma_i(\mu) = \min(\max(0, \gamma_i), u_\gamma)$.

5. Application to the EOQ Model

In this section, the storage capacity models developed thus far are applied to the AS/R system in which all items are ordered based on the standard EOQ inventory model. In this case, inventory level is uniform between ai and bi such that

$$a_i = 0 \text{ and } b_i = (2\xi d_i)^{1/2} \qquad \forall i \qquad (19)$$

Where ξ is the ratio of ordering cost to holding cost of item i, which is assumed, for simplicity, to be constant for all items.

To investigate the effects of various item demand rates on the storage capacity, we represent the demand rate of item i by the function

$$d_i = D_0 f_d(i)$$

$$= D_0 p(1-p)^{i-1}/(1-(1-p)^n) \qquad i = 1, \dots, n \qquad (20)$$

where D_0 and p are the total demand per period measured in full pallet loads and the shape parameter of the distribution, respectively. Note that $f_d(i)$ in (20) is a truncated geometric probability function which is assumed to approximate the ABC curve of the inventory items.

From (19) and (20), the maximum storage requirement of item i will be:

$$b_i = [2\xi D_0 p(1-p)^{(i-1)}/(1-(1-p)^n)]^{1/2}.$$

Note that in this case, the inventory level of item i, X_i, can be considered to follow the uniform distribution, $U(0,b_i)$ and $w_i = b_i$ for all i.

In order to examine the effects of change in item demand rate on the solution pattern and the storage capacity, example problems are solved under the following conditions:

$n = 10$, $D_0 = 100$, $\xi = 1$, $u_\beta = 0.05$, $\alpha = 0.1$,
$\lambda_1 = 1$, $\lambda_2 = 0.1, 1, 10, 50$,
$p = 0.0075, 0.0448, 0.1088$.

Table 1 lists the summary of the optimal solutions. From the Table, the following observations can be made:

1) The obtained solutions behave exactly following Theorem 3 where the general form of the optimal

Table1. The optimal values of and their corresponding storage capacities.

P	0.0075				0.0448				0.1088			
λ	10	1	0.1	0.02	10	1	0.1	0.02	10	1	0.1	0.02
β_1	0.0248	0.0188	0.0118	0.0106	0.0500	0.0500	0.0187	0.0114	0.0500	0.0500	0.0278	0.0127
β_2	0.0215	0.0169	0.0115	0.0106	0.0367	0.0367	0.0169	0.0112	0.0500	0.0405	0.0238	0.0123
β_3	0.0181	0.0151	0.0112	0.0106	0.0164	0.0164	0.0152	0.0110	0.0027	0.0125	0.0197	0.0118
β_4	0.0147	0.0132	0.0109	0.0105	0.0000	0.0000	0.0133	0.0108	0.0000	0.0000	0.0153	0.0113
β_5	0.0113	0.0114	0.0106	0.0105	0.0000	0.0000	0.0115	0.0106	0.0000	0.0000	0.0107	0.0108
β_6	0.0079	0.0095	0.0103	0.0105	0.0000	0.0000	0.0096	0.0104	0.0000	0.0000	0.0059	0.0103
β_7	0.0045	0.0076	0.0100	0.0104	0.0000	0.0000	0.0077	0.0101	0.0000	0.0000	0.0008	0.0097
β_8	0.0011	0.0058	0.0097	0.0104	0.0000	0.0000	0.0058	0.0099	0.0000	0.0000	0.0000	0.0091
β_9	0.0000	0.0039	0.0093	0.0103	0.0000	0.0000	0.0038	0.0097	0.0000	0.0000	0.0000	0.0085
β_{10}	0.0000	0.0020	0.0090	0.0103	0.0000	0.0000	0.0017	0.0095	0.0000	0.0000	0.0000	0.0078
$S(\alpha_0)$	44.248	44.248	44.249	44.249	44.123	44.124	44.142	44.154	43.563	43.564	43.593	43.652

solutions are given. For example, when p=0.1088 and λ=10, the solution certainly has the general form (case 4) with k=2 and l=4. Also, when p=0.0075 and λ=10, the solution belongs to case 3 with k=0 and l=9.

2) As the value of demand distribution parameter p increases, the shortage probability gets bigger for the highly frequent items such as item 1 and 2. Also observed is that the number of items having nonzero shortage probability becomes smaller. This implies that the higher shortage probabilities need to be selectively assigned to high turnover-frequency items when the skewness of demand curve becomes larger.
3) The effect of λ on the shortage probabilities for each item appears to be significant. Especially for the highly frequent items, the shortage probabilities dramatically increase as the value of λ gets larger, which is intuitively expected from the beginning.
4) As the skewness of the demand curve increases, the required storage capacity seems to decrease. The same observation can be made for the change in λ. However, their effect on the storage capacity is not significant

6. Conclusions

This paper considers a storage sizing problem of a unit-load AS/RS under the dedicated storage policy, the full turnover-based assignment. We first formulate the problem as a nonlinear optimization model. For the model, an iterative search method is suggested to determine the optimal storage capacity such that the total cost of storage space and space shortage is minimized while satisfying a given service level. In the model, inventory levels of storage items are treated to be statistically independent random variables each of which follows a uniform distribution. Due to the dynamic conditions and statistical dependence among items that typically exist in real situations, it is very difficult to determine exactly the storage requirements. Therefore, where possible, the distribution of the aggregate storage requirement should be developed directly from the historical data.

Nevertheless, we believe that the results obtained using the statistical approach presented here can be used in determining bounds or approximations for the first-cut design of the storage. In addition, since the previous research on storage sizing is very limited, the suggested approach could be a fundamental basis for further studies in this area.

References

[1] Bafna, K. M. (1983) Procedures given for determining AS/RS system size and preparing specs. *Industrial Engineering*, 15, August, 76-81.

[2] Hausman, W. H., Schwarz, L. B., and Graves, S. C. (1976) Optimal storage assignments in automated warehousing systems. *Management Science*, 22, 629-638.

[3] Francis, R. L., McGinnis, L. F. Jr., and White, J. A. (1992) *Facility layout and location: an analytical approach*. Prentice Hall, New Jersey, USA.

[4] Graves, S. C., Hausman, W. H., and Schwarz, L. B. (1997) Storage-retrieval interleaving in automatic warehousing systems. *Management Science*, 23, 935-945.

[5] Jacoby, S. L. S., Kowalik, J. S., and Pizzo, J. T. (1972) *Iterative methods for nonlinear optimization problems*, Prentice-Hall, New Jersey, USA.

[6] Kim, K. H. (1993) A Joint Determination of Storage Locations and Space Requirements for Correlated Items in a Miniload Automated Storage-Retrieval System., *International Journal of Production Research*, 31(11), 2649-2659.

[7] Ko, C. S. and Hwang, H. (1992) The storage space versus expected travel time of storage assignment rules in an automated warehousing system. *J. of the Korean Institute of Industrial Engineers*, 18(2), 23-29.

[8] Mullens, M. A. (1981) Use a computer to determine the size of a new warehouse, particularly in storage

and retrieval areas. *Industrial Engineering*, 13, June, 24-32.

[9] Rosenblatt, M. J. (1989) Issues in design and control of AS/RS. *Advanced Information Technologies for Industrial Material Flow Systems*, Editors: Nof, S.Y. and Moodie, C. L. Springler-Verlag, Berlin Heidelberg.

[10] Rosenblatt, M. J. and Roll, Y. (1984) Warehouse design with storage policy considerations. *International Journal of Production Research*, 22, 809-821.

[11] Rosenblatt, M. J. and Roll Y. (1988) Warehouse capacity in a stochastic enviroment. *International Journal of Production Research*, 26, 1847-1851.

[12] Yang, M. H. (1992) Optimization of layout design in an AS/RS for maximizing its throughput rate. *J. of the Korean Institute of Industrial Engineers*, 18 (2), 109-121.

[13] Zangwill, I. W. (1969) Nonlinear programming: a unified approach. Englewood Cliffs: Prentice-Hall, Inc.

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