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An Algorithm for Performance Index of Telecommunications Network

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Abstract

Performance index is a measure of telecommunications network integrating reliability and capacity simultaneously. This paper suggests a computerized algorithm evaluating a performance index for telecommunications network and compares this computerized algorithm with the algorithm[1] by experimenting on several benchmarks. A computerized algorithm proposed by this paper is superior to the algorithm[1] with respect to the computation time for most of the benchmarks.

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1. Introduction

Any telecommunications network can be modeled as a graph G = (V, E), where V is the set of nodes (or vertices) and E is the set of links (or edges). There are two measures, network reliability and network capacity to evaluate the network performance. Traditionally, these two measures are used independently while neither is a true measure of the performance of the telecommunications network. Recent studies [1,2,3,5,7] have suggested a composite performance index of a network, integrating the important measures of reliability and capacity.

In this paper a manual technique which is proposed by Rushdi[5] has been computerized and a comparison of this computerized algorithm(henceforth computerized algorithm) with the Aggarwal's algorithm[1] has been carried out by experimenting on published benchmarks. The computerized algorithm proposed in this paper outperforms the algorithm [1] with respect to the computation time for most of the benchmarks.

Section 2 introduces the assumptions and notations used in this paper. Section 3 presents a computerized algorithm to evaluate performance index of telecommunications network and section 4 provides an example to illustrate the computerized algorithm proposed in this paper. Section 5 shows the comparison of two algorithms. We conclude the paper in Section 6.

2. Assumptions

a. The telecommunications network is modeled by a graph G = (V, E) whose nodes are perfectly reliable and of unlimited capacities.

b. Each network link can have only two states, good or failed. The link failures are statistically independent.

c. Each network link is assigned specific values for its reliability and capacity. The link capacity is the upper bound on the link flow in either direction.

Notations

s, t source, terminal node

n number of links of the network

m number of minimal paths of the network

k number of minimal cutsets of the network

MC_j Minimal cutset j of the network

MP_i Minimal path i of the network

p_l reliability of link l

c_l capacity of link l

C_{max} maximum capacity of interconnection from the source node s to the terminal node t:

$$C_{max} = \min_j \{ \sum_{l \in MC_j} c_l \}$$

PI Performance Index; the mean value of the source to terminal capacity normalized by its maximum:

$$PI = E\{C_{st}\} / C_{max} \quad \text{where } C_{st} \text{ is a source to terminal network capacity}$$

3. Algorithm

A manual technique[5] using a generalized cutset procedure has been computerized as follows:

Main Algorithm

1. Find all minimal cutsets $\{MC_j \mid j = 1, 2, \cdots, k\}$ of the network[6].

2. Find all minimal paths $\{MP_i \mid i = 1, 2, \cdots, m\}$ of the network. It is better (but not necessary) if the paths are enumerated in order of their cardinality [4].

3. Define an n-dimensional vector $E_i = (e_{i1}, e_{i2}, \cdots, e_{in}),\ i = 1, \cdots, m$ corresponding to MP_i such that

$$\begin{cases} e_{il} = 1 & \text{if } l \in MP_i \\ e_{il} = 0 & \text{otherwise} \end{cases}$$

4. Define an n-dimensional vector $V_j^* = (v_{j1}^*, v_{j2}^*, \cdots, v_{jn}^*),\ j = 1, \cdots, k$ corresponding to MC_j such that

$$\begin{cases} v_{jl}^* = c_l & \text{if } l \in MC_j \\ v_{jl}^* = 0 & \text{otherwise} \end{cases}$$

5. persum= 0
6. For i=1 To m

   7. Separate each minimal path to disjoint paths (use sum of disjoint products method: see Appendix) : let d be the number of disjoint paths $P_1, P_2, \cdots, P_d$

   8. For u=1 To d

      9. For j=1 To k

         modified n+1 dimensional vector $V_j = (b_{j1}, v_{j1}, v_{j2}, \cdots, v_{jn})$ is derived from

         V_j^* as follows:

$$\begin{cases} b_{j1} = \sum_{l \in D_u} v^*_{jl} \\ v_{jl} = 0 \quad \text{if } l \in D_u \cup \overline{D}_u \end{cases}$$

$$v_{jl} = v^*_{jl} \quad \text{if } l \notin D_u \cup \overline{D}_u$$

where D_u is a set of links themselves belonging to P_u
$\overline{D}_u$ is a set of links whose complement is belong to P_u

Next j

10. persum = persum + SUB($\boldsymbol{V}$) * Probability of disjoint path P_u

where $\boldsymbol{V} = (V_1, V_2, \cdots, V_k)^t$

Next u

Next i

11. Calculate C_{max}
12. PI = persum/ C_{max}

SUB($\boldsymbol{V}$)

1. per = $\min_j\{b_{j1}\}$ and $b_{j1} = b_{j1} - \min_j\{b_{j1}\}$, $j = 1, 2, \cdots, k$
2. g=0
3. If there are V_j's such that $v_{jl} = 0$ for all $l = 1, 2, \cdots, n$ Then
take one V_{j_0} with $b_{j_0 1} = \min_j\{b_{j1}\}$ among them and
delete all V_j such that $b_{j_0 1} \le b_{j1}$ (In this case, V_j may have nonzero entries v_{jl})
and g= the number of deleted V_j (i.e. $g = |V_j|$)

End If
4. $k = k - g$
5. If $k > 1$ Then

1) If there is V_j such that $b_{j1} = 0$ and all entries v_{jl} are zero except one v_{jh} $(1 \le h \le n)$ Then

take the reliability p_h of link h and

per = per + $p_h *$ SUB($\boldsymbol{W}$)

where $W_j = (b_{j1}, v_{j1}, v_{j2}, \cdots, v_{jn}),\ j = 1, \cdots, k$

is modified from V_j as follows:

$$\begin{cases} b_{j1} = b_{j1} + v_{jh} \\ v_{jh} = 0 \\ v_{jl} = v_{jl},\ l \neq h, l = 1, 2, \cdots, n \end{cases}$$

2) Else

take the reliability p_{l_0} of any link l_0 which $v_{jl_0} \neq 0$ and

per = per + $p_{l_0} *$ SUB($\boldsymbol{W}$) + $(1 - p_{l_0}) *$ SUB($\overline{\boldsymbol{W}}$)

where $W_j = (b_{j1}, v_{j1}, v_{j2}, \cdots, v_{jn}),\ j = 1, \cdots, k$

is modified from V_j as follows:

$$\begin{cases} b_{j1} = b_{j1} + v_{jl_0} \\ v_{jl_0} = 0 \\ v_{jl} = v_{jl},\ l \neq l_0, l = 1, 2, \cdots, n \end{cases}$$

and $\overline{W_j} = (b_{j1}, v_{j1}, v_{j2}, \cdots, v_{jn}),\ j = 1, \cdots, k$

is modified from V_j as follows:

$$\begin{cases} b_{j1} = b_{j1} \\ v_{jl_0} = 0 \\ v_{jl} = v_{jl},\ l \neq l_0, l = 1, 2, \cdots, n \end{cases}$$

End If

End If

6. If $k = 1$ Then

per:= per + $b_{j_0 1}$

End If

7. SUB=per

4. Example

We now illustrate the preceding algorithm using bridge network of Fig.1, where the links are numbered as shown. Their probabilities and capacities are:

$$p_l = 0.9 \text{ for all } l, \quad c_1 = 10, \quad c_2 = 4, \quad c_3 = 5, \quad c_4 = 3, \quad c_5 = 4$$

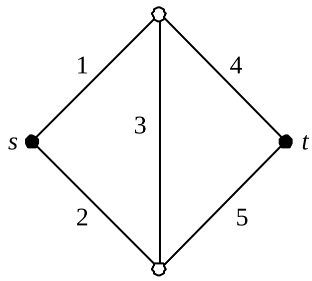

Fig.1. Bridge Network

The network of Fig. 1 has 4 minimal cutsets

$MC_1 = \{1, 2\}$, $MC_2 = \{4, 5\}$, $MC_3 = \{1, 3, 5\}$, $MC_4 = \{2, 3, 4\}$

and 4 minimal paths

$MP_1 = \{1, 4\}$, $MP_2 = \{2, 5\}$, $MP_3 = \{1, 3, 5\}$, $MP_4 = \{2, 3, 4\}$.

Define *5*-dimensional vectors

$E_1 = (1, 0, 0, 1, 0)$, $E_2 = (0, 1, 0, 0, 1)$, $E_3 = (1, 0, 1, 0, 1)$, $E_4 = (0, 1, 1, 1, 0)$ and

$V_1^* = (10, 4, 0, 0, 0)$, $V_2^* = (0, 0, 0, 3, 4)$, $V_3^* = (10, 0, 5, 0, 4)$, $V_4^* = (0, 4, 5, 3, 0)$.

Let persum = 0

For i=1

Separate minimal path $MP_1 = \{1, 4\}$ to disjoint paths:

(In case i=1, there is no predecessor minimal path to compare and skip step 7).

Write MP_1 for P_1; i.e. $P_1 = \{1, 4\}$

Then modified 5+1 dimensional vectors $\boldsymbol{V} = (V_1, V_2, V_3, V_4)^t$ are

$V_1 = (10, 0, 4, 0, 0, 0)$,

$V_2 = (3, 0, 0, 0, 0, 4)$,

$V_3 = (10, 0, 0, 5, 0, 4)$,

$V_4 = (3, 0, 4, 5, 0, 0)$

persum = $0 + \text{SUB}(\boldsymbol{V})^* \, p_1 p_4$

SUB($\boldsymbol{V}$)

per = 3 and $b_{11} = 7$, $b_{21} = 0$, $b_{31} = 7$, $b_{41} = 0$

$$V_1 = (7,\ 0,\ 4,\ 0,\ 0,\ 0),$$
$$V_2 = (0,\ 0,\ 0,\ 0,\ 0,\ 4),$$
$$V_3 = (7,\ 0,\ 0,\ 5,\ 0,\ 4),$$
$$V_4 = (0,\ 0,\ 4,\ 5,\ 0,\ 0)$$

Since there is no V_j deleted from $\boldsymbol{V}$, g =0 and k =4.

In V_2, all entries are zero except $v_{25} = 4$.

per = per + $p_5 *$ SUB($\boldsymbol{V}$) where
$$V_1 = (7,\ 0,\ 4,\ 0,\ 0,\ 0),$$
$$V_2 = (4,\ 0,\ 0,\ 0,\ 0,\ 0),$$
$$V_3 = (11,\ 0,\ 0,\ 5,\ 0,\ 0),$$
$$V_4 = (0,\ 0,\ 4,\ 5,\ 0,\ 0)$$

SUB($\boldsymbol{V}$)

per = 0 and $b_{11} = 7$, $b_{21} = 4$, $b_{31} = 11$, $b_{41} = 0$

$$V_1 = (7,\ 0,\ 4,\ 0,\ 0,\ 0),$$
$$V_2 = (4,\ 0,\ 0,\ 0,\ 0,\ 0),$$
$$V_3 = (11,\ 0,\ 0,\ 5,\ 0,\ 0),$$
$$V_4 = (0,\ 0,\ 4,\ 5,\ 0,\ 0)$$

In V_2, $v_{2l} = 0$ ($l = 1,\ 2,\ 3,\ 4,\ 5$) except $b_{21} = 4$.

Thus delete V_1, V_3 and g =2 and k =2.

Take the reliability p_2 of link 2 which $v_{42} \neq 0$

per = 0 + $p_2 *$ SUB($\boldsymbol{V}$) + $(1 - p_2) *$ SUB($\overline{\boldsymbol{V}}$)

where $\begin{cases} V_2 = (4,\ 0,\ 0,\ 0,\ 0,\ 0) \\ V_4 = (4,\ 0,\ 0,\ 5,\ 0,\ 0) \end{cases}$ and $\begin{cases} \overline{V_2} = (4,\ 0,\ 0,\ 0,\ 0,\ 0) \\ \overline{V_4} = (0,\ 0,\ 0,\ 5,\ 0,\ 0) \end{cases}$

SUB($\boldsymbol{V}$)

per =4

$$V_2 = (0,\ 0,\ 0,\ 0,\ 0,\ 0)$$
$$V_4 = (0,\ 0,\ 0,\ 5,\ 0,\ 0)$$

Now take V_2 and delete V_4 and g =1 and k =1.

per = 4 + 0 = 4

SUB($\overline{\boldsymbol{V}}$)

per =0 and g =0 and k =2.

per = 0+ $p_3 * \text{SUB}(\overline{\boldsymbol{V}})$

where $\overline{V_2}$ = (4, 0, 0, 0, 0, 0)

$\overline{V_4}$ = (5, 0, 0, 0, 0, 0)

SUB($\overline{\boldsymbol{V}}$)

per =4

$\overline{V_2}$ = (0, 0, 0, 0, 0, 0)

$\overline{V_4}$ = (1, 0, 0, 0, 0, 0)

Now take $\overline{V_2}$ and delete $\overline{V_4}$ and g =1 and k =1.

per = 4+ 0 = 4

$$
\begin{aligned}
\text{Thus per} &= 3 + p_5 * \text{SUB}(\boldsymbol{V}) \\
&= 3 + p_5 *(p_2 * \text{SUB}(\boldsymbol{V}) + (1-p_2) * \text{SUB}(\overline{\boldsymbol{V}})) \\
&= 3 + p_5 *(p_2 * 4 + (1-p_2) *(p_3 * \text{SUB}(\overline{\boldsymbol{V}}))) \\
&= 3 + p_5 *(p_2 * 4 + (1-p_2) *(p_3 * 4)) \\
&= 3 + 4p_5 (p_2 + (1-p_2) p_3)
\end{aligned}
$$

persum = 0 + (3 + $4p_5(p_2 + (1-p_2)p_3)$) $p_1 p_4$

For i=2

Separate minimal path $MP_2 = \{2, 5\}$ to disjoint paths $P_1 = \{\bar{1}, 2, 5\}$ and $P_2 = \{1, \bar{4}, 2, 5\}$.

For d=1

modified 5+1 dimensional vectors $\boldsymbol{V} = (V_1, V_2, V_3, V_4)^t$ are

$V_1 = (4, 0, 0, 0, 0, 0)$,

$V_2 = (4, 0, 0, 0, 3, 0)$,

$V_3 = (4, 0, 0, 5, 0, 0)$,

$V_4 = (4, 0, 0, 5, 3, 0)$

persum = (3 + $4p_5(p_2 + (1-p_2)p_3)$) $p_1 p_4$ + SUB($\boldsymbol{V}$)* $(1-p_1)p_2 p_5$

SUB($\boldsymbol{V}$)

per =4

V_1 = (0, 0, 0, 0, 0, 0),

V_2 = (0, 0, 0, 0, 3, 0),

V_3 = (0, 0, 0, 5, 0, 0),

V_4 = (0, 0, 0, 5, 3, 0)

Now take V_1 and delete V_2, V_3, V_4 and g =3 and k =1.

per = 4 + 0 = 4

persum = (3 + $4p_5(p_2+(1-p_2)p_3))\,p_1p_4 + 4(1-p_1)p_2p_5$

For d=2

modified 5+1 dimensional vectors $\boldsymbol{V} = (V_1, V_2, V_3, V_4)^t$ are

V_1 = (14, 0, 0, 0, 0, 0),

V_2 = (4, 0, 0, 0, 0, 0),

V_3 = (14, 0, 0, 5, 0, 0),

V_4 = (4, 0, 0, 5, 0, 0)

persum = (3 + $4p_5(p_2+(1-p_2)p_3))\,p_1p_4 + 4(1-p_1)p_2p_5$ + SUB($\boldsymbol{V}$)* $p_1(1-p_4)p_2p_5$

SUB($\boldsymbol{V}$)

per =4

V_1 = (10, 0, 0, 0, 0, 0),

V_2 = (0, 0, 0, 0, 0, 0),

V_3 = (10, 0, 0, 5, 0, 0),

V_4 = (0, 0, 0, 5, 0, 0)

Now take V_2 and delete V_1, V_3, V_4 and g =3 and k =1.

per = 4 + 0 = 4

persum = (3 + $4p_5(p_2+(1-p_2)p_3))\,p_1p_4 + 4(1-p_1)p_2p_5 + 4\,p_1(1-p_4)p_2p_5$

After the iterations i=3,4, we get

persum = (3 + $4p_5(p_2+(1-p_2)p_3))\,p_1p_4 + 4(1-p_1)p_2p_5 + 4\,p_1(1-p_4)p_2p_5$

$+\ 4\,p_1(1-p_2)p_3(1-p_4)p_5 + 3(1-p_1)p_2p_3p_4(1-p_5)$

C_{max} = min{ 10+4, 3+4, 10+5+4, 4+5+3 } = 7

PI = persum / 7 = 0.854781

5. Comparison of Algorithms

We implemented the two algorithms, computerized algorithm and Aggarwal's algorithm[1], in the visual basic language and executed them in IBM 586 PC using benchmarks as shown in Fig. 2. The performance of two algorithms are compared in terms of computation time. Table 1 shows that computerized algorithm proposed in this paper outperforms Aggarwal's algorithm[1] in most of the benchmarks, as shown in Fig.2. We do not present the input data (link reliabilities, link capacities) of the benchmarks here; Details can be obtained by the authors.

Table 1
Comparison of Computation Times (seconds) for two algorithms

Algorithm / Network	Aggarwal's Algorithm[1]	Computerized Algorithm
A	0.11	0.17
B	2.08	0.50
C	2.75	0.71
D	1.04	0.38
E	13.02	2.58
F	20.10	2.80
G	81.45	9.28
H	43.17	4.06
I	1,525.61	339.93
J	27.14	2.30
K	44,695.70	11,169.38

Computation times can be changed a bit depend on the windows environment. We have tried ten times for each benchmark and the computation times on the Table 1 are the average of them.

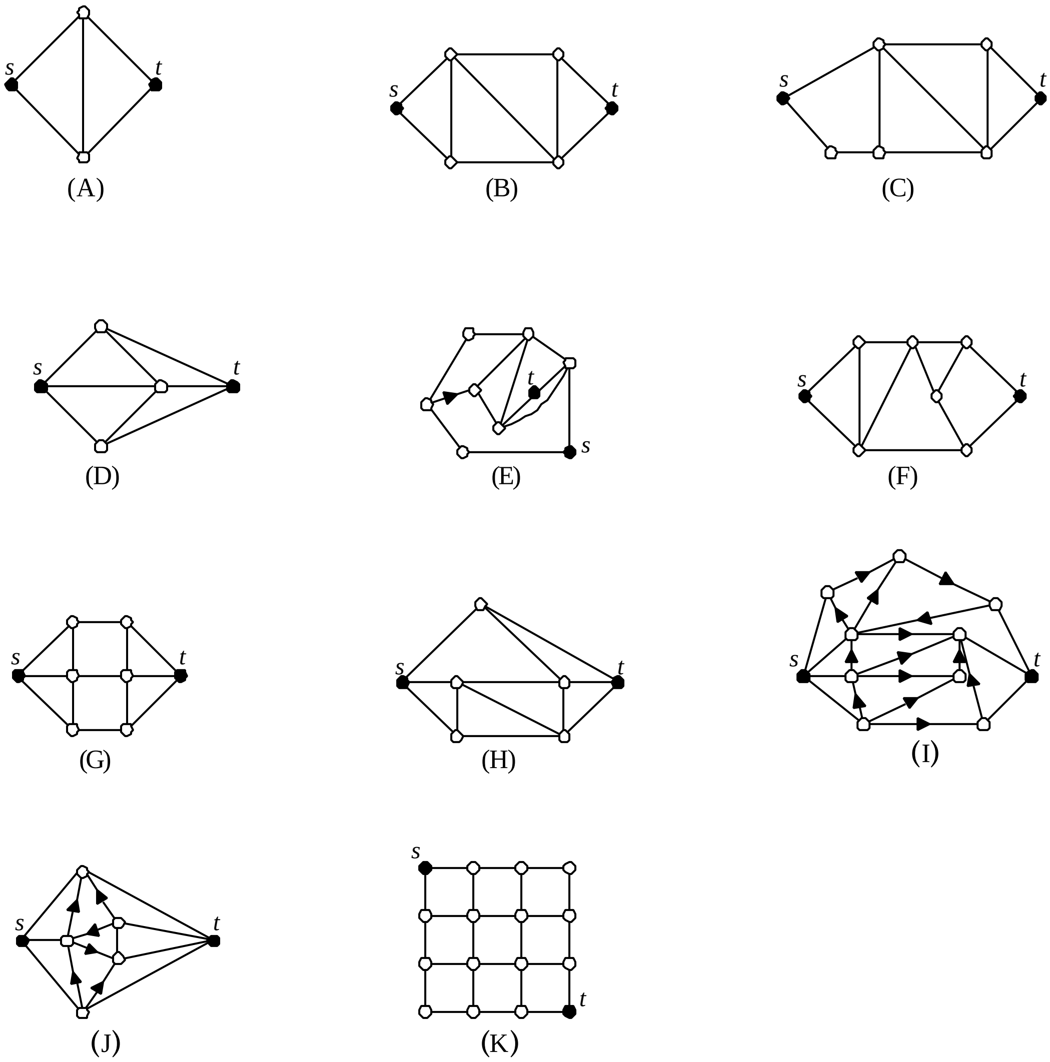

Fig.2. Benchmarks

6. Conclusion

Performance index is an important measure of telecommunications network integrating reliability and capacity simultaneously. This paper presents a computerized algorithm which evaluates a performance index for telecommunications network and compares this computerized algorithm with the Aggarwal's algorithm[1] in terms of computation time. The results of running eleven problems in Table 1 show that even though the worst-case analysis of the number of subproblems (sum of disjoint product terms) in computerized algorithm has exponential time it is more efficient than [1] in that less computation times are needed. This computerized algorithm is quite useful for evaluating a performance index of telecommunications network.

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Appendix

The procedure of main algorithm #7 (sum of disjoint products method) has been added to make the paper self-contained.

7. If there are any nonzero entries in E_f $(f = 1, \cdots, i-1)$, corresponding to zero entries in E_i

1) Form a list of complemented products (as indicated by continuous overbars) for each f = 1, …, i-1.
2) Multiply the complemented products successively from left to right, using Boolean algebra theorems for simplification after each multiplication; let the disjoint terms obtained by the above be $R_1, R_2, \cdots, R_d$.
3) Disjoint terms $R_1, R_2, \cdots, R_d$ are multiplied by MP_i and obtain the disjoint product terms $MP_i R_1, MP_i R_2, \cdots, MP_i R_d$.

write $MP_i R_u$ for P_u in convenience for each $u = 1, \cdots, d$.

Example

We will explain the above procedure with Fig. 1.

5-dimensional vectors E_i corresponding to 4 minimal paths

$MP_1 = \{1, 4\}$, $MP_2 = \{2, 5\}$, $MP_3 = \{1, 3, 5\}$, $MP_4 = \{2, 3, 4\}$ are:

$E_1 = (1, 0, 0, 1, 0)$, $E_2 = (0, 1, 0, 0, 1)$, $E_3 = (1, 0, 1, 0, 1)$, $E_4 = (0, 1, 1, 1, 0)$

In case i =1, there is no predecessor 5-dimensional vector to compare with E_1.

So we do not separate $MP_1 = \{1, 4\}$.

In case i =2, nonzero entries in E_1 corresponding to zero entries in E_2 are {1, 4}.

Form a complemented product $\overline{14}$ and $\overline{14} = \bar{1} + 1\bar{4}$ by boolean algebra.

Thus disjoint terms $\{\bar{1}\}$, $\{1, \bar{4}\}$ are multiplied by MP_2 and

obtain the disjoint product terms $P_1 = \{\bar{1}, 2, 5\}$ and $P_2 = \{1, \bar{4}, 2, 5\}$.

In case i =3, nonzero entries in E_1 corresponding to zero entries in E_3 are {4} and

nonzero entries in E_2 corresponding to zero entries in E_3 are {2}.

Multiply the complemented products $\{\bar{4}\}\ \{\bar{2}\}$ and $\{\bar{4}\}\ \{\bar{2}\}$ is multiplied by MP_3 and

obtain the disjoint product terms $P_1 = \{\bar{4}, \bar{2}, 1, 3, 5\}$.

In case i =4, nonzero entries in E_1 corresponding to zero entries in E_4 are {1} and

nonzero entries in E_2 corresponding to zero entries in E_4 are {5} and

nonzero entries in E_3 corresponding to zero entries in E_4 are {1, 5}

Multiply the complemented products $\{\bar{1}\}\{\bar{5}\}\{\overline{15}\}$ and $\{\overline{15}\}$ is deleted by boolean algebra.

$\{\bar{1}\}\ \{\bar{5}\}$ is multiplied by MP_4 and obtain the disjoint product terms $P_1 = \{\bar{1}, \bar{5}, 2, 3, 4\}$.