〈논 문〉 SAE NO. 98370121

The Application of a Direct Coupled BEM-FEM Model to Predict the TL Characteristics of Simple Expansion Silencers with Vibratory Walls

진동 벽면을 가진 단순 확장형 소음기 모델의 투과손실 특성 해석을 위한 DIRECT BEM-FEM 연성 모델의 적용

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ABSTRACT

A directly coupled Boundary Element and Finite Element Model was applied to the dynamic analysis of a coupled acoustic silencer with vibratory wall. In this coupled BEM-FEM muffler model, the BEM model was used to discretize the acoustic cavity and the FEM model was used to discretize the vibratory wall structure. Then the BEM model was coupled with the FEM model. The results of the coupled BEM-FEM model for the dynamic analysis of the simple expansion type reactive muffler configurations with flexible walls were verified by comparing the predicted results to analytical solutions. In order to investigate the effects of the muffler's structural flexibility on its transmission loss(TL) characteristics, the results of the coupled BEM-FEM model in conjunction with the four-pole parameter theory were utilized. The muffler's TL characteristics using the BEM-FEM coupled model with flexible walls as compared to other muffler configurations was studied. Finally the muffler's TL values with respect to different wall's thickness are predicted and compared.

Key Words : Boundary Element Method, Finite Element Method, Transmission Loss, Four-Pole Parameter, Plane Wave Theory

1. Introduction

Exhaust noise has long been considered as one of the major noise sources in automotive system. Even though many studies on acoustic silencers have been conducted in this area, the number published on silencer systems that include structure flexibility is very limited. In most of these studies, the silencer's performance has been studied assuming a rigid wall and the structural effects

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of end plates and the surrounding shell have been ignored.

However the side walls and the end plate of the silencer chamber element with thin outer shell, should also be considered as flexible elements so that the acoustic cavity and shell structure can interact with each other. In predicting the acoustic performance of mufflers, the coupling effect of the structure and acoustic cavity needs to be considered for more efficient analysis and design of the system.

Among the previous works on flexible mufflers, the end plate vibration effects for an expansion chamber and low frequency acoustic transmission through the walls of rectangular duct for flexible walls were studied by Cummings.[1] In addition, Young and Crocker[2] modeled the chamber-plate vibratory system using a two dimensional FEM model to investigate the effect of end plate vibrations on sound transmission in the expansion chamber. However the effect of side wall vibrations on the transmission loss characteristics in a flexible wall muffler was not investigated in this area, particularly using a 3-D directly coupled model.

Since the FEM model is ideally suited for the structural analysis and the BEM model is better suited for the acoustic cavity analysis, the use of the coupled BEM-FEM model for the analysis of an acoustic silencer with flexible walls seems very appropriate.

There are several proposed methods for a BEM-FEM coupling analysis, which are the direct method and the indirect method such as the modal method or the mobility mothod[7]

In the direct method, the equations of structural motion are directly coupled with BEM acoustic equations so that the resulting coupled equation should be solved in the form of one single system equation. In the modal method and the mobility method, the dynamic characteristics of the structures are represented by modal parameters and the pressure loading is expressed by using the mobility matrix of structures, respectively. In this study, the direct coupled method is applied to a structural acoustic problem.

In order to match the corresponding nodes for the FEM and BEM elements during the coupling process, triangular elements are utilized to create the BEM in discretizing the Helmholz integral equation for the acoustic cavity model. For the FEM, triangular flat shell elements are utilized, which are created by combining bending and plane stress elements. Then the two models are coupled at the common interfaces where they have the same number of elements and node points.

In this paper, the directly coupled BEM-FEM model in conjunction with the four pole parameter theory[3,4] is utilized to calculate the transmission loss(TL) of an expansion-chamber reactive silencer configuration with flexible walls. The results are compared with those of a rigid walled muffler model and a basic plane wave theory model.

2. Coupled BEM-FEM Formulation

Consider an acoustic cavity with vibratory flexible walls on its surface. The interior of the cavity is filled with an inviscid fluid such as air. Then the sound pressure p in the interior of the acoustic cavity must satisfy the boundary integral equation derived using Green's integral theorem and the Laplace transformed Helmholtz equation, or

$$C p(\bar{y}, s) + \int\int_s p(\bar{x}, s)\frac{\partial G}{\partial n}(\bar{x}, \bar{y}, s)dS(\bar{x}) = \int\int_s G(\bar{x}, \bar{y}, s)\frac{\partial p}{\partial n}(\bar{x}, s)dS(\bar{x}) \quad (1)$$

where s is the Laplace transform parameter, S is the surface area, the function G is the fundamental solution of the inhomogeneous Helmholtz equation, $\overline{X}$ is a field point and $\overline{Y}$ is a source point. The coefficient C for the interior problem is given as

$$C = \int\int_s \frac{\partial}{\partial n}\left(\frac{1}{4\pi r}\right) dS \tag{2}$$

where r is the distance between the source point and the field point. In order to discretize Eq. (1), triangular shell elements were utilized. Then the response of the pressure, p and its normal derivative, $\frac{\partial p}{\partial n}$ are expressed in terms of the quantities at node k as follows:

$$p = \sum_{k=1}^{3} \phi^k p^k, \quad \frac{\partial p}{\partial n} = \sum_{k=1}^{3} \phi^k \frac{\partial p^k}{\partial n} \tag{3}$$

where ϕ^k is the shape functions or interpolation functions for the BEM elements. Discretizing Eq. (1) into N elements(or NG nodes), using Eq. (3), Eq. (1) may be rewritten as

$$Cp_i + \sum_{e=1}^{N} [\hat{H}]^{(ie)} \{p\}^{(e)} = \sum_{e=1}^{N} [G]^{(je)} \left\{\frac{\partial p}{\partial n}\right\}^{(e)} \tag{4}$$

where $i = 1, 2, \cdots, NG$.

The components of Eq. (4) are given as

$$[\hat{H}]^{(ie)} = \left\{\int_0^1 \int_0^{1-\xi_1} \frac{\partial G}{\partial n} [\phi^{(1)} \phi^{(2)} \phi^{(3)}] J d\xi_1 d\xi_2\right\}^{(ie)},$$

$$[G]^{(ie)} = \left\{\int_0^1 \int_p^{1-\xi_1} \overline{G} [\phi^{(1)} \phi(2) \phi(3)] J d\xi_1 d\xi_2\right\}^{(ie)},$$

$$\{\overline{p}\}^{(e)} = \begin{Bmatrix} \overline{p}^{(1)} \\ \overline{p}^{(2)} \\ \overline{p}^{(3)} \end{Bmatrix}^{(e)}, \quad \left\{\frac{\partial \overline{p}}{\partial n}\right\}^{(e)} = \begin{Bmatrix} \frac{\partial \overline{p}}{\partial n}^{(1)} \\ \frac{\partial \overline{p}}{\partial n}^{(2)} \\ \frac{\partial \overline{p}}{\partial n}^{(3)} \end{Bmatrix}^{(e)} \tag{5}$$

In Eq. (5), J is the Jacobian of the transformation from the global coordinate system into the local coordinate system. In matrix form, Eq. (4) can be rewritten as

$$[H]\{p\} - [G]\left\{\frac{\partial p}{\partial n}\right\} = 0 \tag{6}$$

where $[H] = [\hat{H}] + C[1]$ and [1] is the identity matrix.

The Laplace transformed FEM dynamic system equation for modeling the flexible walls, assuming no damping, subjected to the fluid in the acoustic cavity can be written as

$$(s^2[M] + [K])\{\overline{a}\} + [R]\{\overline{p}\} = \{\overline{q}^f\} \tag{7}$$

where $\overline{a}$ is the transformed nodal displacement vector and $[R]$, the fluid loading matrix, or $[R] = \sum_e \int\int_{S_e} N\phi n dS$.

The external force loading on the structure is given as $\{\overline{q}^f\} = \int\int_s [N]^T \{p^f\} dS$, where $[N]$ is the FEM element shape function and p^f is the external pressure. For this FEM model, the stiffness matrix $[K]$ and mass matrix $[M]$ consist of flat shell elements created by combining plate bending elements and plane stress elements.

For the coupling of BEM and FEM, the compatibility and equilibrium conditions are satisfied at the common interface, or

$$\overline{a}^a \cdot n^a = \overline{a} \cdot n, \quad f_n = -q^a \tag{8}$$

where $\overline{a}^a$ and n^a are the Laplace transformed acoustic nodal displacement and outward normal vector for the acoustic cavity, respectively, and $\overline{a}$ and n are the Laplace transformed nodal displacement and outward normal vector for the structural component, respectively. f_n is the normal force produced by the vibrating structure at the interface and q^a is the total force related to the acoustic pressure. Using Eq. (8), the resulting BEM-FEM coupled system equation can be written as

$$\begin{bmatrix} [A_s] & [R] & 0 & 0 \\ -[\tilde{G}_N]_I & [H_u]_I & [H_u]_B & -[\tilde{G}_u]_B \end{bmatrix} \begin{Bmatrix} \{\bar{a}_u\}_I \\ \{\bar{p}_u\}_I \\ \{\bar{p}_u\}_B \\ \{\bar{a}^a_u\}_B \end{Bmatrix} = \begin{bmatrix} [N^T] & 0 & 0 \\ 0 & -[H_k] & [\tilde{G}_k] \end{bmatrix} \begin{Bmatrix} \bar{f} \\ \bar{p}_k \\ \bar{a}^a_k \end{Bmatrix} \quad (9)$$

where $[A_s]=s^2[M]+[K]$, $[G_N]_I=-\rho_0 s^2[G][N]n$, $[H_u]_I$, $[G_N]_I$ are matrices consisting of columns of $[H]$ and $[G]$ corresponding to unknown interface boundary values of $\{\bar{p}_u\}_I$ and $\{\bar{a}_n\}_I$. $[H_u]_B$, $[G_N]_B$ are matrices consisting of columns of $[H]$ and $[G]$ corresponding to unknown acoustic boundary values of $\{\bar{p}_u\}_B$ and $\{\bar{a}_n\}_B$. $\bar{p}_k$, $\bar{a}^a_k$ are known pressure and nodal displacement on the boundary, and $\bar{p}_u$, $\bar{a}^a_u$ are unknowns on the boundary.

3. Transmission Loss of a Silencer

The TL of a silencer element can be computed using a four pole parameter matrix theory.[3,4] The input pressure amplitude and particle velocity amplitude can be described by

$$\begin{Bmatrix} p_1 \\ u_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} p_2 \\ u_2 \end{Bmatrix} \quad (10)$$

where A, B, C and D are the four-pole parameters, p, u are pressure and particle velocity, respectively and index 1, 2 idicate inlet and outlet of the muffler.

At dynamic equilibrium, the four-pole parameters are given as follow;

$$A=\left.\frac{p_1}{p_2}\right|_{u_2=0},$$

$$B=\left.\frac{p_1}{u_2}\right|_{p_2=0}=-i\left.\frac{\rho_0\omega p_1}{\partial p_2/\partial n}\right|_{p_2=0},$$

$$C=\left.\frac{u_1}{p_2}\right|_{u_2=0}=-i\left.\frac{\partial p_2/\partial n}{\rho_0\omega p_2}\right|_{u_2=0},$$

$$D=\left.\frac{u_1}{u_2}\right|_{p_2=0}=-i\left.\frac{\partial p_1/\partial n}{\partial p_2/\partial n}\right|_{p_2=0}$$

where ω is the circular frequency, the subscript $u_2=0$ indicates that the velocity at the outlet tube is zero and the subscript $p_2=0$ indicates that the pressure at the outlet tube is zero. More details on the four pole parameter theory are given in the reference.[3~5]

The TL of the muffler system, with the assumptions that the inlet and outlet cross-sectional areas are the same and the outlet is non-reflecting $z_0=\rho_a c$, is given by

$$TL=20\log_{10}\left\{\frac{1}{2}\left|A+\frac{B}{z_0}+Cz_0+D\right|\right\}$$

where z_0 is the acoustic characteristic impedance at the outlet.

4. Numerical Results

As an example of applying the developed coupled BEM-FEM model, the analysis of a flat simple expansion chamber muffler model is selected as shown in Fig.1. The input excitation is that of a piston exhibiting simple

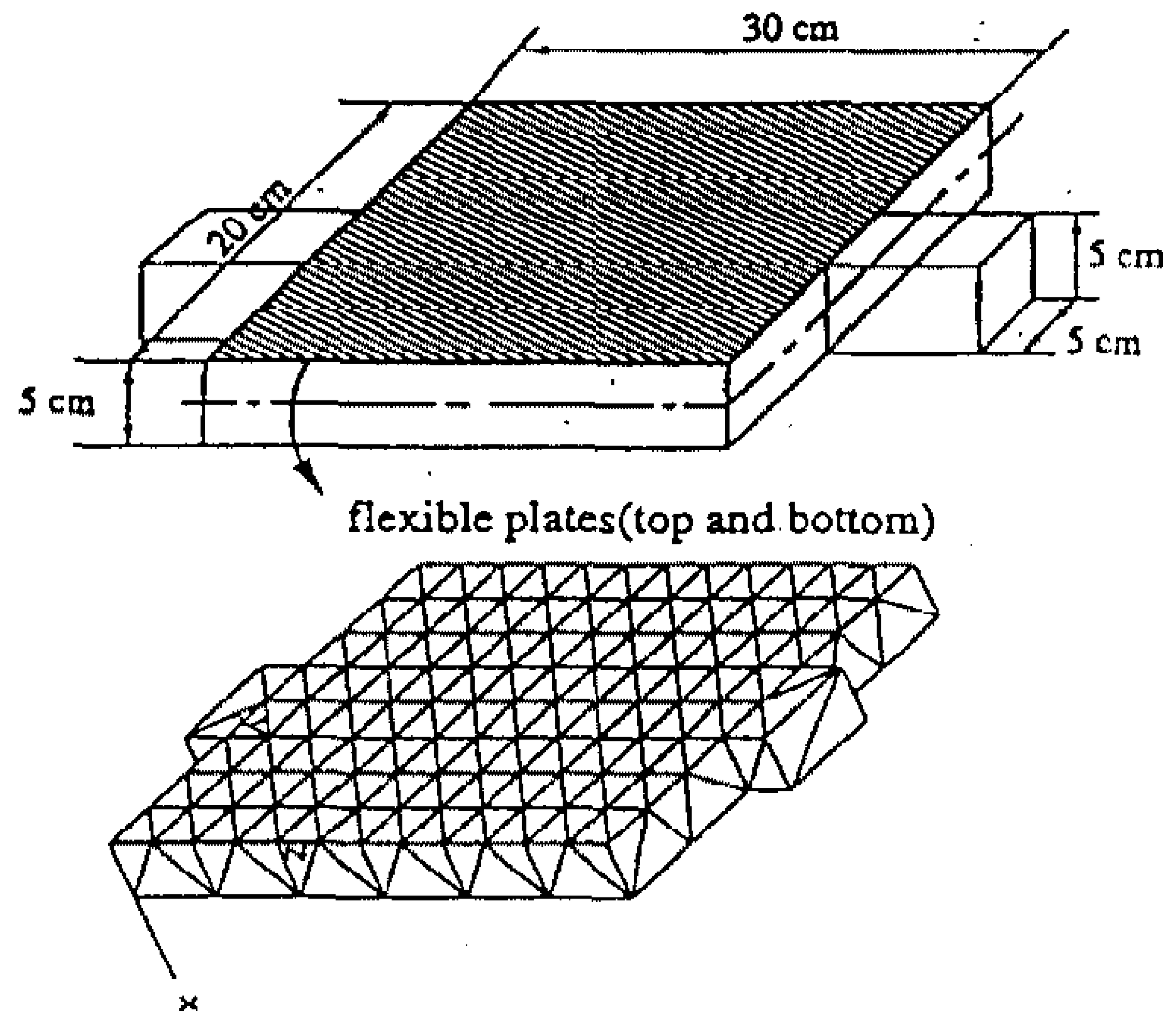


Fig.1 Schematic diagram of a flat chamber muffler model and its discretized upper half BEM-FEM mesh model

harmonic motion at the beginning of the inlet tube. The structure's internal volume is discretized using a BEM model.

As shown in Fig.1, the top and the bottom plates are considered flexible walls which are modeled using flat shell elements.

The resulting BEM-FEM models are then coupled at the interface positions. The chamber length, L=300mm and the expansion ratio, m=4 are selected for the muffler chamber. The plate structures are made of carbon steel, the thickness of the plate is selected to be 1.27mm and the FEM boundary conditions along its edges are assumed clamped.

The material properties used herein are as follow:

$$E = 2.07\times10^{11}Pa,\ \rho = 7.8\times10^{3}kg/m^{3},$$
$$\upsilon = 0.3,\ \rho_a = 1.21kg/m^{3},\ c_a = 343m/\sec$$

where E is the Young's modulus of the muffler's side plates, ρ is the mass density of the plates, υ is Poisson's ratio, ρ_a is the mass density of the air, and c_a is the speed of sound in air.

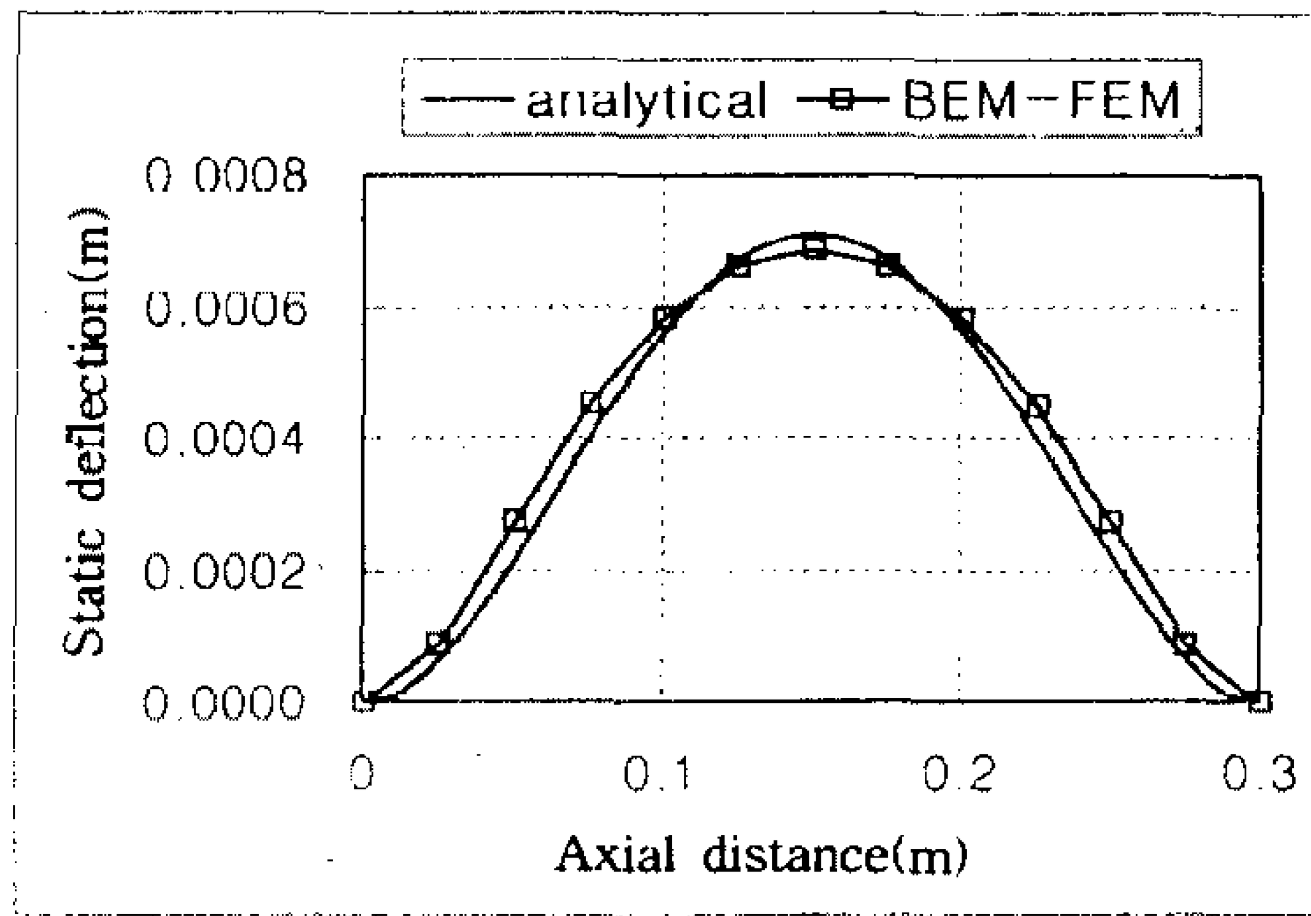


Fig.2 Comparison between the static deflections predicted by the BEM-FEM coupled method and analytical solution

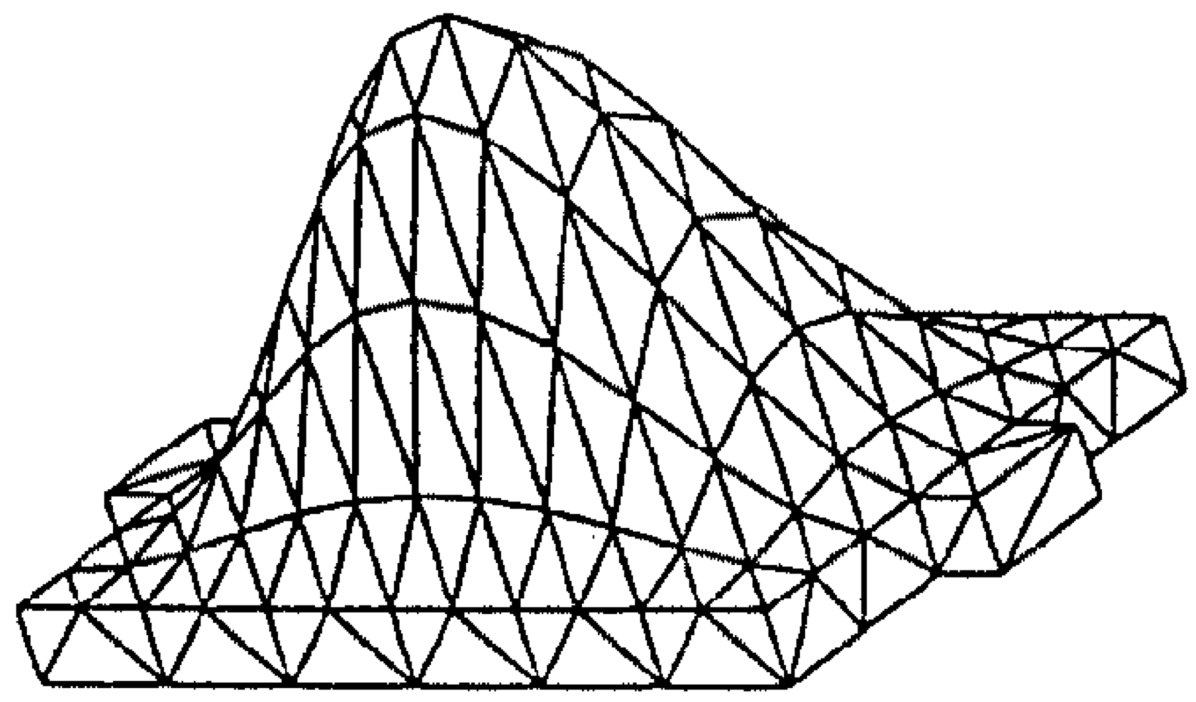
Table 1 Uncoupled natural frequencies predicted by FEM for the upper half meshed model of the flat expansion reactive muffler

Wall thickness	Uncoupled natural frequency, symmetric modes only		
	$1^{st}(1,1)$	$2^{nd}(3,1)$	$3^{rd}(1,3)$
1.27mm	209Hz	530Hz	844Hz

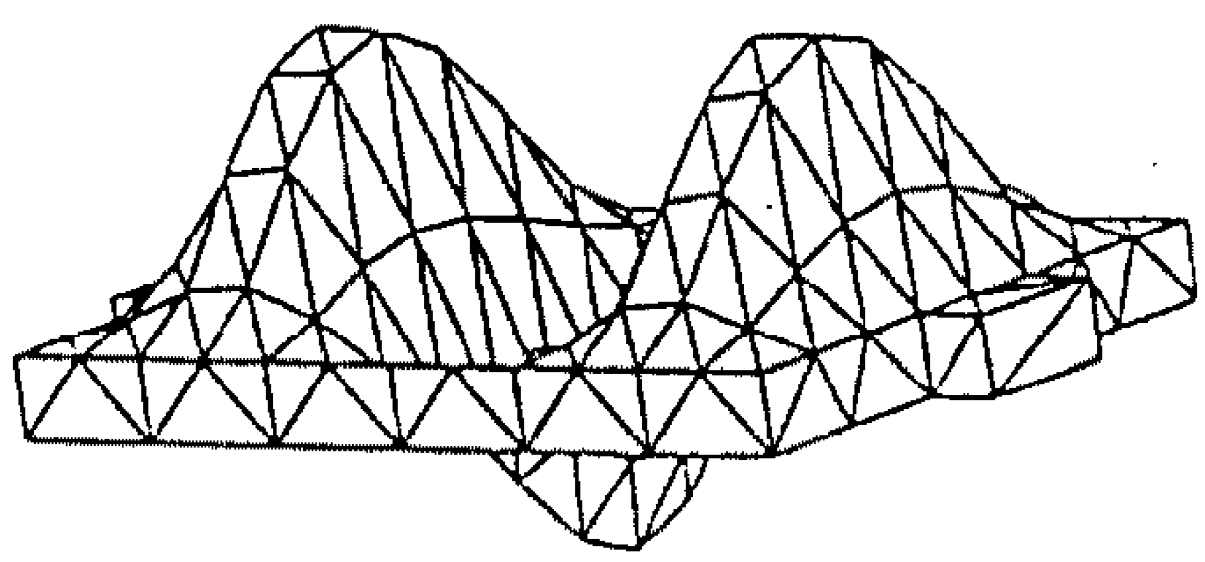
In order to check the model's ability to predict the deflection of the flexible wall as discretized by the three node flat shell elements, the structure's static deflections predicted by the BEM-FEM coupled method are compared with the analytical static deflection for an assumed constant internal pressure. As shown in Fig.2, good agreement between the analytical and BEM-FEM model's predicted static deflections is obtained.

The uncoupled eigen-frequencies predicted by the shell FEM model are presented in Table 1.

Fig.3 shows the mode shapes of the muffler's vibrating walls that corresponds to the first and second coupled structural symmetric resonant frequencies. The TL for the flat muffler with flexible walls, as predicted by the BEM-FEM coupled model and four



1st mode(1,1) at 226 Hz



2nd mode(3,1) at 532 Hz

Fig.3 Mode shapes at the 1st and 2nd coupled structural symmetric eigen-frequencies

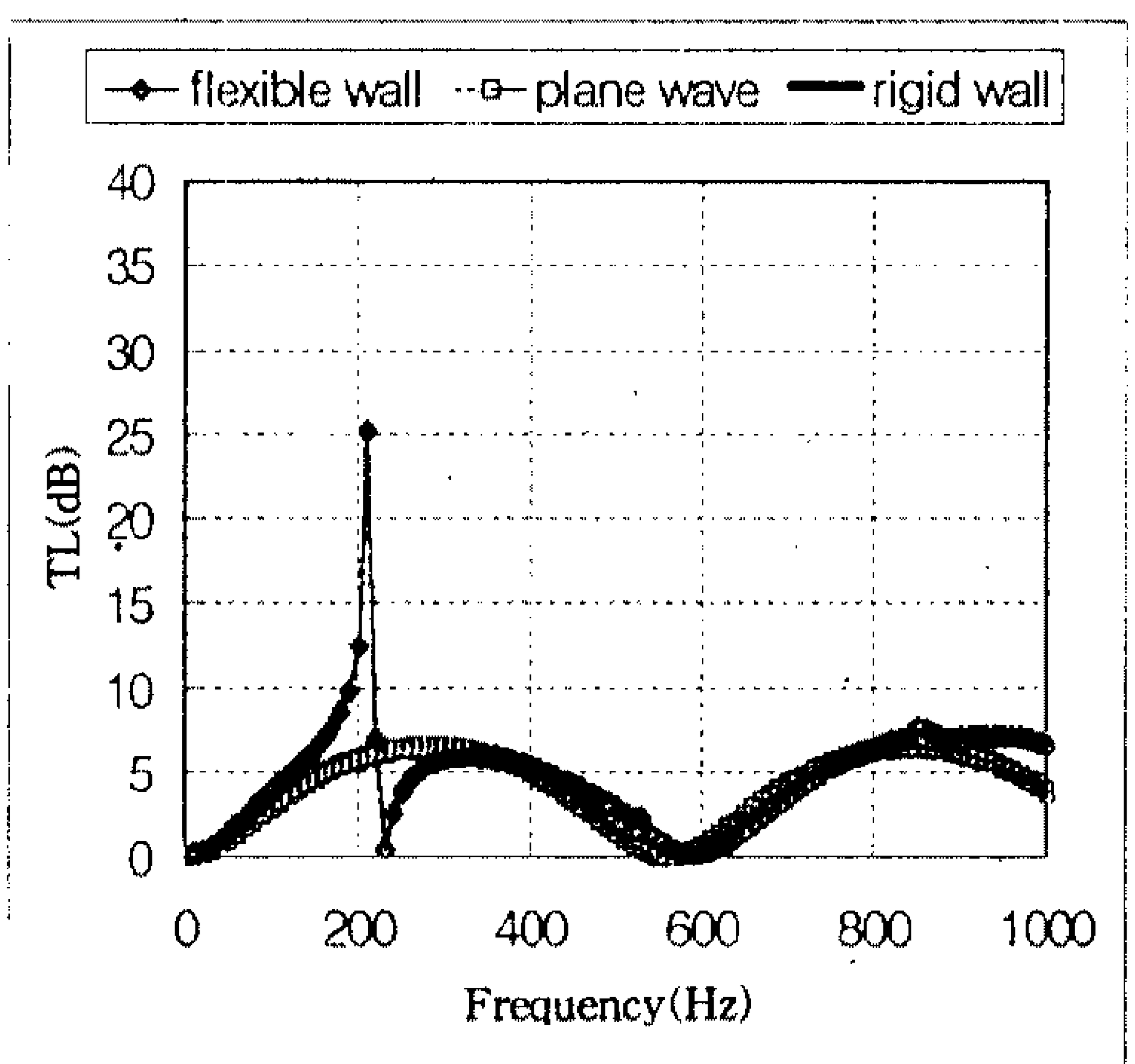


Fig.4 Comparison between the TL's for the muffler with flexible and rigid walls and the TL using the plane wave theory

pole parameter theory is shown in Fig.4

This figure presents the TL values for the three conditions of plane wave propagation, BEM model prediction (for rigid wall) and the coupled BEM-FEM model.

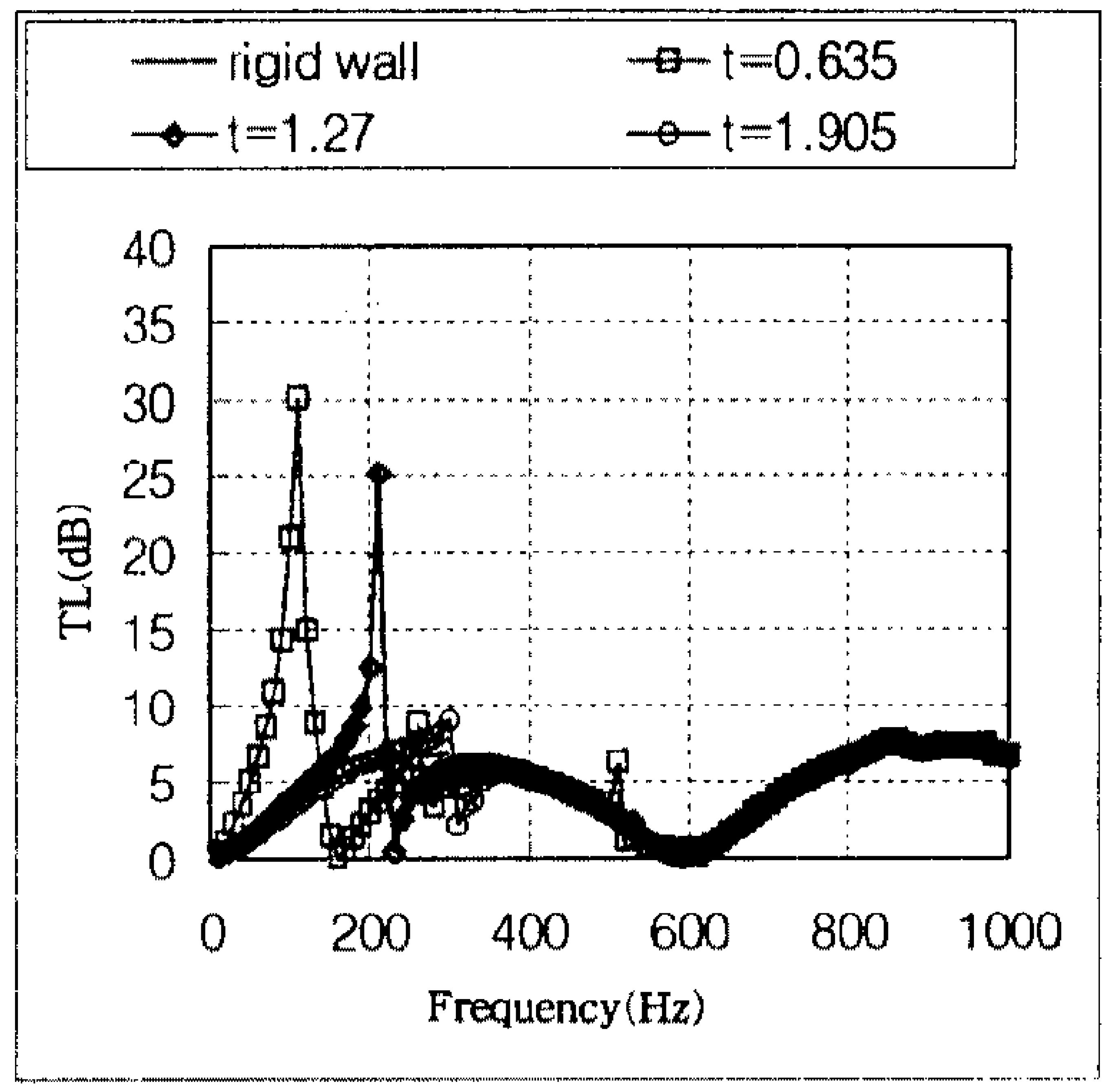


Fig.5 Comparison between the TL's for the mufflers with different wall thickness

Good agreement between the basic plane wave theory and BEM model prediction is achieved. The effects on TL by the coupled system's structural resonance corresponding to the first symmetrical mode are evident at around 226Hz. The coupled frequency has been shifted up from the uncoupled eigen-frequencies(209Hz) because the coupled structural resonant frequencies were more affected by the stiffness loading rather than mass loading of the air added to the system. The effects on TL by the second symmetrical structural resonant mode are also shown at around 532Hz in Fig.4 although they do not give a significant effect on the TL and wall's deflection. Fig.5 shows the differences in TL values with respect to different wall's thickness.

As the wall thickness of a muffler increases, the TL value tends to go to that of the rigid-walled muffler. In this analysis, no structural damping was added to the FEM structural formulations. With damping, better TL results can be predicted at the peak frequencies since a real-world structure has daming in itself.

5. Conclusions

A directly coupled BEM-FEM model has been applied to the dynamics of acoustic silencers exhibiting flexible surfaces. In this coupled BEM-FEM formulation, the BEM model was used to discretize the acoustic cavity. This BEM model was coupled to a flat shell FEM model. The formulated model was then used to predict the performance of various silencer configurations. The results of the coupled BEM-FEM model as applied to the dynamic analysis of a simple expansion type reactive muffler configuration with flexible walls has been verified by comparing to known analytical solutions.

In order to investigate the effects of the muffler's structural flexibility on its transmission loss (TL) characteristics, the results of the coupled BEM-FEM model in conjunction with the four-pole parameter theory were studied. The muffler's TL results using the BEM-FEM coupled model with flexible walls have been compared to other muffler configurations.

In this analysis, there is no structural damping added for the FEM part. If the structural damping is considered for the coupled system analysis, the results would provide a better prediction for the real-world model.

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