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A Performance Comparison between Operation Strategies for Idle Vehicles in Automated Guided Vehicle System

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Abstract

An Automated Guided Vehicle System (AGVS) with a unidirectional loop guide path is modeled as a discrete-time stationary Markov chain. It is discussed how to estimate the mean response time, the utilization, and the cycle time of AGV for a delivery order. Three common operation strategies for idle vehicles - central zone positioning rule, circulatory loop positioning rule and point of release positioning rule - are analyzed. These different operation strategies are compared with each other based on the performance measures.

1. Introduction

The Automated Guided Vehicle System (AGVS) is chosen in many industrial fields because of its flexibility, reliability, safety, and it increases the productivity. But the performance of the material handling system is significantly influenced by several operation policies. One of the important operation policies is the positioning strategy of idle vehicles on the guide path[2,5]. In this paper, the problem of idle vehicle positioning is addressed when a unidirectional loop guide path is used.

When an AGV completes a delivery task, it stands idle. One of the operational issues on AGVS that needs additional research is where to locate or position the idle vehicles in anticipation of a future pickup call.

Studies on AGV positioning methodologies have been performed by [2] and [5]. Egbelu [2] proposed three considerations when

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determining the home position of idle vehicles:

1) *Minimization of maximum vehicle response time.*
2) *Minimization of mean vehicle response time.*
3) *Even distribution of idle vehicles in the network.*

Egbelu[2] formulated a mathematical model (linear programming) based on the objective of minimizing the maximum vehicle travel time to reach the pickup location from the home location. Both bi-directional and unidirectional loops with single and multiple vehicles were studied. Kim[5] suggested a methodology to determine the home location of idle vehicles in an AGVS with a loop guide path. In his paper, the mean response time for a pickup call is minimized. Static and dynamic central zone positioning strategies were analyzed. Both the cases of bi-directional and unidirectional guide path were considered.

According to literatures on AGVS[1,2], the following rules are commonly used when positioning idle vehicles:

1) *Central zone positioning rule:* One or more staging areas on the guide path network are designated for buffering all the idle vehicles. When a vehicle becomes idle, it is routed to the staging area regardless of the location of the idle vehicle at that time.
2) *Circulatory loop positioning rule:* One or more loops on the guide path network are designated as circulatory positioning loops where idle vehicles continue to circulate until they are reassigned to new tasks.
3) *Point of release positioning rule:* When a vehicle becomes idle, it remains at the point of release until it is reassigned to a new task.

AGVS with a loop guide path layout is studied by several researchers[7,8]. The single loop AGVS is frequently utilized in practice because of its operational simplicity. Tanchoco *et al.*[8] suggested a guide path layout configuration of a single closed loop and presented a procedure of designing the optimal single loop guide path for a given facility layout. Sinriech *et al.*[7] considered the problem of determining the pick-up and delivery stations in the design of single-loop AGV systems.

Several researchers viewed the movement of AGV as a stochastic process[3,6]. Hodgson *et al.*[3] modeled an AGVS using Markov Decision Processes to obtain generalized rules for scheduling. Malmborg[6] applied an imbedded Markov chain to describe the dynamics of each control zone within an AGVS. Estimates of vehicle-blocking times are obtained directly from the probability distribution of the number of vehicles which are moving on and waiting to enter each control zone k.

Co and Tanchoco[1] classified the three states of AGV as "loaded", "assigned but empty", and "unassigned and empty". In this paper, the assigned but empty travel time is designated as the response time which corresponds to the travel time of the AGV from the starting position, where it receives a delivery order from the central controller, to the load pickup point that requests a delivery.

In most actual applications of manufacturing systems, the average service time of AGVS is the more popular criteria to evaluate operation strategies of AGVS than the maximum service time that is often validated in the cases of emergency services[5]. And since the service time consists of the assigned but empty travel(response) time, the loaded travel time and the transfer time and the latter two items are fixed, the expected response time is a reasonable criterion for determining the home location of vehicles. In this paper, the three positioning rules of idle vehicles are compared with each other based on the performance measures which are functions of the mean response time.

In the next section, AGVS is formulated as a Markov process for the three positioning rules. In section 3, a numerical example is provided in order to compare the performance measures. In the final section, concluding remarks are suggested.

2. Mathematical modeling

In order to formulate a mathematical model, the followings are assumed:

① The guide path of AGVS constitutes a loop. A loop layout is a guide path network that is made of a single circuit and around which all the workstations served by the vehicles are located.

② A single vehicle is used.

③ The inter-arrival time between the delivery orders follows the exponential distribution.

④ Unidirectional guide path is assumed.

⑤ The number of waiting delivery orders is limited. The reason why the number of waiting orders is restricted is to reduce the number of states in the stochastic analysis. And since the probability of the state with a large number of waiting orders becomes very low in practice, this approximation makes little difference in the numerical results.

In order to formulate the response time for the vehicle to arrive at the delivery-requesting station after issuing the order, the following notations will be introduced:

n = the number of nodes (pickup/delivery station) on the guide path and the station is indexed counter-clockwise from reference node 1 to last node n (node $(h\text{-}1)$ represents the previous node to node h, that is, if $h=1$, then node $(h\text{-}1)$ corresponds to node n),

T = a complete turn-around time of the vehicle on the loop guide path,

f_{ij} = the travel frequency of the vehicle from node i to node j (expressed in a relative value),

t_{ij} = the travel time of the vehicle from node i to node j,

p_i = the probability that an arbitrary delivery order is issued by station i,

e_i = the probability that the destination of an arbitrary order is i,

λ = the arrival rate of the delivery orders,

d = the load transfer time at pickup & delivery station,

$F(k,t)$ = the probability that k orders arrive during the time interval $(0,t)$,

$F(k,t_1,t_2)$ = the probability that no order arrives during $(0,t_1)$ and k orders arrive during the time interval (t_1,t_2),

r_{ij} = the probability that an arbitrary order whose delivery location is station j is issued by a station on the way from station i to station j (including station i),

$p(i,j)$ = the transition probability from state i to state j,

π_j = the state probability of state j,

q_j = the probability that an assigned but empty travel starts at station j.

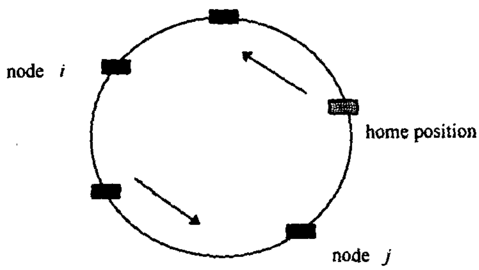
In order to define the Markov chain, an event to describe the change of the state of AGVS has to be selected. The selected event is the beginning of the response to a specific delivery order.

A cycle of the delivery task is defined as the time interval from the beginning time of response to a specific order to the one for the next order. And the initial state of the k-th cycle is defined by a vector with two elements, $X_k=(x,y)$, where x is the number of orders waiting excluding the k-th order, and y the station number on which the vehicle responds to the k-th order. Then, it can be shown that the stochastic process $\{X_k\}$, $k=1,2,\ldots$, satisfies the Markov property and that it is a discrete-time stationary Markov chain [4].

Note that $x \in \{0,1,2,\ldots\}$, and $y \in \{1,2,\ldots,n\}$. In the following, the transition matrix is defined. Let $p(i,j)$ be the (i,j) element of the transition matrix and (x_i, y_i) and (x_j, y_j) be the elements of the i-th and j-th state, respectively.

The strategy of central zone positioning

Central positioning zone is one or more staging areas on the guide path network which are designated for buffering of all the idle vehicles. When a vehicle becomes idle, it



[Figure 1] Loop guide path for AGVS

is routed to the staging area regardless of the location of the idle vehicle at that time.

A vehicle responds to a delivery order in three ways depending on the status of the vehicle when the order arrives. (1) In case one or more orders are idle, after the vehicle completes an order, the vehicle begins to carry out the next order immediately. (2) In case there are no waiting delivery orders, the vehicle proceeds to the positioning location. The vehicle can get a delivery call on the way to the positioning location. (3) Otherwise, it will have a call after it arrives at the positioning location. Each of the above three cases is defined as a possible type of response.

This paper considers only the case when one parking area is provided. A typical loop layout is represented in Figure 1. And, let B_i be the set of stations whose locations are on the way from the parking location to station i.

In defining the discrete-time stationary Markov process, the types of state transitions may be classified into 7 cases. The state transition probability from state (x_i,y_i) to state (x_j,y_j), $p(i,j)$, in each case can be estimated as followings:

1) the case that $x_i=0$, $y_i=g$, and $x_j=0$, $y_j=h$ (where h is not the parking area):

$$p(i,j)=e_h r_{gh} F(1,2d+t_{gh})+e_h(1-r_{gh})F(1,2d+T+t_{gh})+\sum_{k\in B_h}\{e_k r_{gk} F(1,2d+t_{gk}+t_{k(h-1)},2d+t_{gk}+t_{kh})+e_k(1-r_{gk})F(1,2d+T+t_{gk}+t_{k(h-1)},\ 2d+T+t_{gk}+t_{kh})\}. \quad \text{.........(1)}$$

There are two cases that (station h becomes the location where the vehicle is activated for the next order (station h becomes the starting location of a cycle): the first case is when the previous order is completed at station h and the next order is ready to be served. The second case occurs when the previous order is completed at a station in B_h and an order is issued during the vehicle travels on the path segment between stations $(h-1)$ and h. Both in the above and following formulations, station $(h-1)$ is designated as the adjacent station preceding station h and the counter-clockwise travel on the loop is assumed. The first two terms in the above expression correspond to the former case while the last two terms are related to the latter case.

2) the case that $x_i=0$, $y_i=g$, and $x_j=v(>0)$, $y_j=h$ (where h is not the parking area):

$$p(i,j)=e_h r_{gh} F(v+1,2d+t_{gh})+e_h(1-r_{gh})F(v+1,2d+T+t_{gh})+\sum_{k\in B_h}\{e_k r_{gk} F(v+1,2d+t_{gk}+t_{k(h-1)},2d+t_{gk}+t_{kh})+e_k(1-r_{gk})F(v+1,2d+T+t_{gk}+t_{k(h-1)},\ 2d+T+t_{gk}+t_{kh})\}. \quad \text{.........(2)}$$

The difference of this case from the case 1) is that the number of orders issued during the corresponding interval is $(v+1)$ which is greater than 1.

3) the case that $x_i=u(>0)$, $y_i=g$, and $x_j=v(\geq u)$, $y_j=h$:

$$p(i,j)=e_h r_{gh} F(v-u+1,2d+t_{gh})+e_h(1-r_{gh})F(v-u+1,2d+T+t_{gh}). \quad \text{.........(3)}$$

In this case, one or more orders are waiting to be served. Thus, the vehicle receives the next order at station h only when it completes the previous command at station h.

4) the case that $x_i=u(>1)$, $y_i=g$, and $x_j=u-1(\geq 0)$, $y_j=h$:

$$p(i,j)=e_h r_{gh} F(0,2d+t_{gh})+e_h(1-r_{gh})F(0,2d+T+t_{gh}). \quad (4)$$

Since there are one or more orders waiting, the vehicle receives the next order immediately when it completes the previous order at station h. And no order is issued during the cycle.

5) the case that $x_i=u(>0)$, $y_i=g$, and $x_j=v(<u-1)$, $y_j=h$:

$$p(i,j)=0. \quad (5)$$

Note that the number of waiting orders can not be reduced by more than one.

6) the case that $x_i=0$, $y_i=g$, and $x_j=0$, $y_j=h$(where station h is the parking area):

$$p(i,j)=e_h r_{gh} F(1,2d+t_{gh})+e_h(1-r_{gh})F(1,2d+T+t_{gh})+\sum_{k=1(\neq g)}^{n}\{e_k r_{gk} F(0,2d+t_{gk}+t_{k(h-1)}) \times [F(0,2d+t_{gk}+t_{k(h-1)},2d+t_{gk}+t_{kh}) + F(1,2d+t_{gk}+t_{k(h-1)},2d+t_{gk}+t_{kh})] + e_k(1-r_{gk})F(0,2d+T+t_{gk}+t_{k(h-1)}) \times [F(0,2d+T+t_{gk}+t_{k(h-1)},2d+T+t_{gk}+t_{kh}) + F(1,2d+T+t_{gk}+t_{k(h-1)},2d+T+t_{gk}+t_{kh})]\}. \quad (6)$$

When station h is the home position, the vehicle is activated at the home position in either of the following two cases: (1) when the home position is the destination of the previous call and (2) when the idle vehicle waits at the home position because no waiting order exists. In order that $x_j=0$ for the latter case, no order should be issued until the vehicle passes station $(h-1)$ and at most one order is issued before it arrives at the home position.

7) the case that $x_i=0$, $y_i=g$, and $x_j=v(>0)$, $y_j=h$(where station h is the parking area):

$$p(i,j)=e_h r_{gh} F(v+1,2d+t_{gh})+e_h(1-r_{gh})F(v+1,2d+T+t_{gh})+\sum_{k=1(\neq g)}^{n}\{e_k r_{gk} F(0,2d+t_{gk}+t_{k(h-1)}) \times F(v+1,2d+t_{gk}+t_{k(h-1)},2d+t_{gk}+t_{kh})+e_k(1-r_{gk}) F(0,2d+T+t_{gk}+t_{k(h-1)}) \times F(v+1,2d+T+t_{gk}+t_{k(h-1)},2d+T+t_{gk}+t_{kh})\}. \quad (7)$$

The difference of this case from the previous case is that the number of orders issued is $(v+1)$ instead of one.

All the enumerated cases are summarized in Table 1.

<Table 1> Summary of state transition cases under central zone positioning

x_i	y_i	x_j	y_j	case
0	g	0	h(no home position)	1)
0	g	$v(>0)$	h(no home position)	2)
$u(>0)$	g	$v(\geq u)$	h	3)
$u(>1)$	g	$u-1(\geq 0)$	h	4)
$u(>0)$	g	$v(<u-1)$	h	5)
0	g	0	h(home position)	6)
0	g	$v(>0)$	h(home position)	7)

By the assumption that the inter-arrival time between delivery orders follows the exponential distribution, $F(k,t)$ and $F(k,t_1,t_2)$ are evaluated as follows:

$$F(k,t) = \frac{e^{-\lambda t}(\lambda t)^k}{k!},\ k=0,1,2..... \quad ....(8)$$

$F(k,t_1,t_2)$ implies the probability that no order arrives before t_1 and k orders arrive between t_1 and t_2.

Thus,

$$F(k,t) = e^{-\lambda t_1}\frac{e^{-\lambda(t_2-t_1)}[\lambda(t_2-t_1)]^k}{k!}$$

$$= \frac{e^{-\lambda t_2}[\lambda(t_2-t_1)]^k}{k!},\ k=0,1,2,...$$

....(9)

In order to construct the transition probability matrix [p(i,j)], (x_i,y_i) is sequenced so that (x_i,y_i) proceeds (x_j,y_j) if $x_i < x_j$ or if $x_i = x_j$ and $y_i < y_j$. And, assign the natural number (1,2,......) to (x_i,y_i) in turn which represents the state number.

Thus, the transition probability matrix results in these forms:

	1 (0,1)	2 (0,2)	3 (0,3).............	j (x_j,y_j) ...
1=(0,1)	$p(1,1)$	$p(1,2)$	$p(1,3)$..............	$p(1,j)$ ...
2=(0,2)	$p(2,1)$	$p(2,2)$	$p(2,3)$..............	$p(2,j)$ ...
		.		
$i=(x_i,y_i)$	$p(i,1)$	$p(i,2)$	$p(i,3)$..............	$p(i,j)$ ...
.		.		
.		.		

[Figure 2] The transition probability

From the above transition probability, the state probability can be obtained by solving following simultaneous equations[3]:

$$\sum_i \pi_i p(i,j) = \pi_j,\ j=1,2,3,...... \quad ....(10)$$

$$\sum_j \pi_j = 1. \quad ....(11)$$

Let q_j be the probability that an assigned but empty travel starts at station j. Then, it may be calculated as follows:

$q_j = \sum_u \pi_u$, where the index, u, can take all the values which corresponds to (x_u,y_u) such that $y_u = j$.

The expected response (assigned but empty travel) time, ***Eresp***, may be evaluated by the following formula:

$$E_{resp} = \sum_i\sum_j q_i p_j t_{ij}. \quad ....(12)$$

If the expected working cycle time(E_{cycle}) is defined as the loaded travel time, the transfer time, and the assigned but empty travel time to carry out one delivery command, it may be estimated as

$$\{\sum_i\sum_j f_{ij}t_{ij} / \sum_i\sum_j f_{ij}\} + 2d + E_{resp}. \quad ....(13)$$

And the utilization(total working time to complete orders arriving per unit time, T_{util}) of the vehicle may be expressed as

$$\sum_i\sum_j \{\lambda f_{ij} / \sum_i\sum_j f_{ij}\}(t_{ij} + 2d + E_{resp}). \quad .....(14)$$

Based on the suggested formula, the expected response time for several alternative locations of the parking area can be evaluated and one location which has the least response time can be selected.

In the next section, the procedure using a numerical example is illustrated.

The strategy of point of release positioning

In this case, if the vehicle completes a task and no task is assigned, then it remains at the point of release until it is reassigned to a new task. Then, under the assumption that delivery orders are independent from each other, the state is the station number where the vehicle begins a new cycle. Thus, it is defined by the station number alone and the transition probability $p(i,j)$ is e_j for all i. And $\pi_j = e_j$ is the solution of (10) and (11), and $q_j = e_j$.

The strategy of circulatory loop positioning

The transition probability can be defined in the same way as the central zone positioning. $p(i,j)$ for each case can be estimated as follows:

1) the case that x_i=0, y_i=g, and x_j=0, y_j=h:

$$p(i,j)=e_h r_{gh} F(1,2d+t_{gh})+e_h(1-r_{gh}) F(1,2d+T+t_{gh})+\sum_{k=1}^{n}\sum_{s=0}^{\infty}\{e_h r_{gh} F(1,2d+sT+t_{gk}+t_{k(h-1)},2d+sT+t_{gk}+t_{kh})+ek(1-r_{gk})F(1,2d+(s+1)T+t_{gk}+t_{k(h-1)},\ 2d+(s+1)T+t_{gk}+t_{kh})\}. \quad \text{(15)}$$

The difference of above formula from the one of the same case in the central zone positioning strategy is the last two terms. Under the strategy of circulating loop positioning, no home position is designated and so the probabilities, each of which corresponds to every cyclic travel of the vehicle on the loop until it receives an order, have been added.

2) the case that x_i=0, y_i=g, and x_j=v(>0), y_j=h:

$$p(i,j)=e_h r_{gh} F(v+1,2d+t_{gh})+e_h(1-r_{gh})F(v+1,2d+T+t_{gh})+\sum_{k=1}^{n}\sum_{s=0}^{\infty}\{e_k r_{gk} F(v+1,2d+sT+t_{gk}+t_{k(h-1)},2d+sT+t_{gk}+t_{kh})+e_k(1-r_{gk})F(v+1,2d+(s+1)T+t_{gk}+t_{k(h-1)},\ 2d+(s+1)T+t_{gk}+t_{kh})\}. \quad \text{(16)}$$

The difference of this case from case 1) is that the number of orders issued during the corresponding interval is (v+1) which is greater than 1.

3) the case that x_i=u(>0), y_i=g, and x_j=v(≥u), y_j=h:

$$p(i,j)=e_h r_{gh} F(v-u+1,2d+t_{gh})+e_h(1-r_{gh})F(v-u+1,2d+T+t_{gh}). \quad \text{(17)}$$

This case is the same as the case 3) in the central zone positioning strategy.

4) the case that x_i=u(>1), y_i=g, and x_j=u-1(≥0), y_j=h:

$$p(i,j)=e_h r_{gh} F(0,2d+t_{gh})\ e_h(1-r_{gh})F(0,2d+T+t_{gh}). \quad \text{(18)}$$

This case is the same as the case 4) in the central zone positioning strategy.

5) the case that x_i=u(>0), y_i=g, and x_j=v(<u-1), y_j=h:

$$p(i,j)=0. \quad \text{(19)}$$

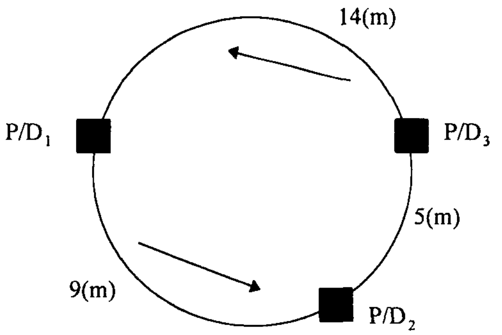
The state probabilities and the expected response time under this strategy can be estimated by the same formulas as those in central zone positioning strategy.

3. Numerical example

central zone positioning strategy

In this section, it is illustrated how to locate an optimal central parking space in which every vehicle is directed when it is idle.

Consider a loop layout as in Figure 3. It is assumed that both the pickup and delivery tasks may be performed at every station and the guide path direction is counter-clockwise. To reduce the number of states, it is assumed that no more than two orders stand idle in the system. It is implied that any additional delivery order is rejected and excluded from further consideration.



[Figure 3] Loop layout of the example problem

Then, the transition probability may be modified as follows:

$\tilde{p}((x_i,y_i),(2,y_j)) = \sum_{x_j=2}^{\infty}(x_i,y_i),(x_j,y_j))$ for all y_i and y_j, x_i=0,1,2,

$\tilde{p}((x_i,y_i),(x_j,y_j)) = p((x_i,y_i),(x_j,y_j))$ for x_i=0 or 1 and x_j=0 or 1,

$\tilde{p}((x_i,y_i),(x_j,y_j)) = 0$ for $x_i > 2$ or $x_j > 2$.

Table 2 shows Form/To flow requirements between workstations. Let p_i denote the probability that an arbitrary delivery order is issued by a specific station i. It may be estimated by the column-wise summation of elements in the corresponding row divided by the total sum of elements in From/To flow quantity matrix demonstrated in the last column of Table 2. The probability(e_i) that the destination of an arbitrary order is i may be estimated by the row-wise summation of elements in the corresponding column divided by the total sum of elements in From/To flow quantity matrix as in the last row of Table2. Figure 3 show the configuration of the guide path and a number on the arc represents the distance between workstations.

The following data are used in the example:

- order arrival rate : λ=0.002 ~ 2.0/sec
- pickup & delivery time : $2\times d$=10 sec
- vehicle speed=1.2m/sec

〈Table 2〉 From/To flow matrix(f_{ij})

to / from	P/D1	P/D2	P/D3	total	p_i
P/D1	-	40	60	100	0.5
P/D2	40	-	20	60	0.3
P/D3	20	20	-	40	0.2
total	60	60	80	200	-
e_i	0.3	0.3	0.4	-	-

〈Table 3〉 From/To travel time matrix (t_{ij} :sec)

from \ to	1 (P/D_1)	2 (P/D_2)	3 (P/D_3)
1(P/D_1)	0	7.50	11.67
2(P/D_2)	15.83	0	4.17
3(P/D_3)	11.67	19.17	0

〈Table 4〉 r_{ij} matrix

from \ to	1 (P/D_1)	2 (P/D_2)	3 (P/D_3)
1(P/D_1)	1.0	2/3	1.0
2(P/D_2)	1.0	1.0	1/4
3(P/D_3)	1/3	1.0	1.0

By the assumption that the number of waiting orders may not exceed two, the number of states becomes nine and they may be listed as follows: state(0,1), state(0,2),..., state(2,3). From Figure 3 and vehicle speed, the travel time between workstations can be evaluated. It may be summarized as Table 3. And the r_{ij} matrix may be constructed easily as the one in Table 4.

In the following, the evaluation process of various performance measures suggested in the previous section is illustrated:

Let the home position be determined as P/D_1 and λ be 0.002. Then $B_1=\phi$, B_2={P/D_2}

The transition probability may be evaluated by (1)-(9) and the results are listed in Table 5.

In order to get the state probability π_j, the simultaneous equations (10) and (11) are solved and the solution is the corresponding elements in Table 6.

As you can see in Table 6, state probability π_1 of state (0,1) is the largest at order arrival rate λ=0.002 when the home position is located at P/D_1 . This value decreases for the same home location as order arrival rate λ increases. It is due to the first that more arriving orders become to wait at the high

〈Table 5〉 The transition probability $\tilde{p}(i,j)$ when the home position is at P/D_1 and λ =0.002

i \ j	1 (0,1)	2 (0,2)	3 (0,3)	4 (1,1)	5 (1,2)	6 (1,3)	7 (2,1)	8 (2,2)	9 (2,3)
1 (0,1)	0.64	0.01	0.02	0.10	0.10	0.13	0.00	0.00	0.00
2 (0,2)	0.65	0.01	0.03	0.10	0.09	0.13	0.00	0.00	0.00
3 (0,3)	0.67	0.02	0.01	0.09	0.09	0.12	0.00	0.00	0.00
4 (1,1)	0.29	0.29	0.38	0.01	0.01	0.02	0.00	0.00	0.00
5 (1,2)	0.28	0.29	0.38	0.01	0.01	0.02	0.00	0.00	0.00
6 (1,3)	0.28	0.28	0.39	0.02	0.02	0.01	0.00	0.00	0.00
7 (2,1)	0.00	0.00	0.00	0.29	0.29	0.38	0.01	0.01	0.02
8 (2,2)	0.00	0.00	0.00	0.28	0.29	0.38	0.02	0.01	0.02
9 (2,3)	0.00	0.00	0.00	0.28	0.28	0.39	0.01	0.02	0.01

〈Table 6〉 The state probabilities for various cases

home position	state / λ	(0,1) π_1	(0,2) π_2	(0,3) π_3	(1,1) π_4	(1,2) π_5	(1,3) π_6	(2,1) π_7	(2,2) π_8	(2,3) π_9
P/D1	0.002	0.55	0.08	0.11	0.08	0.08	0.10	0.00	0.00	0.00
	0.02	0.28	0.12	0.17	0.12	0.11	0.15	0.02	0.02	0.02
	0.2	0.00	0.00	0.00	0.01	0.02	0.03	0.27	0.28	0.38
	0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.30	0.40
P/D2	0.002	0.09	0.53	0.11	0.08	0.08	0.10	0.00	0.00	0.00
	0.02	0.15	0.24	0.16	0.12	0.11	0.15	0.02	0.02	0.02
	0.2	0.00	0.00	0.00	0.01	0.02	0.03	0.28	0.28	0.38
	0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.30	0.40
P/D3	0.002	0.10	0.10	0.49	0.09	0.09	0.12	0.00	0.00	0.00
	0.02	0.12	0.14	0.27	0.16	0.12	0.12	0.02	0.02	0.02
	0.2	0.00	0.00	0.00	0.01	0.02	0.03	0.27	0.28	0.38
	0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.30	0.40

arrival rate of orders. As π_i of state (x_i,y_i), x_i=0,1, decreases at a higher arrival rate, π_j of state $(2,y_j)$ increases. It approaches to e_j but it may not exceed e_j for each workstation.

〈Table 7〉 Results of q_i for the various arrival rates and the home positions

home position	node / λ	1 P/D_1	2 P/D_2	3 P/D_3
P/D1	0.002	0.63	0.16	0.21
	0.02	0.41	0.25	0.34
	0.2	0.29	0.29	0.42
	0.5	0.30	0.30	0.40
P/D2	0.002	0.17	0.61	0.22
	0.02	0.29	0.38	0.34
	0.2	0.29	0.29	0.42
	0.5	0.30	0.30	0.40
P/D3	0.002	0.19	0.19	0.61
	0.02	0.27	0.28	0.45
	0.2	0.29	0.29	0.42
	0.5	0.30	0.30	0.40

From the obtained state probability π_j's, q_i, the probability that an assigned empty travel starts at station i, can be evaluated. They are summarized in Table 7.

〈Table 8〉 The evaluated values of E_{resp}, E_{cycle}, and T_{util}

arrival rate(λ)	home position	E_{resp}	E_{cycle}	T_{util}
0.002	P/D_1	6.72	28.39	0.057
	P/D_2	8.66	30.32	0.061
	P/D_3	9.68	31.35	0.063
0.02	P/D_1	7.98	29.65	0.593
	P/D_2	8.51	30.18	0.604
	P/D_3	8.90	30.56	0.611
0.2	P/D_1	8.72	30.39	1.000
	P/D_2	8.72	30.39	1.000
	P/D_3	8.72	30.39	1.000

The expected response time, E_{resp}, can be evaluated by the formula (12), the expected working cycle time to perform one delivery command, E_{cycl}, by the equation (13) and the utilization of the vehicle, T_{util}, by equation

(14). The results are summarized in Table 8.

The objective is to select one location which has the smallest response time. When $\lambda \in (0.002,\ 0.05)$, P/$D_1$ is the better candidate for the home position than the other two stations. But, for a higher value of λ than 0.05, the expected response time shows no difference for three candidate home positions, which is shown in Figure 4.

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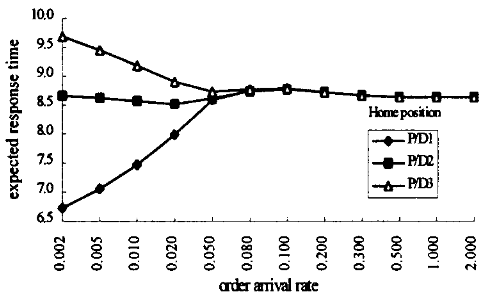
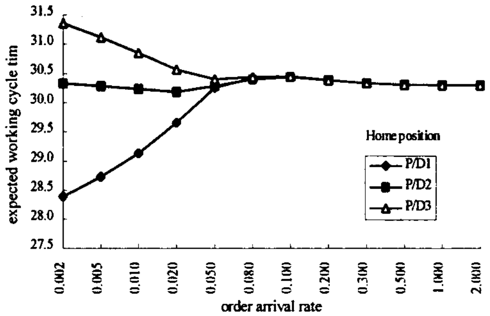


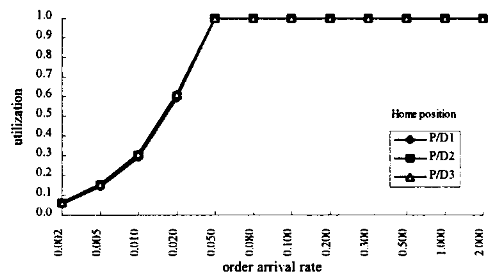
[Figure 4] Expected response time for each home location

It results from the fact that at a higher value of λ there exists less chance for AGV to visit the home position and start the next cycle there. Thus, in the case of a higher λ value, it is not important where to locate the home position.

And for the expected working cycle time, the same argument holds, which is illustrated in Figure 5. For the utilization, cases with different home positions showed no difference between each other, which is shown in Figure 6.



[Figure 5] Expected working cycle time for each home location



[Figure 6] Utilization of AGV for each home location

point of release positioning strategy

As stated in previous section, the number of states are 3 each of which corresponds to the station number. Therefore, π_j equals to e_j, $j=1,2,3$. And so, from the equation (10) and (11), $\pi_1=0.3$, $\pi_2=0.3$, $\pi_3=0.4$. From the equation (12) and (13), $E_{resp}=8.63$, $E_{cycle}=30.30$ for all values of λ and T_{util} may also be evaluated easily using equation (14).

circulatory loop positioning strategy

The transition probability, $p(i,j)$, is evaluated in Table 9 using (14)-(19). The states with a higher value of y_i than 2 are truncated for the same reason as in the case of central zone positioning. After evaluating the $p(i,j)$'s, π_j's can be obtained by the simultaneous equations (10), (11). The results of state probability π_j's obtained are summarized in Table 10. The results of E_{resp}, E_{cycle} and T_{util} are summarized in Table 11.

comparisons of the three positioning strategies

Three positioning strategies are compared with each other at various levels of order arrival rate.

〈Table 9〉 The transition probability under the strategy of circulatory loop positioning (λ =0.002)

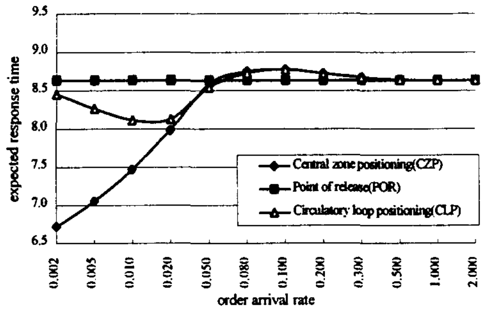
i \ j	1 (0,1)	2 (0,2)	3 (0,3)	4 (1,1)	5 (1,2)	6 (1,3)	7 (2,1)	8 (2,2)	9 (2,3)
1 (0,1)	0.32	0.31	0.37	0.00	0.00	0.00	0.00	0.00	0.00
2 (0,2)	0.32	0.30	0.38	0.00	0.00	0.00	0.00	0.00	0.00
3 (0,3)	0.33	0.31	0.36	0.00	0.00	0.00	0.00	0.00	0.00
4 (1,1)	0.29	0.29	0.38	0.01	0.14	0.17	0.00	0.00	0.00
5 (1,2)	0.28	0.29	0.38	0.01	0.01	0.02	0.00	0.00	0.00
6 (1,3)	0.28	0.28	0.39	0.02	0.02	0.01	0.00	0.00	0.00
7 (2,1)	0.00	0.00	0.00	0.29	0.29	0.38	0.01	0.01	0.02
8 (2,2)	0.00	0.00	0.00	0.28	0.29	0.38	0.02	0.01	0.02
9 (2,3)	0.00	0.00	0.00	0.28	0.28	0.39	0.02	0.02	0.01

〈Table 10〉 The state probabilities for various λ

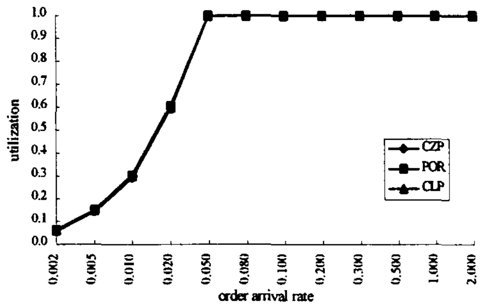
state λ	(0,1) π_1	(0,2) π_2	(0,3) π_3	(1,1) π_4	(1,2) π_5	(1,3) π_6	(2,1) π_7	(2,2) π_8	(2,3) π_9
0.002	0.32	0.30	0.38	0.00	0.00	0.00	0.00	0.00	0.00
0.02	0.30	0.26	0.26	0.05	0.04	0.05	0.01	0.01	0.01
0.2	0.00	0.00	0.00	0.01	0.02	0.03	0.27	0.28	0.38

〈Table 11〉 Results of E_{resp}, E_{cycle}, and T_{util}

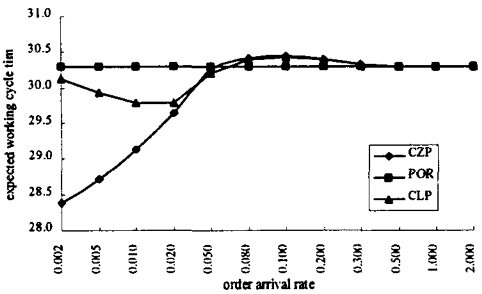
λ	E_{resp}	E_{cycle}	T_{util}
0.002	8.45	30.11	0.060
0.02	8.13	29.79	0.596
0.2	8.72	30.39	1.000



[Figure 7] Comparisons of expected response time for three positioning strategies



[Figure 9] Comparisons of utilization for three positioning strategies



[Figure 8] Comparisons of expected working cycle time for three positioning strategies

The expected response time is compared between positioning strategies in Figure 7. When order arrival rate is lower than 0.05, the central zone positioning strategy gives the best performance among all the strategies. However, as the arrival rate grows higher, the difference between strategies becomes less significant. Comparisons are made for the expected working cycle time in Figure 8, which shows a similar pattern as the case of the expected response time. There are no differences in the utilization between strategies as in Figure 9.

4. Conclusions

Three operation strategies for idle vehicle are analyzed. They are central zone positioning, staying on the point of release, and circulatory positioning around the loop guide path. Mathematical models for the expected response time, the expected working cycle time, and the utilization are formulated using a discrete-time stationary Markov chain. For the strategy of central zone positioning, the problem of optimally positioning a home location is also treated.

Based on the analytic formulation, performances of an example are evaluated for these three strategies. In the example case, the strategy of central zone positioning is shown to outperform the other two strategies.

Utilizing the results of this study, formulas for other performance measures may be derived which are very effective tools in evaluating proposed alternatives for the operation and the design of AGVS without the help of the time-consuming simulation.

REFERENCES

[1] Co, G. Christine and Tanchoco, J. M. "A., A review of research on AGVS vehicle management," *Engineering Costs and Production Economics*, Vol.21(1991), pp.35-42.

[2] Egbelu, P. J., "Positioning of automated guided vehicles in a loop layout to improve response time," *European Journal of Operational Research*, Vol.71(1993), pp.32-44.

[3] Hodgson, T. J., King, R. E., Monteith, S. K., and Schultz, S. R., "Developing control rules for an AGVS using Markov Decision Processes, *Material Flow*, Vol.4(1987), pp.85-96.

[4] Isaacson, D.L. and Madsen, R. W., *Markov chains: Theory and Applications*, John Wiley & Sons, New York, 1976

[5] Kim, K. H., "Positioning of automated guided vehicles in a loop layout to minimize the mean vehicle response time, *International Journal of Production Economics*, Vol.39(1995), pp.201-214.

[6] Malmborg, C. J., "A model for the design of zone control automated guided vehicle system, *International Journal of Production Research*, Vol.28, No.10(1990), pp.1741-1758.

[7] Sinriech, D. and Tanchoco, J. M. A., "The Centroid Projection Method for Locating Pick-Up and Delivery Stations in Single-Loop AGV Systems, *Journal of Manufacturing Systems*, Vol.11, No.4(1992), pp.297-307.

[8] Tanchoco, J. M. A. and Sinriech, D., "OSL-optimal single loop guide paths for AGVS, *International Journal of Production Research*, Vol.30, No.3(1992), pp.665-681.