

## 철근콘크리트 부재의 파괴거동에 대한 크기 효과

### Size Effect in the Fracture Behavior of Reinforced Concrete Members

김 동 백\* · 김 운 학\* · 백 신 원\*\*

Dong-Baik Kim · Woon-Hak Kim · Shin-Won Paik

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#### ABSTRACT

In this study, the size effect in flexural and shear behaviors of reinforced concrete beams with stirrup has been studied. The specimens of different size with same longitudinal reinforcement ratio are tested. The major variables of test include the size(relative depth) of the members as well as the longitudinal reinforcement ratios. The nominal resistances in flexure and shear are obtained for various sizes and steel ratios. It is found from the present study that the size effect is also very pronounced for the flexural resistance in reinforced concrete structures.

The prediction formulas for the size effect of reinforced concrete beams in flexure and shear are proposed. The proposed equations agree relatively well with experimental data. The present study will provide useful bases for more accurate analysis and design of reinforced concrete structures.

#### 국 문 요 약

본 연구에서는 철근콘크리트 보의 휨 및 전단거동에 대한 크기효과를 실험적으로 연구하고 그 결과를 다른 연구와 비교했다. 주철근비는 같고 크기가 다른 부재를 하나의 시리즈로 실험을 했으며, 주요 실험변수는 주철근비와 부재의 상대높이이다. 휨 및 전단강도의 크기효과는 주철근비에 따라 유효높이가 일정하게 변하는 부재를 실험함으로써 측정했다.

본 연구의 결과를 살펴보면 철근콘크리트 보의 휨 및 전단에 대한 크기효과는 상당히 크다는 사실을 알았다. 철근콘크리트 부재의 휨 및 전단강도에 대한 예측식을 제안했으며, 실험치와 비교적 잘 일치했다. 따라서 본 연구의 결과는 철근콘크리트 구조물의 보다 정밀한 해석 및 설계에 유용할 것으로 기대된다.

\* 안성산업대학교 토목공학과

\*\* 안성산업대학교 안전공학과

## 1. Introduction

Concrete is a brittle material, and thus its mechanical behavior is critically influenced by crack growth. Recently, the understanding of fracture mechanics in continuous material has significantly increased. The study of concrete fracture may provide the fundamental bases of concrete structures, and also the study of size effect of concrete or reinforced concrete fracture will provide the exact bases in the design of concrete structures.

It is generally recognized that crack initiation in concrete occurs in opening mode (mode I), therefore we can take many fracture parameters from mode I fracture analysis, i.e., fracture toughness  $K$ , fracture energy  $G$ , etc<sup>1)</sup>.

Recently, there some analytical and experimental models are proposed a system for mode I fracture and the size effect of reinforced concrete. These studies say that the size effect may be extremely large for large, unreinforced, prenotched concrete beams, and the size effect may be decreased rapidly, as the amount of reinforcement increases, but these are only theoretical results, i.e., these have not experimental evidences. Therefore, the main subject of this paper is the verification of analytical results, the next is the comparison of results of two studies, and the third is derivation of new factors for the size effect law of reinforced concrete by regression analysis.

Many analytical and experimental studies about concrete have been proceeded recently and many studies about reinforced concrete are proceeding now, but most studies of reinforced concrete are analytical studies. Most experimental studies are not real size structure but small specimens which can be tested easily in laboratory and most subjects of these studies are not structural behavior but

rather material properties. Especially, there are few studies about size effect of reinforced concrete because of much research expenses and huge equipment. The study about effect of mode I fracture of reinforced concrete is the base of fracture of reinforced concrete but can not be tested easily in laboratory, because the size of specimen in mode I became large in proportion to the increase of effective depth of specimen. Pure mode I fracture needs value exceeds 6 of  $a/d$ (shear span) but became bigger the value of  $a/d$ , became much longer the length of specimen(in this study the value of  $a/d$  let 4). The objectives of this study are the investigation of size effect of reinforced concrete according to variation of reinforcement ratio and of shear stirrups. The reinforcement ratio in a series are  $\rho=0$ ,  $\rho=\rho_{\min}$ ,  $\rho=0.5\rho_{\max}$ ,  $\rho=\rho_{\max}=0.75\rho_b$ , respectively and shear stirrup(U-stirrup) has been designed on each section according as the fracture load. The width of all designed section are constant as 20cm, the effective depth of sections are, 10cm, 20cm, 30cm, 40cm, the effective length(span) of specimens are 80cm, 160cm, 240cm, 320cm, and total length of specimens are 90cm, 170cm, 260cm, 340cm, respectively.

## 2. Size Effect Tests on Reinforced Concrete Beams

### 2.1 General Remarks

Throughout the experience of tests it is difficult to fabricate the test beams in the laboratory. For the required compressive strength of concrete cannot be obtained because of bad condition of laboratory. Mix design of concrete is made by Koreyo Remicon Company using specified compressive strength, 280 kg/cm<sup>2</sup>. Mix proportion of concrete is shown in Table 1. Dimension of test specimens and

specification of test specimens are shown in Table 2, Table 3, respectively.

Table 1 Mix proportion of concrete

Cement (kg/m <sup>3</sup> )	Fine Aggregate (kg/m <sup>3</sup> )	Coarse Aggregate (kg/m <sup>3</sup> )	Water (kg/m <sup>3</sup> )	Entrained Air (%)
402	611	1163	178	1.5

Table 2 Dimension of test specimens

Type of Specimen	Length, L (cm)	Span, S (cm)	Width, b (cm)	Effective Depth, d (cm)	Height, h (cm)
A	90	80	20	10	13
B	180	160	20	20	25
C	260	240	20	30	35
D	340	320	20	40	45

Table 3 Specification of test specimens

Specimen Index	Longitudinal Reinforcement Ratio, R	Shear Reinforcement Ratio	Effective Depth of Specimen, D
R0D10	0	0	10
R0D20	"	"	20
R0D30	"	"	30
R0D40	"	"	40
R1D10	$\rho_{min}$	0.0015	10
R1D20	"	"	20
R1D30	"	"	30
R1D40	"	"	40
R2D10	$0.5 \rho_{max}$	0.0019	10
R2D20	"	"	20
R2D30	"	"	30
R2D40	"	"	40
R3D10	$\rho_{max}$	0.0030	10
R3D20	"	"	20
R3D30	"	"	30
R3D40	"	"	40

About the specimen index, the number after the character R means the longitudinal reinforcement increment step and the number after the character D means the effective depth of section. For example, R0D10 is the speci-

men that has no longitudinal reinforcement and 10cm height. R3D40 is the specimen that has maximum longitudinal reinforcement ratio and 40cm height.

## 2.2 Test Results

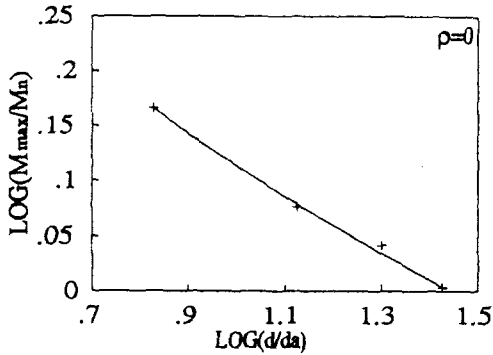
It is known generally that the size effect of bending capacity in reinforced concrete is less susceptible than that of shear capacity. But this fact must be verified by experiments, because they are the results of theoretical analysis or of regression analysis of data which reported by many other studies.

To observe the size effect experimentally in flexural behavior of reinforced concrete beam, the logarithmic value of relative moment  $\log(M_{max}/M_n)$  and relative beam depth  $\log(d/d_a)$  have been arranged as following Fig. 1.

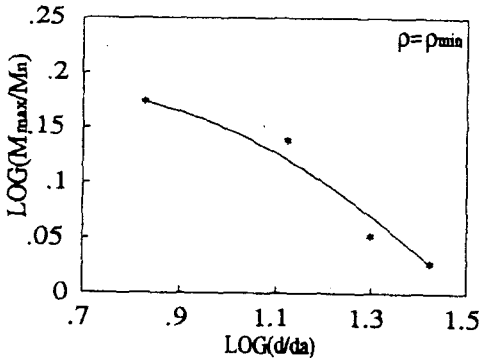
The size effect in flexural behavior decreases with the increment of longitudinal reinforcement ratio, but the size effect of heavily reinforced beams is higher than that of normally reinforced concrete beams. The size effects in flexural capacity of unreinforced and lightly reinforced concrete beams are very pronounced and also the size effect of normally or heavily reinforced concrete beams more pronounced than that of other studies. These results are very different from those of other studies. The reason why this phenomenon occurs can be explained as follows. Here,  $M_{max}$  is measured value of the test and  $M_n$  is nominal moment strength calculated by the design code of Korean Society of Civil Engineering(KSCE), and  $d$  is depth of specimen,  $d_a$  is maximum aggregate size,  $\rho_{max}$  is  $0.75 \rho_b$ .

To observe the size effect in shear capacity, the logarithmic value of relative shear capacity  $\log(V_{max}/V_n)$  and relative beam depth  $\log(d/d_a)$  have been arranged as following Fig. 2. The size effect in shear capacity of rein-

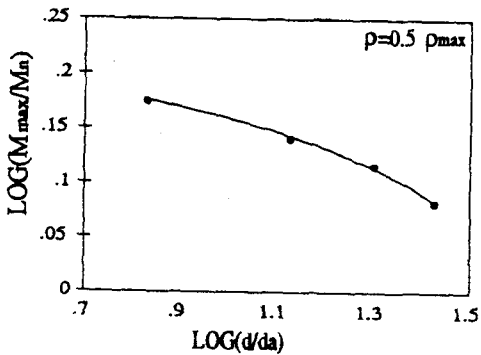
forced concrete member decreases with the increment of longitudinal reinforcement ratio, but in the case of  $\rho_{max}$  the size effect is lightly higher than that of  $0.5\rho_{max}$ (Fig. 2). Here,  $V_{max}$  is measured value of the test and  $V_n$  is nominal value of shear capacity.



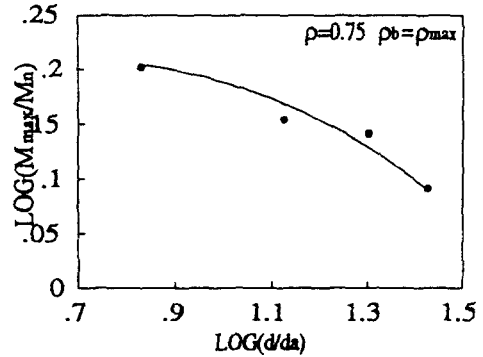
(a)  $\rho = 0$



(b)  $\rho = \rho_{min}$



(c)  $\rho = 0.5\rho_{max}$



(d)  $\rho = \rho_{max}$

Fig. 1 Size effect in flexural capacity of R.C. beams

The typical crack pattern of each test specimen has been arranged as following Fig. 3 ( $\rho = \rho_{max}$ ). In the case of heavily reinforced concrete beams, there exist web shear cracks although the depth of beam is short as 10cm. The compressive cracks increase gradually with the increment of beam depth, especially there exist many shear cracks and web shear cracks in the specimen R3D30 and R3D40 and perfect compressive failure occurs in the test of these specimens.

### 2.3 Derivation of Size Effect Law

From the size effect law of Bazant<sup>2)</sup>, we assume the size effect law in moment strength of reinforced concrete beams as follow.

$$M_{max} = M_n \left( \frac{\alpha_0}{\sqrt{1 + d/(\lambda_0 d_a)}} \right) \dots\dots\dots (1)$$

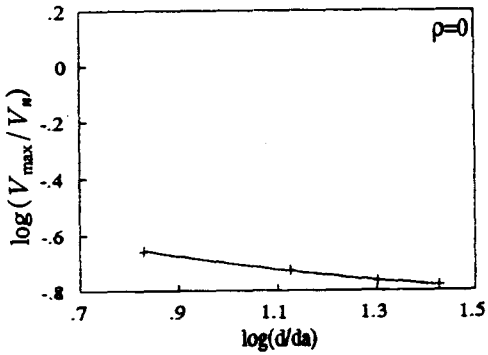
Here,  $M_{max}$  is ultimate moment strength at failure and  $M_n$  is nominal moment strength of designed section. The coefficients  $\alpha_0$  and  $\lambda_0$  are function of the longitudinal reinforcement ratio  $\rho$ , and they are assumed as follows.

$$\alpha_0 = a_1 + a_2\rho + a_3\rho^2 \dots\dots\dots (2a)$$

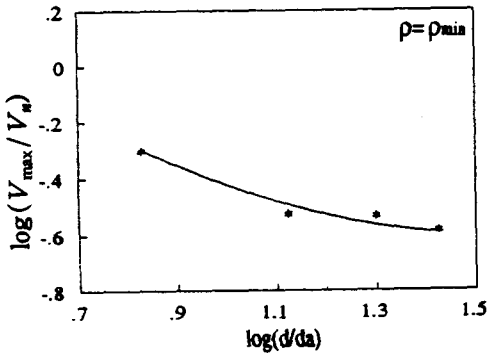
$$\lambda_0 = b_1 + b_2\rho + b_3\rho^2 \dots\dots\dots (2b)$$

From the measured values of specimens, the values of  $\alpha_0$  and  $\lambda_0$  at each longitudinal

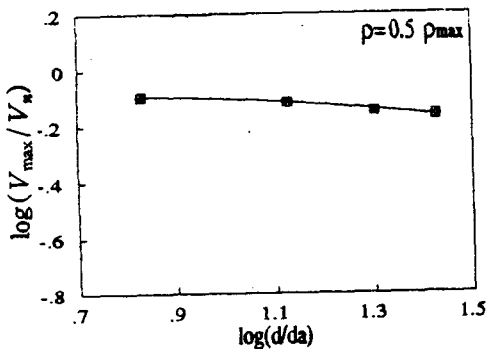
reinforcement ratio can be calculated. And then, the unknown in Eq.(2a) and Eq.(2b) can be calculated by the regression of  $\rho$ ,  $\alpha_0$ ,  $\lambda_0$ . Therefore Eq.(2a) and Eq.(2b) may be modified as follows.



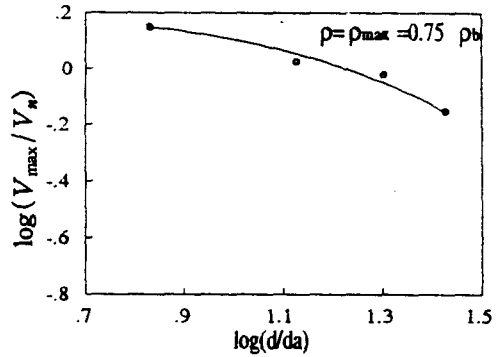
(a)  $\rho = 0$



(b)  $\rho = \rho_{min}$

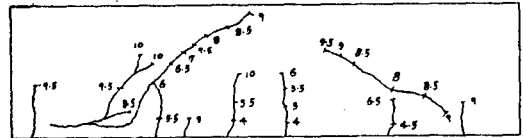


(c)  $\rho = 0.5 \rho_{max}$



(d)  $\rho = \rho_{max}$

Fig. 2 Size effect in shear capacity of R.C. beams



(a) R3D10



(b) R3D20



(c) R3D30



(d) R3D40

Fig. 3 Crack pattern of specimens ( $\rho = \rho_{max}$ )

$$\alpha_0 = 0.433 + 279 \rho - 8168 \rho^2 \dots\dots\dots (3a)$$

$$\lambda_0 = 2.719 - 133 \rho + 3229 \rho^2 \dots\dots\dots (3b)$$

To derive the size effect law in shear capacity of reinforced concrete beams, we use again the Bazant's size effect law and then the relative shear strength will be described as follow.

$$\frac{V_{max}}{V_n} = \frac{\alpha_0}{\sqrt{1 + d/(\lambda_0 d_a)}} \dots\dots\dots (4)$$

Here,  $V_{max}$  is the ultimate shear strength at failure and  $V_n$  is the nominal shear strength of designed section. The coefficients  $\alpha_0$  and  $\lambda_0$  are function of the longitudinal reinforcement ratio  $\rho$ , and they are assumed as follows.

$$\alpha_0 = a_1 + a_2(\rho / \rho_v) \dots\dots\dots (5a)$$

$$\lambda_0 = b_1 + b_2(\rho / \rho_v) + b_3(\rho / \rho_v)^2 \dots (5b)$$

From the measured values of specimens, the values of  $\alpha_0$  and  $\lambda_0$  at each longitudinal reinforcement ratio can be calculated. And then, the unknown in Eq.(5a) and Eq.(5b) can be calculated by the regression of  $\rho$ ,  $\alpha_0$ ,  $\lambda_0$ . Therefore Eq.(5a) and Eq.(5b) may be modified as follows.

$$\alpha_0 = 0.484 - 0.0768(\rho / \rho_v) \dots\dots\dots (6a)$$

$$\lambda_0 = 0.21 + 0.1835(\rho / \rho_v) - 0.0048(\rho / \rho_v)^2 \dots\dots\dots (6b)$$

If we want to measure the size effect in shear capacity of reinforced concrete beams,

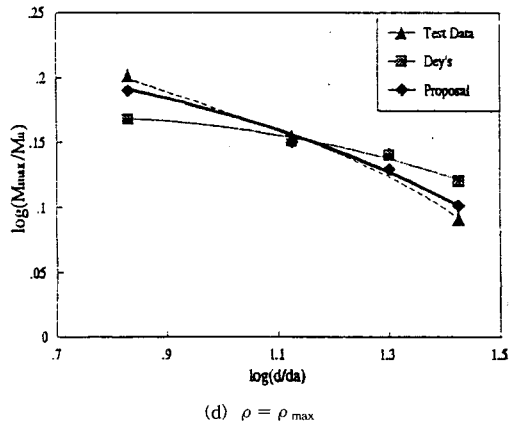
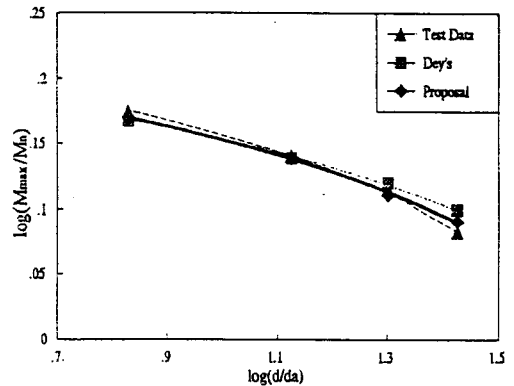
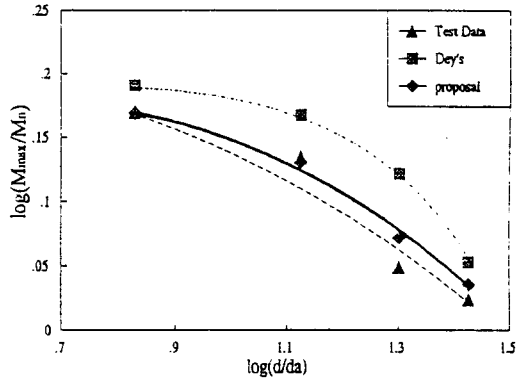
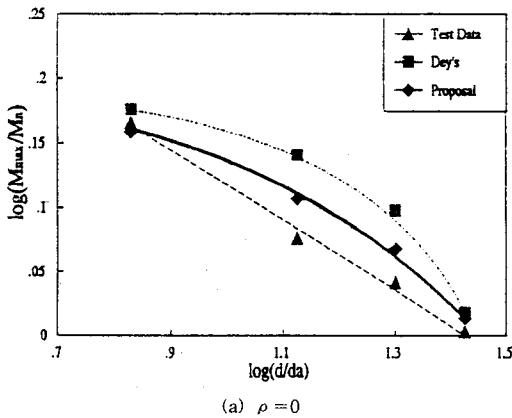


Fig. 4 Comparison of size effect in flexure with Dey's study

longitudinal reinforcement ratio  $\rho$ , shear rein-

forcement ratio  $\rho_v$ , and relative beam depth  $d/d_a$  is needed.

### 2.4 Comparison of Size Effect

Fig. 4 show comparison of size effect in flexure with Dey's study according to longitudinal reinforcement ratio. The size effect in flexural behavior of reinforced concrete members decreases with the increment of longitudinal reinforcement ratio and this trend is similar with the results of Dey's study and with the knowledge which is generally known, but in the case of heavily reinforced member the size effect decreases. The results of proposed formula agree with those of tests and the trend of size effects is similar with those

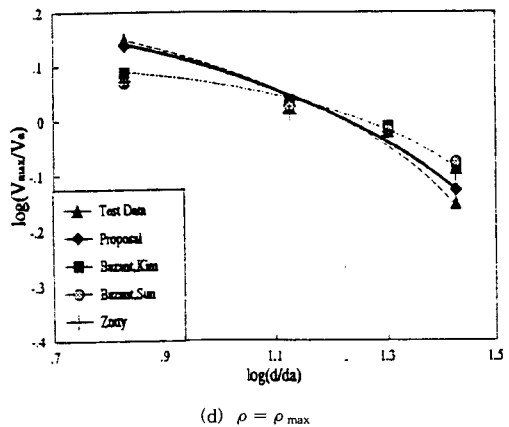
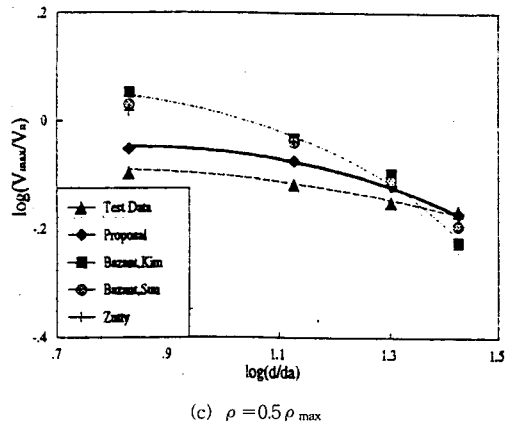
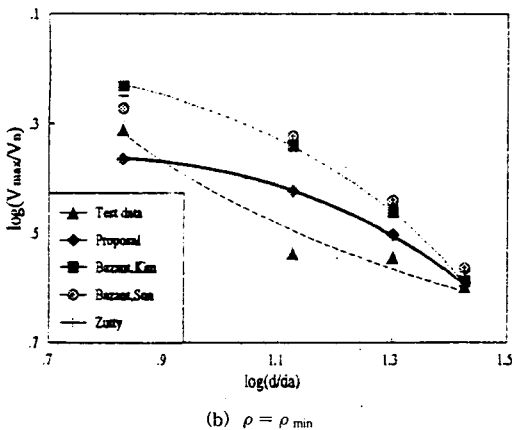
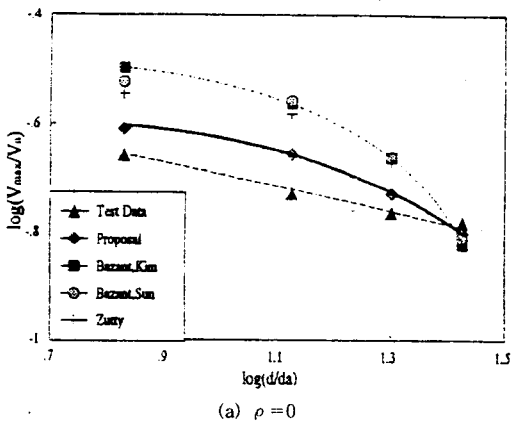


Fig. 5 Comparison of size effect in shear with Dey's study

of other studies.

In the case of unreinforced or lightly reinforced concrete members, the trends of size effect in shear capacity are a little different from those of other studies(Fig. 5 (a), (b)), but agree well with those of proposed formula (Fig. 5).

### 3. Conclusions

In this study, the size effect in flexural behavior of reinforced concrete with stirrups have been studied by experimentally. The specimens of different size with same longitudinal

reinforcement ratio are tested as a series. The major experimental variables include the relative depth  $d$  of the specimens as well as the longitudinal reinforcement ratio  $\rho$ . The nominal resistance in flexure is obtained by various sizes and reinforcement ratios.

From the results of this study, it known that the size effect of reinforced concrete in flexural resistance is very pronounced. The size effect laws for flexural behavior of reinforced concrete have been derived, and the derived formulas agree relatively well with experimental data. The trends of size effect in experiment are similar to those of other studies, but in a case of heavily reinforced concrete members a little different from other studies.

The size effect of flexural behavior decreases with the increment of longitudinal reinforcement ratio  $\rho$ , but in the case of heavily reinforced concrete members ( $\rho = \rho_{max}$ ), the size effect is higher than that of normally reinforced concrete members ( $\rho = 0.5 \rho_{max}$ ).

The size effect laws for shear behavior of reinforced concrete have been derived, and the derived formulas agree relatively well with experimental data. The trends of size effect in experiment are similar to those of other studies, but in a case of heavily reinforced concrete members a little different from other studies.

The trends of size effect in shear capacity are similar to those of flexural behavior, and the size effect of heavily reinforced concrete member is lightly higher than that of normally reinforced concrete members.

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