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MULTIVARIATE DISTRIBUTIONS WITH SELFDECOMPOSABLE PROJECTIONS

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ABSTRACT. A random vector X on $\mathbb{R}^d$ with the following properties is constructed: the distribution of X is infinitely divisible and not selfdecomposable, but every linear transformation of X to a lower-dimensional space has a selfdecomposable distribution.

1. Results and background facts

Denote the characteristic function of a distribution μ on the d-dimensional Euclidean space $\mathbb{R}^d$ by $\widehat{\mu}(z)$, $z \in \mathbb{R}^d$. A distribution μ on $\mathbb{R}^d$ is called *selfdecomposable* (or of class L) if, for any $b \in (0,1)$, there is a distribution ρ_b such that

$$\widehat{\mu}(z) = \widehat{\mu}(z)\widehat{\rho}_b(z), \quad z \in \mathbb{R}^d. \tag{1.1}$$

Denote the class of selfdecomposable distributions, the class of stable distributions, and the class of infinitely divisible distributions on $\mathbb{R}^d$ by $L(\mathbb{R}^d)$, $S(\mathbb{R}^d)$, and $I(\mathbb{R}^d)$, respectively. Then

$$S(\mathbb{R}^d) \subset L(\mathbb{R}^d) \subset I(\mathbb{R}^d). \tag{1.2}$$

Characterization of these classes in the theory of limit distributions for sums of independent random variables is well-known. See [4, 13]. A random variable with selfdecomposable distribution is also called selfdecomposable.

We identify the Euclidean spaces with the collections of real column vectors. Thus a linear transformation from $\mathbb{R}^d$ to $\mathbb{R}^l$ is represented by an $l \times d$ matrix. The distribution of a random variable $X = (X_j)_{1\le j\le d}$ on $\mathbb{R}^d$ is denoted by P_X. If a random variable X on $\mathbb{R}^d$ is selfdecomposable,

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then, for any $l \times d$ matrix A, AX is selfdecomposable on $\mathbb{R}^l$. This fact is easy to see, because

$$\widehat{P_{AX}}(z) = \widehat{P_X}(A'z), \quad z \in \mathbb{R}^l.$$

Here we denote by A' the transpose of the matrix A. We shall show that the converse in some sense of this fact does not hold. That is, we shall show the following.

THEOREM 1.1. *Let $d \geq 2$. There is a random variable on $\mathbb{R}^d$ satisfying the following three conditions:*

(a) *P_X is infinitely divisible.*
(b) *P_X is not selfdecomposable.*
(c) *For any $l \leq d-1$ and any $l \times d$ matrix A, P_{AX} is selfdecomposable.*

The condition (c) *implies that, for any projector A (that is $d \times d$ matrix satisfying $A^2 = A$) with the dimension of the range being $\leq d-1$, AX is selfdecomposable.*

The background facts are as follows. If a random variable $X = (X_j)_{1 \leq j \leq d}$ on $\mathbb{R}^d$ is Gaussian, stable, or infinitely divisible, respectively, then, for any linear transformation A, AX is Gaussian, stable, or infinitely divisible, respectively. It is easy to see that a random variable $X = (X_j)_{1 \leq j \leq d}$ on $\mathbb{R}^d$ is Gaussian if and only if every linear combination $\sum_{j=1}^d a_j X_j$ with $a_1, \ldots, a_d \in \mathbb{R}$ is Gaussian.

1. (Lévy [6]) Let $Z_k = \begin{pmatrix} Z_{k1} \\ Z_{k2} \end{pmatrix}$, $k = 1, \ldots, n$, are independent random variables on $\mathbb{R}^2$, each Gaussian distributed with mean $\binom{0}{0}$ and covariance $\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$ and let $X_1 = \sum_{k=1}^n {Z_{k1}}^2$, $X_2 = \sum_{k=1}^n {Z_{k2}}^2$, $X_3 = \sum_{k=1}^n Z_{k1} Z_{k2}$. Then the random variable $X = (X_j)_{1 \leq j \leq 3}$ on $\mathbb{R}^3$ is not infinitely divisible, while $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$, $\begin{pmatrix} X_2 \\ X_3 \end{pmatrix}$, and $\begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ are infinitely divisible on $\mathbb{R}^2$.

2. (Dwass and Teicher [2]) Let $X = (X_j)_{1 \leq j \leq 3}$ be the same as above. Then, any linear combination $\sum_{j=1}^3 a_j X_j$ is infinitely divisible.

3. (Ibragimov [5] for $d = 2$ and Linnik and Ostrovskii [7] for general d) There is a random variable $X = (X_j)_{1 \leq j \leq d}$ on $\mathbb{R}^d$ such that X is not infinitely divisible, but every linear combination $\sum_{j=1}^d a_j X_j$ is infinitely divisible. Giné and Hahn [3] remarks that the X of Linnik and Ostrovskii [7] can be chosen so that every $(d-1)$-dimensional projection of X is infinitely divisible.

4. A random variable $X = (X_j)_{1\le j\le d}$ on $\mathbb{R}^d$ is strictly stable if and only if every linear combination $\sum_{j=1}^d a_jX_j$ is strictly stable.

5. A random variable $X = (X_j)_{1\le j\le d}$ on $\mathbb{R}^d$ is stable with index $\alpha \in [1,2]$ if and only if every linear combination $\sum_{j=1}^d a_jX_j$ is stable with index in $[1,2]$.

The two results above are by Dudley and Kanter [1] and Samorodnitsky and Taqqu [10]. See the book [11] of Samorodnitsky and Taqqu.

6. (Marcus [9]. Proof improved by Samotij and Żak [12]) For any $\alpha \in (0,1)$ there is a random variable $X = (X_j)_{1\le j\le 2}$ on $\mathbb{R}^2$ such that X is not stable but any linear combination $a_1X_1 + a_2X_2$ is stable with index α.

7. (Giné and Hahn [3]) Let $d \ge 2$ and let X be a random variable on $\mathbb{R}^d$. If every 2-dimensional projection of X is infinitely divisible and if every linear combination $\sum_{j=1}^d a_jX_j$ is stable, then X is stable.

Our Theorem 1.1 shows that the statement 7 with "stable" replaced by "selfdecomposable" is false.

In the case of finite-dimensional distributions of processes with independent increments, a related problem is treated in [8].

2. Preliminaries on selfdecomposability

Denote the Euclidean norm of $x = (x_j)_{1\le j\le d} \in \mathbb{R}^d$ by $|x| = \left(\sum_{j=1}^d {x_j}^2\right)^{1/2}$. The σ-algebra of Borel sets in $\mathbb{R}^d$ is $\mathcal{B}(\mathbb{R}^d)$. The indicator function of a set B is $1_B(x)$. For a signed measure ρ defined on $\mathcal{B}(\mathbb{R}^d)$, the measure of total variation of ρ is denoted by $|\rho|$. We use the following facts on selfdecomposability of infinitely divisible distributions.

PROPOSITION 2.1. *Let $d \ge 1$. If μ is a selfdecomposable distribution on $\mathbb{R}^d$, then there is a unique measure ρ on $\mathbb{R}^d$ satisfying*

$$\rho(\{0\}) = 0 \quad \text{and} \quad \int_{|x|\le 2} |x|^2\rho(dx) + \int_{|x|>2} \log|x|\rho(dx) < \infty \tag{2.1}$$

such that the Lévy measure ν of μ is expressed as

$$\nu(B) = \int_{\mathbb{R}^d} \rho(dx) \int_0^\infty 1_B(e^{-t}x)dt \quad \text{for any } B \in \mathcal{B}(\mathbb{R}^d). \tag{2.2}$$

Conversely, if ρ *is a measure on* $\mathbb{R}^d$ *satisfying* (2.1), *then the measure* ν *defined by* (2.2) *satisfies*

$$\nu(\{0\}) = 0 \quad \text{and} \quad \int_{\mathbb{R}^d} (1 \wedge |x|^2)\nu(dx) < \infty \tag{2.3}$$

and is the Lévy measure of a selfdecomposable distribution on $\mathbb{R}^d$.

This is reformulation in Sato and Yamazato [14] of the result of Urbanik [15].

PROPOSITION 2.2. *Let* $d \geq 1$ *and let* μ *be an infinitely divisible distribution on* $\mathbb{R}^d$ *with Lévy measure* ν. *Suppose that there is a signed measure* ρ *on* $\mathbb{R}^d$ *such that* (2.1) *is satisfied with* $|\rho|$ *in place of* ρ *and that* ν *is represented by* (2.2) *with* ρ. *Then* ρ *is uniquely determined by* ν. *If* $\rho(B) < 0$ *for some* $B \in \mathcal{B}(\mathbb{R}^d)$, *then* μ *is not selfdecomposable.*

Proof. To prove the uniqueness of ρ, suppose that signed measures ρ_1 and ρ_2 serve as ρ to express the Lévy measure ν. For $k = 1, 2$, let $\rho_k = \rho_k^+ - \rho_k^-$ be the Jordan decomposition of ρ_k, where ρ_k^+ and ρ_k^- are the upper and lower variations of ρ_k, respectively. Then (2.1) holds with ρ_k^+ and ρ_k^- in place of ρ. Define

$$\nu_k^+(B) = \int_{\mathbb{R}^d} \rho_k^+(dx) \int_0^\infty 1_B(e^{-t}x)dt \quad \text{for } B \in \mathcal{B}(\mathbb{R}^d)$$

and similarly ν_k^- by ρ_k^- in place of ρ_k^+. Then we have $\nu = \nu_1^+ - \nu_1^- = \nu_2^+ - \nu_2^-$. Define $\widetilde{\nu} = \nu_1^+ + \nu_2^- = \nu_2^+ + \nu_1^-$. Consider an infinitely divisible distribution with Lévy measure $\widetilde{\nu}$. Then $\widetilde{\nu}$ has an expression similar to (2.2) using $\rho_1^+ + \rho_2^-$ and $\rho_2^+ + \rho_1^-$. Hence, by the uniqueness in Proposition 2.1, we have $\rho_1^+ + \rho_2^- = \rho_2^+ + \rho_1^-$. Therefore $\rho_1 = \rho_2$.

Suppose that μ is selfdecomposable. Then, by Proposition 2.1, the Lévy measure ν of μ has the representation (2.2) by a measure. It follows from the uniqueness just proved that, in this case, our signed measure ρ is actually a measure. Hence the last sentence in our proposition is true. □

3. Construction

Let us construct an infinitely divisible distribution on $\mathbb{R}^d$ such that the random variable that has this distribution satisfies the requirements of Theorem 1.1. Let $D_1 = \{x \in \mathbb{R}^d\colon 1 < |x| \le 2\}$ and $D_2 = \{x \in \mathbb{R}^d\colon |x| \le a\}$ with $0 < a \le 1$. Let

$$\rho(dx) = (1_{D_1}(x) - b1_{D_2}(x))dx \tag{3.1}$$

with $0 < b \le 1$. This is a signed measure absolutely continuous with respect to the Lebesgue measure. Define ν by (2.2) using this signed measure ρ. Then, we can prove the following.

LEMMA 3.1. *The signed measure* ν *just defined is in fact a measure on* $\mathbb{R}^d$.

LEMMA 3.2. *Let* A *be an* $l \times d$ *matrix with* $l \le d-1$. *Then the signed measure* ρ_A *defined by*

$$\rho_A(B) = \rho(\{x \in \mathbb{R}^d\colon Ax \in B\}) \quad \text{for } B \in \mathcal{B}(\mathbb{R}^l) \tag{3.2}$$

is in fact a measure on $\mathbb{R}^l$.

These two lemmas yield the following proposition, which shows Theorem 1.1.

PROPOSITION 3.3. *Let* ν *be defined as above. If* μ *is an infinitely divisible distribution on* $\mathbb{R}^d$ *whose Lévy measure coincides with* ν, *then the random variable* X *on* $\mathbb{R}^d$ *with distribution* μ *satisfies* (a), (b), and (c) *in* Theorem 1.1.

Proof. Notice that ν is a measure by Lemma 3.1. Since $|\rho|$ satisfies (2.1), ν satisfies $\nu(\{0\}) = 0$ and $\int(1 \wedge |x|^2)\nu(dx) < \infty$. Hence there is an infinitely divisible distribution μ on $\mathbb{R}^d$ having ν as its Lévy measure. We choose the Gaussian part of μ arbitrarily. Then μ is not selfdecomposable by Proposition 2.2. Let X be a random variable on $\mathbb{R}^d$ with distribution μ. Then, for any $l \times d$ matrix A, AX is infinitely divisible on $\mathbb{R}^l$ and its Lévy measure ν_A is such that $\nu_A(\{0\}) = 0$ and

$$\nu_A(B) = \nu(\{x \in \mathbb{R}^d\colon Ax \in B\}) \quad \text{for } B \in \mathcal{B}(\mathbb{R}^l \setminus \{0\}).$$

We have

$$\nu_A(B) = \int_{\mathbb{R}^d \setminus \{0\}} \rho_A(dx) \int_0^\infty 1_B(e^{-t}x)dt \quad \text{for } B \in \mathcal{B}(\mathbb{R}^l \setminus \{0\}).$$

If $l \le d-1$, then, using Lemma 3.2, we see that ρ_A is a measure and it satisfies

$$\int_{|x| \le 2} |x|^2 \rho_A(dx) + \int_{|x|>2} \log|x| \rho_A(dx) < \infty.$$

Consequently, by Proposition 2.1, AX is selfdecomposable if $l \leq d-1$. □

Proof of Lemma 3.1. Denote the Lebesgue measure on $\mathbb{R}^d$ by λ_d. Assume that $a = b = 1$. It is enough to prove the lemma in this case. We have, for $B \in \mathcal{B}(\mathbb{R}^d)$,

$$\nu(B) = \int_0^\infty (\lambda_d(e^t B \cap D_1) - \lambda_d(e^t B \cap D_2))dt.$$

Hence ν is absolutely continuous with respect to λ_d. Since ρ is rotation invariant, so is ν. Thus, ν has a rotation invariant density, that is, $\nu(dx) = g(|x|)dx$ with a Borel measurable function g on $[0, \infty)$. Therefore, in order to prove that $\nu \geq 0$, it is enough to show

$$\int_{r_1<|x|\leq r_2} \nu(dx) \geq 0 \quad \text{for } 0 \leq r_1 < r_2 < \infty. \tag{3.3}$$

Let c_d be the surface measure of the unit sphere in $\mathbb{R}^d$. Let

$$f_0(r) = 1_{(1,2]}(r) - 1_{(0,1]}(r). \tag{3.4}$$

We have

$$\begin{aligned}\int_{r_1<|x|\leq r_2} \nu(dx) &= \int_{\mathbb{R}^d} \rho(dx) \int_0^\infty 1_{\{\log(|x|/r_2)\leq t<\log(|x|/r_1)\}} dt \\ &= c_d \int_0^\infty f_0(r) r^{d-1} \Big(\Big(0 \vee \log \frac{r}{r_1}\Big) - \Big(0 \vee \log \frac{r}{r_2}\Big)\Big) dr.\end{aligned}$$

Denote the last integral by I. Then

$$I = \int_{r_1}^{r_2} f_0(r) r^{d-1} \log \frac{r}{r_1} dr + \int_{r_2}^\infty f_0(r) r^{d-1} \log \frac{r_2}{r_1} dr. \tag{3.5}$$

Let us show that $I \geq 0$ for $0 \leq r_1 < r_2 < \infty$. It is enough to consider the following four cases. 1: $r_1 > 1$. 2: $r_1 < r_2 \leq 1$. 3: $r_1 \leq 1$ and $1 < r_2 \leq 2$. 4: $r_1 \leq 1$ and $r_2 > 2$.

Case 1. The negative term in (3.4) is irrelevant. Hence $I \geq 0$.

Case 2. Noticing that

$$1 < \frac{r_2}{r_1} \leq \frac{1}{r_1} < \frac{2}{r_1},$$

we have, from (3.5),

$$I = -\int_{r_1}^{r_2} r^{d-1}\log\frac{r}{r_1}dr - \int_{r_2}^{1} r^{d-1}\log\frac{r_2}{r_1}dr + \int_{1}^{2} r^{d-1}\log\frac{r_2}{r_1}dr$$
$$= {r_1}^d\left[-\int_{1}^{r_2/r_1} s^{d-1}\log s\,ds - \int_{r_2/r_1}^{1/r_1} s^{d-1}\log\frac{r_2}{r_1}ds + \int_{1/r_1}^{2/r_1} s^{d-1}\log\frac{r_2}{r_1}ds\right].$$

Hence,

$$(3.6)\quad I = {r_1}^d\left[-\int_{1}^{1/r_1} s^{d-1}\log\Big(s\wedge\frac{r_2}{r_1}\Big)ds + \int_{1/r_1}^{2/r_1} s^{d-1}\log\Big(s\wedge\frac{r_2}{r_1}\Big)ds\right].$$

Since the function $s^{d-1}\log(s\wedge\frac{r_2}{r_1})$ of $s \geq 1$ is increasing and nonnegative, this shows that $I \geq 0$.

Case 3. $I = -\int_{r_1}^{1} r^{d-1}\log\frac{r}{r_1}dr + \int_{1}^{r_2} r^{d-1}\log\frac{r}{r_1}dr + \int_{r_2}^{2} r^{d-1}\log\frac{r_2}{r_1}dr.$

Case 4. $I = -\int_{r_1}^{1} r^{d-1}\log\frac{r}{r_1}dr + \int_{1}^{2} r^{d-1}\log\frac{r}{r_1}dr.$

Hence, in Cases 3 and 4, we get the same expression (3.6). It follows that $I \geq 0$, completing the proof. □

Proof of Lemma 3.2. Again we assume that $a = b = 1$. It suffices to prove the lemma in this case. Let A be an $l\times d$ matrix with $l \leq d-1$. We can choose $v^{(1)} \in \mathbb{R}^d$ such that $v^{(1)} \neq 0$ and $Av^{(1)} = 0$. Then choose $v^{(2)},\dots,v^{(d)} \in \mathbb{R}^d$ in such a way that $\{v^{(1)},\dots,v^{(d)}\}$ is a linearly independent system. Let $v^{(k)} = (v_{jk})_{1\leq j\leq d}$. Any $x = (x_j)_{1\leq j\leq d} \in \mathbb{R}^d$ is expressed as $x = \sum_{k=1}^{d} y_k v^{(k)}$ with some $y = (y_j)_{1\leq j\leq d} \in \mathbb{R}^d$. That is, $x_j = \sum_{k=1}^{d} v_{jk}y_k$ for $1 \leq j \leq d$. For any $B \in \mathcal{B}(\mathbb{R}^l)$, $Ax \in B$ is equivalent to $\sum_{k=2}^{d} y_k Av^{(k)} \in B$. Let

$$f(x) = 1_{D_1}(x) - 1_{D_2}(x) = 1_{(1,4]}(|x|^2) - 1_{(0,1]}(|x|^2).$$

Then

$$\rho_A(B) = \int_{\{x:\ Ax\in B\}} f(x)dx$$
$$= c\int_{\{\sum_{k=2}^{d} y_k Av^{(k)}\in B\}} dy_2\cdots dy_d \int_{-\infty}^{\infty} f(x)dy_1,$$

where c is the absolute value of the determinant of (v_{jk}). In order to show that $\rho_A(B) \geq 0$ for any B, it is enough to prove that

$$(3.7)\qquad \int_{-\infty}^{\infty}\big(1_{(1,4]}(|x|^2) - 1_{(0,1]}(|x|^2)\big)dy_1 \geq 0$$

for every fixed $y_2, \dots, y_d$. Notice that

$$|x|^2 = \sum_{j=1}^{d}\Big(\sum_{k=1}^{d} v_{jk}y_k\Big)^2 = \sum_{j=1}^{d}(p_j{}^2\xi^2 + 2p_jq_j\xi + q_j{}^2),$$

where $\xi = y_1$, $p_j = v_{j1}$, and $q_j = \sum_{k=2}^{d} v_{jk}y_k$. Hence

$$|x|^2 = P\xi^2 + 2R\xi + Q \quad \text{with } P = \sum_{j=1}^{d} p_j{}^2,\ Q = \sum_{j=1}^{d} q_j{}^2,\ R = \sum_{j=1}^{d} p_jq_j.$$

We have $P > 0$, since (v_{jk}) is an invertible matrix. Let

$$J = \int_{-\infty}^{\infty} \big(1_{(1,4]}(P\xi^2 + 2R\xi + Q) - 1_{(0,1]}(P\xi^2 + 2R\xi + Q)\big)d\xi.$$

Then (3.7) is equivalent to that $J \geq 0$. If the equation $P\xi^2+2R\xi+Q = 1$ does not have two real roots, then clearly $J \geq 0$. Suppose that the equation $P\xi^2 + 2R\xi + Q = 1$ has two real roots $\alpha_1 < \alpha_2$. Then, the equation $P\xi^2 + 2R\xi + Q = 4$ has two real roots $\beta_1 < \beta_2$ and we have $\beta_1 < \alpha_1 < \alpha_2 < \beta_2$. Thus

$$J \geq (\alpha_1 - \beta_1) + (\beta_2 - \alpha_2) - (\alpha_2 - \alpha_1) = \beta_2 - \beta_1 - 2(\alpha_2 - \alpha_1).$$

Since $\alpha_2-\alpha_1 = 2\sqrt{R^2 - P(Q-1)}/P$ and $\beta_2-\beta_1 = 2\sqrt{R^2 - P(Q-4)}/P$, the nonnegativity of J follows if $\sqrt{R^2 - P(Q-4)} \geq 2\sqrt{R^2 - P(Q-1)}$. This is equivalent to $PQ \geq R^2$. This inequality is true, because this is the Cauchy-Schwarz. Hence $J \geq 0$. □

We remark that Ibragimov [5] and Linnik and Ostrovskii [7] both use the argument which we can call the method of "signed Lévy measure", in order to show that a distribution is not infinitely divisible. The method is explained in Gnedenko and Kolmogorov [4], p. 81. Our proof of the theorem is suggested by this method and the ρ in (3.1) is the signed measure employed in [7].

ADDED IN PROOF. O. E. Barndorff-Nielsen drew the author's attention to the relevance of a paper by D. N. Shanbhag and M. Sreeharai (An extension of Goldie's result and further results in infinite divisibility, Zeit. Wahrsch. Verw. Gebiete **47** (1979), 19-25). They remark that a random variable on $\mathbb{R}^d$ with some hyperbolic distribution is infinitely divisible and not selfdecomposable, but every linear combination of its components is selfdecomposable. The author stresses that the construction in this paper would be of methodological interest and the random

variable constructed has the stronger property (c). He thanks O. E. Barndorff-Nielsen for his kind comments.

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