A Mixed Integer Programming Model for Bulk Cargo Ship Scheduling with a Single Loading Port

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Abstract

This paper concerns a bulk or semibulk cargo ship scheduling problem with a single loading port. This type of ship scheduling problem is frequently needed in real world for carrying minerals or agricultural produce from a major single production zone to many destinations scattered over a large area of the world. The first optimization model for this problem was introduced by Ronen (1986) as a nonlinear mixed integer program. The model developed in this paper is an improvement of his model in the sense that nonlinearities and numerous unnecessary integer variables have been eliminated. By this improvement we could expect real world instances of moderate sizes to be solved optimal solutions by commercial integer programming software. Similarity between the ship scheduling model and the capacitated facility location model is also discussed.

1. Introduction

This paper presents an improved formulation for a ship scheduling model with a single loading port, first developed by Ronen (1986) as a nonlinear mixed integer program. This model can frequently be applied to ship scheduling problems of transporting minerals or agricultural produce, which usually have a single supply zone of a large production capacity enough to satisfy the demands at many places widely separated from one another.

Comprehensive surveys of many models recently developed for various ship scheduling problems can be found in (Ronen 1993). Typical optimization models for bulk cargo ship scheduling

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problems (Appelgren 1971, Fisher and Rosenwein 1989, Perakis and Bremer 1992, Bremer and Perakis 1992) are described as pure 0-1 integer programming problems using the framework of set covering, partitioning, or packing. Unlike these, Ronen (1986) gives his model more flexibility by adding continuous variables.

This paper gives a more efficient, but equivalent, alternative for Ronen's model. This is an improvement over Ronen's original formulation, since it does not require nonlinearitiy and greatly reduces the number of the integer variables. It is also interesting to see that the resulting model can be interpreted as a generalization of the capacitated facility location problem (Aikens 1985, Sridharan 1995).

In the next section, we briefly describe Ronen's original formulation as a preliminary step for our discussion. In section 3, our new formulation is given. In section 4, our new formulation is compared with the capacitated facility location problem.

2. Original Formulation

Suppose we have a finite number of ships currently available for operation over the planning horizon, which may already have been determined by long-range planning or by current operation of the fleet. To each ship $k \in K$ (the set of ships), one of a finite set of possible schedules ($n \in N_k$) should be assigned. Since the considered scheduling problem has a fairly short planning horizon, a schedule of a ship can be seen simply as a feasible combination of unloading ports with a shortest route through them. We have a single loading port and a finite number of unloading ports ($j \in J$). The estimates of cargo demands at the unloading ports are given as constants. We assume that the total capacity of the available fleet is greater than the total cargo demand (which is usually the case). Each ship is allowed to carry cargoes to more than one port, and the shipment to a specific port can be spilt into more than one ship. More detailed assumptions, related cost, solution methods and computational experiments can be found in (Ronen, 1986).

The following notation will be used to describe the model:

Notation for data

C_k: Capacity of ship k

a_{kj}: Unit shipping cost to port j by ship k

f_{kn}: Cost of ship k taking route n

u_j: Unloading rate per day at port j

p_k: Daily cost of stay for ship k at each unloading port

D_j: Demand of cargo at port j

N_k: Set of possible schedules for ship k

K: Set of available ships

J: Set of unloading ports

$b_{jn} = \begin{cases} 1 & \text{if schedule } n \text{ visits port } j \\ 0 & \text{otherwise} \end{cases}$

$M_{kj} = \min\{C_k, D_j\}$

Decision Variables

$x_{kn} = \begin{cases} 1 & \text{if ship } k \text{ selects schedule } n \\ 0 & \text{otherwise} \end{cases}$

$z_{kjn} = \begin{cases} 1 & \text{if ship } k \text{ selects schedule } n \text{ and visits port } j \\ 0 & \text{otherwise} \end{cases}$

y_{kj}: amount of cargo to unloading port j shipped by ship k

The nonlinear mixed integer program developed by Ronen is as follows:

$$Min \sum_{k \in K} \sum_{j \in J} \sum_{n \in N_k} a_{kj} y_{kj} z_{kjn} + \sum_{k \in K} \sum_{j \in J} \sum_{n \in N_k} \left(\frac{p_k}{u_j} \right) y_{kj} z_{kjn} + \sum_{k \in K} \sum_{n \in N_k} f_{kn} x_{kn} \quad \cdots\cdots (1)$$

subject to

$$\sum_{j\in J}\sum_{n\in N_k} y_{kj} z_{kjn} \le C_k \qquad \forall k\in K \quad \cdots\cdots (2)$$

$$\sum_{k\in K}\sum_{n\in N_k} y_{kj} z_{kjn} = D_j \qquad j\in J \quad \cdots\cdots (3)$$

$$\sum_{n\in N_k} x_{kn} = 1 \qquad \forall k\in K \quad \cdots\cdots (4)$$

$$\sum_{k\in K}\sum_{n\in N_k} C_k z_{kjn} \ge D_j \qquad \forall j\in J \quad \cdots\cdots (5)$$

$$x_{kn}\in\{0,1\},\ y_{kj}\ge 0,\ z_{kjn}\in\{0,1\}$$

The objective function (1) minimizes the total fleet cost, composed of shipping cost of cargoes, the cost of ships' unloading time, and the cost of taking specific routes. Constraints (2) are the ship capacity constraints, and constraints (3) require that all the demands at destinations should be satisfied. Each ship is guaranteed to have exactly one schedule by constraints (4). In case ship k is set to be idle, it can also be regarded as a specific schedule n. In this case, f_{kn} simply becomes the demurrage cost during the planning horizon. Constraints (5) require that the sum of the capacities of the ships visiting each port should be at least equal to the cargo demand at the port, but are in fact redundant by constraints (2) and (3). Ronen includes these redundant constraints in his model, in order to consider only feasible schedules in the solution heuristics he tried.

3. Improved Formulation

By our new formulation we show that the nonlinearity existing in Ronen's model (Ronen 1986), in fact, is not necessary. The resulting model becomes a mixed integer linear program with 0-1 integer variables.

It is easy to find that y_{kj} can take a positive value only when port j is visited by ship k, that is, only when $\sum_{n\in N_k} z_{kjn} = 1$. This is the only reason why the original model has many nonlinear terms, $y_{kj}z_{kjn}$, in both objective function and constraints. This formulation also requires a great number of integer variables, z_{kjn}, which in fact are found to be unnecessary in our new formulation. In order to make y_{kj} take a positive value only when port j is visited by ship k, we, instead, introduce a set of regulating constraints as follows:

$$y_{kj} - M_{kj}\Big(\sum_{n\in N_k} b_{jn}x_{kn}\Big) \le 0 \qquad \forall (k,j)$$

It is natural that M_{kj} is a upper bound of y_{kj}, that is, the amount of cargo carried to port j cannot exceed the ship capacity C_k nor cargo demand D_j.

Hence the new model formulated in this paper becomes as follows:

$$Min \sum_{k\in K}\sum_{j\in J} a_{kj}y_{kj} + \sum_{k\in K}\sum_{j\in J}\Big(\frac{p_k}{u_j}\Big)y_{kj} + \sum_{k\in K}\sum_{n\in N_k} f_{kn}x_{kn} \quad \cdots\cdots (1')$$

subject to

$$\sum_{j\in J} y_{kj} \le C_k \qquad \forall k\in K \quad \cdots\cdots (2')$$

$$\sum_{k\in K} y_{kj} = D_j \qquad \forall j\in J \quad \cdots\cdots (3')$$

$$\sum_{n\in N_k} x_{kn} = 1 \qquad \forall k\in K \quad \cdots\cdots (4')$$

$$y_{kj} - M_{kj}\Big(\sum_{n\in N_k} b_{jn}x_{kn}\Big) \le 0 \qquad \forall (k,j) \quad \cdots\cdots (5')$$

$$x_{kn}\in\{0,1\},\ y_{kj}\ge 0$$

(1')-(3') are direct replacements for (1)-(3) without complicating nonlinear terms. Constraints (4') are the same as (4), and (5') the regulating constraints described above.

It should be noted that the above new

formulation has not only eliminated nonlinearity from the original formulation, but also greatly reduced the number of integer variables. It is easy to see that the number of integer variables x_{kn} required in the original model is $\sum_{k\in K}|N_k|$ (where $|N_k|$ denotes the cardinality of the set N_k), and that of z_{kjn} is $(\sum_{k\in K}|N_k|)|J|$, and so the total number of required integer variables is $(\sum_{k\in K}|N_k|)(|J|+1)$, while our new model needs only $\sum_{k\in K}|N_k|$ integer variables (x_{kn}). For example, if we are to solve an instance with 9 unloading ports, that is $|J|=9$, then we only have to deal with one tenth as many integer variables as in the original model. Thus we have eliminated both the nonlinearity and unnecessary integer variables.

4. Concluding Remarks

This paper gives an improved reformulation of an existing optimization model. We have given a linear alternative for a nonlinear integer programming model.

Medium-sized versions of this problem are expected to be solved optimally by using commercial software. For example, if 5 ships and 5 ports are involved, and a schedule can visit up to 3 unloading ports, then the maximum number of required integer variables is

$$5\times\left(\frac{5!}{0!5!}+\frac{5!}{1!4!}+\frac{5!}{2!3!}+\frac{5!}{3!2!}\right)=130.$$

For a possible direction for a new solution method of large instances of the model presented in this paper, it is worthwhile to note that our new formulation can be seen as a generalized version of a well-known class of optimization problems called Capacitated Facility Location Problem (CFLP, Aikens 1985). It also should be noted that many efficient methods to exploit the special structure of CFLP have already been developed (Sridharan 1995). CFLP is typically represented as a mixed integer program as follows:

$$\text{Min} \quad \sum_{k\in K}\sum_{j\in J} c_{kj}y_{kj} + \sum_{k\in K} f_k x_k$$

subject to

$$\sum_{j\in J} y_{kj} \le C_k \qquad \forall k\in K$$

$$\sum_{k\in K} y_{kj} = D_j \qquad \forall j\in J$$

$$y_{kj} \le C_k x_k \qquad \forall k\in K$$

$$y_{kj}\ge 0,\ x_k\in\{0,1\}$$

where c_{kj} is the unit shipping cost of cargo carried to demand point j from supply site k, and y_{kj} the cargo amount shipped; f_k is the fixed cost of building the supply facility at candidate site k, and x_k the binary variable to decide to build the supply facility or not; C_k is the capacity of cargo supply at candidate site k, and D_j cargo demand at demand point j.

By setting $c_{kj}=a_{kj}+(p_k/u_j)$, the objective function (1′) has basically the same shape as the one in the above model. Constraints (2′) and (3′) are identical to the first two constraints of CFLP. As a generalization of CFLP, suppose we have a finite set of facility type alternatives ($n\in N_k$) at each candidate site k, and the building costs (f_{kn}) vary with the type selected. Then the resulting CFLP can be exactly represented to be the same as (1′)-(5′). By this inference, the new formulation (1′)-(5′) can be regarded as a generalization of the CFLP. The ordinary CFLP, conversely, can be seen as a special case where each candidate site k has only a single option for facility type.

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