Development of Direct Optimization Algorithms using Radial Basis Functions

방사상 기본 함수를 사용한 직접최적화 알고리즘에 관한 연구

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요 약 : 일반적인 비선형 동역학 최적화문제를 비선형 프로그래밍 문제로 변환하는데 제어변수들을 방사성 기본 함수로 근사화하는 방법이 사용되었다. 방사성 기본 함수의 계수들을 연속적으로 보정하기 위하여 최소수정기법에 기초를 둔 비선형 프로그래밍 알고리즘이 연구되었다. 이러한 알고리즘을 실제적인 다변수 제어 시스템에 적용하여 성능을 검증하였다.

Keywords : direct optimization, parameter optimization, radial basis function(RBF), terminal constraints, minimum norm correction, step size limitation filter

I. Introduction

For open-loop function space optimal control problems, there are two broad classes of approaches to solutions. One class is the indirect methods which require the derivation of the function space necessary conditions for optimality using calculus of variation techniques and result in a two-point boundary value problem (TPBVP)[1], wherein the state and control differential equations are iteratively solved. The second class is the direct methods, which approximate the subset of the unknowns in the continous problem with an assumed parametric model and thereby obtain a parameter optimization problem. This approximation can involve parameterization of the control variables only or both the states and controls can be parameterized. In essence, this paper establishes a useful way to parameterize control functions and also establishes associated methods to optimize the parameters. Many different direct optimization techniques have been proposed and attempted. Direct shooting uses a finitedimensional parameter representation of the control history[2] and explicitly integrates the state trajec- tory. A unique approach was taken by Zondervan et 0al.[3] where a hybrid (direct/indirect) formulation using nonlinear programming for solution was used to solve some transfer problems. A purely direct approach was described by Johnson[4] and later by Hargraves et al.[5] who represented both the control
and state histories using Chebychev orthogonal polynomials and used integral penalty functions to enforce the equations of motion. A different direct approach for solving optimal control problems uses Dickmanns' collocation method with Hermite cubics[6] to convert the optimal control problem into a nonlinear programming problem. As another direct method, we study the direct optimization using radial basis functions. Radial basis functions have been used historically for approximation in a curvefitting and have recently been used as the basis functions for neural and related input/output approximation and learning networks. Sanner and Slotine[7] proposed a direct adaptive tracking control architecture which employs a network of Gaussian radial basis functions to adaptively compensate for the plant nonlinearities. Powell[8], studied radial basis functions and showed they can fit irregularily positioned data points. Girosi and Poggio [9],[10], have recently shown that radial basis function approximation schemes satisfy regularization conditions and have proved existence and uniqueness of the best approximation for radial basis function networks.

This paper will focus on establishing that radial basis functions are well-suited for parameterization of control variables in a dynamical optimal control problem with terminal constraints. Apparently this is the first use of radial basis functions in this setting. The objective of this study is to develop two nonlinear programming methods, based on radial basis function approximation, to optimize $\boldsymbol{u}(t)$ for dynamical systems of the form $\dot{\boldsymbol{x}}= \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u})$, with specified terminal constraints and performance index. The two nonlinear programming methods are based upon i) an evenly spaced radial basis function direct optimization algorithm, and ii) an adaptively spaced radial basis function direct optimization algorithm. We approximate the control variables with radial basis functions which are represented with the prescribed sharpness parameters (σ_i) and center locations (τ_i) for the set of exponential radial basis functions. Starting with

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smaller number of radial basis functions, we try to find the right number of radial basis functions with respect to a tolerance on extremizing the performance index. In a new adaptive RBF optimization algorithm, we check the sensitivity of the terminal constraints and performance index with respect to parameters and add a new radial basis function centered between the most sensitive current radial basis center location and the more sensitive of the two adjacent centers. The result is that the new radial basis functions are recursively "attracted" to regions of high sensitivity. The nonlinear programming algorithms are motivated by an analogous approach [11], [12], in the recent literature. For the application, we consider as a representative multivariable control problem, a minimum energy-loss aeroassisted orbital plane change problem [13]. Since London[14] demonstrated the possibility of using aerodynamic forces to produce an orbital plane change with an expenditure of energy significantly smaller than that associated with an extraatmospheric propulsive maneuver, numerous studies of aeroassisted orbit transfer have been carried out for a wide variety of Earth-orbital and planetary missions[15].

II. RBF direct optimization algorithms

Consider the optimal control problem:
Find $\boldsymbol{u}(t)$ such that the trajectory that ensures from solving the differential equation

$$\dot{\boldsymbol{x}}= f(t,\ \boldsymbol{x},\ \boldsymbol{u}),\ \boldsymbol{x}(t_1)= \boldsymbol{x}_1 \tag{1}$$

extremizes

$$J= \phi(t_f,\ \boldsymbol{x}(t_f))+ \int_{t_1}^{t_f}(t,\ \boldsymbol{x},\ \boldsymbol{u})dt \tag{2}$$

subject to q terminal conditions

$$\boldsymbol{\phi}(t_f,\ \boldsymbol{x}(t_f))=\boldsymbol{0} \tag{3}$$

We first parameterize the control vector $\boldsymbol{u}$ appearing in the equations of motion

$$\dot{\boldsymbol{x}}= f(t,\ \boldsymbol{x},\ \boldsymbol{u}) \tag{4}$$

where $\boldsymbol{x}(t)\in R^n$, $\boldsymbol{u}(t)\in R^m$ with a radial basis function approximation, illustrated here for the case that $\boldsymbol{u}=u$ is a scalar

$$u= \sum_{i=1}^{N} w_i e^{-\frac{1}{2}(\frac{t-\tau_i}{\sigma_i})^2} \tag{5}$$

where (w_i, σ_i, τ_i) are respectively (weight parameter, standard deviation, center location). In this study we consider ω_i as the parameters to be optimized with σ_i and τ_i prescribed. Thus, the system to be solved has the structure

$$\dot{\boldsymbol{x}}= f(t,\ \boldsymbol{x},\ \boldsymbol{w}) \tag{6}$$

where $\boldsymbol{w}\in R^N$ is the vector of RBF weights to be determined by optimization and f has to be appropriately re-defined. More generally, $\boldsymbol{w}$ could be an arbitrary vector of parameters, and might include, for example, the σ_i's and τ_i's. Reference[16] considers a system of the form

$$\dot{\boldsymbol{x}}= f(t,\ \boldsymbol{x},\ \boldsymbol{p}) \tag{7}$$

where $\boldsymbol{p}^T=\{p_1, p_2, \cdots, p_N\}$ is a set of N parameters and we need the matrices of partial derivatives

$$\Phi(t, t_0)=\left[\frac{\partial \boldsymbol{x}(t)}{\partial \boldsymbol{x}(t_0)}\right] \tag{8}$$

and

$$\Psi(t, t_0)=\left[\frac{\partial \boldsymbol{x}(t)}{\partial \boldsymbol{p}}\right] \tag{9}$$

and it is easy to show[16] that Φ and Ψ satisfy the differential equations

$$\frac{d}{dt}[\Psi(t, t_0)]=[B(t)][\Psi(t, t_0)]+\left[\frac{\partial f(t,\ \boldsymbol{x},\ \boldsymbol{p})}{\partial \boldsymbol{p}}\right]$$

$$\frac{d}{dt}[\Phi(t, t_0)]=[B(t)][\Phi(t, t_0)], [\Phi(t_0, t_0)]=[I] \tag{10}$$

$$, [\Psi(t_0, t_0)]=[\ 0] \tag{11}$$

where

$$[B(t)]=\left[\frac{\partial f(t,\ \boldsymbol{x},\ \boldsymbol{p})}{\partial \boldsymbol{x}(t)}\right] \tag{12}$$

Therefore, the dynamics of the original system (1) and the appropriate partial derivatives are governed by the following extended system

$$\dot{\boldsymbol{x}}= f(t,\ \boldsymbol{x},\ \boldsymbol{w}) \tag{13}$$

$$\dot{\boldsymbol{\Psi}}= B\Psi+\frac{\partial f}{\partial \boldsymbol{w}} \tag{14}$$

where

$$\boldsymbol{\Psi}=\left[\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{w}}\right] \tag{15}$$

We can solve the extended system (13)-(15) using, for example, Runge-Kutta methods. When we update the parameters $\boldsymbol{w}$ by a correction $\Delta\boldsymbol{w}$, there are usually more parameters than the terminal constraints, we can seek small $\Delta\boldsymbol{w}$ corrections using the minimum norm correction algorithm[11]. Thus we have a conventional nonlinear programming problem wherein the parameters $\boldsymbol{w}$ defining control variables $\boldsymbol{u}$ are updated iteratively by local linearization, i.e. we recursively solve simultaneous linear algebraic equations

$$\Delta\boldsymbol{y}= A\Delta\boldsymbol{w} \tag{16}$$

with $\Delta\boldsymbol{w}\in R^N$ and $\Delta\boldsymbol{y}\in R^m$. Since we have more parameters than the terminal constraints i.e. $m<N$, the problem is under-determined and we use the minimum norm correction algorithm :

$$\Delta\boldsymbol{w}= A^T(AA^T)^{-1}\Delta\boldsymbol{y} \tag{17}$$

where

$$A=\left[\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{w}}\right]=\left[\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{w}}\right]\left[\frac{\partial \boldsymbol{x}(t_f)}{\partial \boldsymbol{w}}\right]$$

$$= \left[\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}(t_f)} \right] \Psi(t_f, 0) \tag{18}$$

and

$$\Delta \boldsymbol{y} = \begin{bmatrix} \psi_1(t_f) \\ \vdots \\ \psi_q(t_f) \end{bmatrix} \tag{19}$$

while global convergence cannot be guaranteed, we find that convergence is enhanced if we use a step size limitation filter according to the value of $\Delta \boldsymbol{w}$ as follows;

$$\boldsymbol{w}^{\text{new}} = \boldsymbol{w}^{\text{old}} + \Delta \boldsymbol{w} \tag{20}$$

where

$$|\Delta \boldsymbol{w}| = \sqrt{\Delta \boldsymbol{w}^T \Delta \boldsymbol{w}} \tag{21}$$

If $|\Delta \boldsymbol{w}| \le \varepsilon$ for acceptably small ε, then we use the full correction

$$\Delta \boldsymbol{w} = A^T (AA^T)^{-1} \Delta \boldsymbol{y} \tag{22}$$

else if $|\Delta \boldsymbol{w}| \ge \varepsilon$ for acceptably small ε, then we use the scaled correction

$$\Delta \boldsymbol{w} = 2 \Delta \boldsymbol{y} \tag{23}$$

We observe that the explicit minimum norm inverse $\{A^T(AA^T)^{-1}\}$ can for poorly conditioned systems be computed more robustly using the Moore-Penrose[11] generalized inverse based upon the singular value decomposition. After the terminal constraints are met, we have a first feasible solution, but we generally do not know how near the trajectory is to the optimal solution. To drive the performance value toward the optimal, we introduce a homotopy concept[17]. The homotopy process treats the performance index $2(J_o)$ as an additional equality constraint. The object value $2(J_o)$ is made ''portable'' by introducing a one parameter (λ) family of (J_o) values.

$$J_o = \lambda J^* + (1 - \lambda) \cdot J_{current} \tag{24}$$

where J^* is chosen as ''the best value of J one could hope for'' and is usually not actually achievable. We need to find the largest value of λ which meets the terminal constraints for which all constraint violations are reduced to within the acceptable error tolerances within a specified number of local corrections. In this state the $\Delta \boldsymbol{y}$ should be as follows:

$$\Delta \boldsymbol{y} = \begin{bmatrix} \psi_1(t_f) \\ \vdots \\ \psi_q(t_f) \\ J_o - J_{current} \end{bmatrix} \tag{25}$$

In our particular implementation, λ is set to a sequence of values and is incremented after each converged sequence of iterations. When a small increase $(\Delta\lambda_{\min})$ cannot be achieved, while satisfying all constraints within tolerance, the previously converged solution is adopted as the constrained optimum. While it is not possible to rigorously establish that this heuristic approach guarantees convertgence to the global optimum, we find convertgence is comparable, or superior to, conventional gradient projection methods, and is much easier to implement.

1. Evenly spaced RBF direct optimization algorithm

During the optimization process described above, we seek to refine the approximation as the solution progresses and use lower converged dimensioned approximations to establish initial iteratives for each higher dimensioned approximation. To do this, there are several options, all involve increasing the number of RBFs to approximate the optimal control $\boldsymbol{u}(t)$. The simplist strategy is to fix the shape and space the centers evenly over the entire time domain. For each additional radial basis function, we re-space the number of radial basis functions evenly throughout the entire time domain and prescribe the center locations τ_i and the sharpness parameters σ_i and begin from the first step of the algorithm. Fig. 1 shows how an

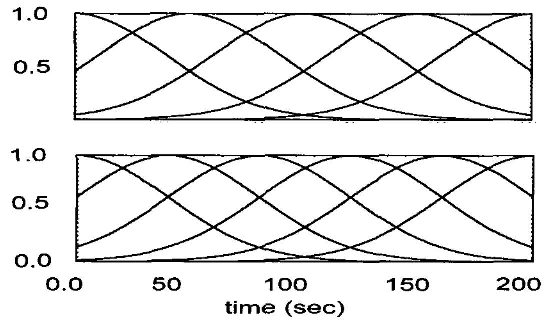

Fig. 1. Evenly spaced radial basis functions.

additional radial basis function isintroduced. This is analogous to "grid refinement" [18] in the finite element method. In Fig. 1 the upper one is for five radial basis functions and the lower one is for six radial basis functions which are spaced evenly throughout the entire time interval.

2. Adaptively spaced RBF direct optimization algorithm

For the adaptively spaced radial basis function algorithm, we have developed a useful heuristic to indicate the most attractive part of the time domain where the approximation needs to be refined. We check the sensitivity of the terminal constraints and the performance index with respect to parameters as follows,

$$A = \begin{bmatrix} \frac{\partial \psi_1}{\partial w_1} & \frac{\partial \psi_1}{\partial w_2} & \cdots & \frac{\partial \psi_1}{\partial w_N} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial \psi_q}{\partial w_1} & \frac{\partial \psi_q}{\partial w_2} & \cdots & \frac{\partial \psi_q}{\partial w_N} \\ \frac{\partial J}{\partial w_1} & \frac{\partial J}{\partial w_2} & \cdots & \frac{\partial J}{\partial w_N} \end{bmatrix} \tag{26}$$

$$= [A_1, A_2, \cdots, A_N]$$

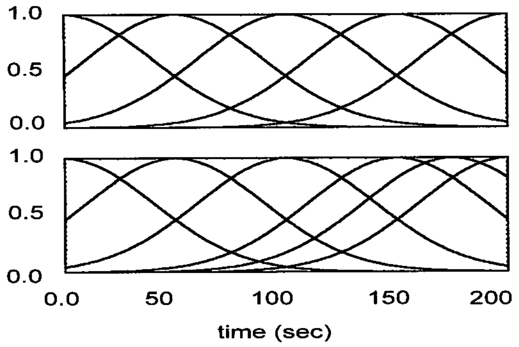

Fig. 2. Adaptively spaced radial basis functions.

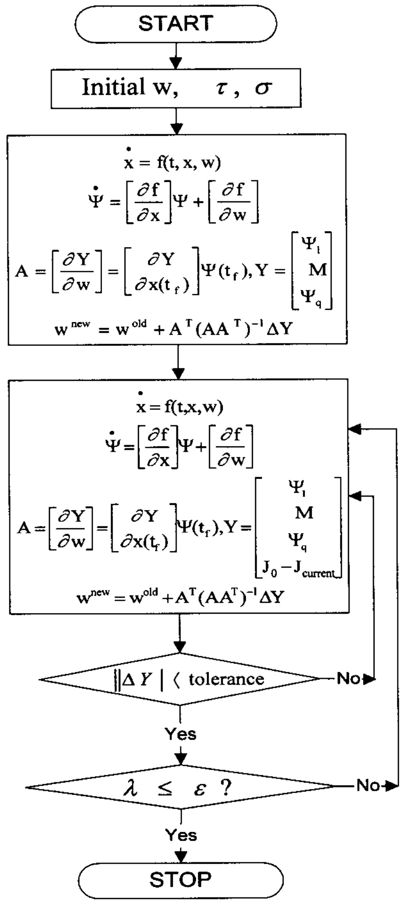

Fig. 3. Flowchart of the RBFs direct optimization algorithms.

we form the augmented Jacobian and we let A_i denote the ith column of the A matrix. The A_i vector is the gradient of the constraint and performance with respect to w_i. We adopt a positive measure of the sensitivity with respect to the i th parameter as $S_i = A_i^T A_i$, and then we introduce a new radial basis function between the most sensitive maximum S_i current radial basis center location and the more sensitive of the two adjacent centers. Fig. 2 illustrates this heuristic approach. In the figure, for example, the sensitivity of the fourth radial basis function is the greatest and the fifth sensitivity is greater than the third. Thus, an additional radial basis function is added in between the fourth and fifth radial basis function. Further notice that we can reduce the parameter proportional to the spacing of the more dense center locations, this will permit sharp localized behavior to be represented more efficiently. Notice important regions of t will "attract" new RBF until the representation of $u(t)$ is locally "shared" by many relatively dense RBFs. We find the sensitivity with respect to any one of these more dense RBFs eventually decreases and then other regions will attract the new adaptively located RBF. With each newly added radial function we increase λ to obtain a new J_o, optimize the weights vector, and follow the same procedure from thereafter. We stop the algorithm when the increase of the performance index is negligible (following the optimization iterations after introduction of each new RBF). We can represent the algorithms in the flowchart shown in Fig. 3.

III. Application

As a multi-variable control problem we consider an aeroassisted plane-change maneuver[13]. A typical aeroassisted plane-change maneuver is depicted in Fig. 4.

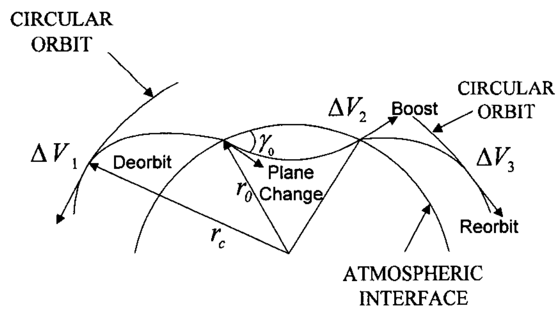

Fig. 4. Aeroassisted plane-change maneuver.

Consider the objective of a transferring from a circular orbit to another circular orbit in a different plane. The maneuver requires three impulses, where each thrust maneuver is idealized as a velocity impulse, ΔV. Initially, the vehicle is in a circular orbit well outside of the Earth's atmosphere. The first impulse, ΔV_1, is applied at the orbital altitude, tangent to the flight path and opposite to the velocity vector. This impulse causes the vehicle to follow an elliptic orbit and enter the atmosphere. The attitude is to be controlled to optimally maneuver the vehicle during the atmospheric portion of the flight to accomplish a maximum change in orbit inclination. Upon entering the atmosphere, the vehicle modulates, via attitude control, the aerodynamic forces to perform the plane

Table 1. The constants of the earth characteristics.

Constant	Value	Dimension
Earth radius r_s	2092642	ft
Gravity acceleration at sea level g_s	32.174	ft/sec^2
Angular velocity ω	0.00007292115	rad/sec

change. The atmospheric turn maneuver is terminated at the point where the vehicle exits the atmosphere. Because of the loss of energy during the atmospheric maneuver, a second impulse, ΔV_2, is required to boost the vehicle to an apogee altitude which is the same as the desired circular altitude. Finally, a third impulse, ΔV_3, is applied at this apogee altitude in the direction of the apogee velocity to re-circularize the orbit. The total ΔV required to complete the maneuver is the sum of ΔV 's from the impulses. The model used in this problem consists of the deorbit equations, the equations of motion during atmospheric flight, and the boost and reorbit equations which will be presented in the following sections.

1. Deorbit equations

Initially, the orbit is in circular orbit at the radius $r_c(=r_s+100nm)$ and the velocity $V_c=(GM/r_c)^{1/2}$ where GM is the gravitational constant of the Earth and r_s is a reference earth radius. Deorbit is accoplished by the impulse ΔV_1 which causes the vehicle to enter the atmosphere at radius r_0 with velocity V_0 and flight path inclination γ_0. For given values of r_c, V_c, ΔV_1, and r_0, theequations for conservation of energy and angular momentum lead to the following expressions for V_0 and γ_0:

$$V_0^2=(V_c-\Delta V_1)^2-2GM\left(\frac{1}{r_c}-\frac{1}{r_0}\right) \quad (27)$$

$$\gamma_0=-\cos^{-1}\left|\frac{r_c(V_c-\Delta V_1)}{r_0 V_0}\right| \quad (28)$$

In order to ensure atmospheric entry, ΔV_1 must exceed the minimum value

$$\Delta V_{1_{\min}}=V_c-\left|\frac{2(V_c^2-GM/r_0)}{1-(r_c/r_0)^2}\right|^{\frac{1}{2}} \quad (29)$$

2. Atmospheric flight equations

The equations of motion[19] are for gliding flight over a nonrotating spherical Earth. If the vehicle is moving from west to east, and if a positive bank angle generates a heading toward the north, these equations are given by

$$\dot{r}=V\sin\gamma \quad (30)$$

$$\dot{V}=-\frac{D}{m}-g\sin\gamma \quad (31)$$

$$\dot{\phi}=\frac{V\cos\gamma\sin\psi}{r} \quad (32)$$

$$\dot{\theta}=\frac{V\cos\gamma\cos\psi}{(r\cos\phi)} \quad (33)$$

$$\dot{\gamma}=\frac{L\cos\mu}{(mV)}+\left(\frac{V}{r}-\frac{g}{V}\right)\cos\gamma \quad (34)$$

$$\dot{\psi}=\frac{L\sin\mu}{(mV\cos\gamma)}-\frac{V}{r}\cos\gamma\cos\psi\tan\phi \quad (35)$$

Here, θ is the longitude, ϕ is the latitude, r is the radial distance from the center of the Earth to the vehicle center of mass, V is the velocity, γ is the flight path angle, ψ is the heading angle, m is the mass, L is the Lift, D is the drag, and μ is the bank angle. The control variables in this application are the angle of attck (α) presented in drag (D) implicitly and the bank angle (μ). The coordinate systems are illustrated in Fig. 5. The earth is assumed to be a sphere whose radius represents mean sea level and is denoted by r_s .

The acceleration of gravity is given by

$$g=g_s\left(\frac{r_s}{r}\right)^2 \quad (36)$$

where g_s is the acceleration of gravity at sea level. The altitude of the vehicle is

$$h=r-r_s \quad (37)$$

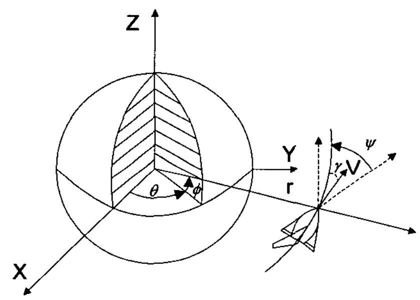

Fig. 5. Coordinate systems.

In order to obtain atmospheric properties as a function of altitude, we adopt an approximate atmosphere (the 1962 U.S. Standard Atmosphere), with the additional assumption that the composition of the atmosphere is constant[13], [20]. The drag and lift forces are modeled using the drag coefficient (C_D) and the lift coefficient (C_L) as follows :

$$D=\left(\frac{1}{2}\right)\rho SV^2C_D,\ L=\left(\frac{1}{2}\right)\rho SV^2C_L \quad (38)$$

where $S\,(125.84ft^2)$ is the aerodynamic reference area. The drag and lift coefficients can be expressed in terms of the axial (C_A) and normal (C_N) force coefficients as ω

$$C_D=C_A\cos\alpha+C_N\sin\alpha,\ C_L=C_N\cos\alpha-C_A\sin\alpha \quad (39)$$

where α is the angle of attack. The osculating orbit inclination ($0 \leq i \leq 180^\circ$) is given by inverting the relation

$$\cos i = \cos\phi\cos\psi \tag{40}$$

Note that the osculating orbit inclination varies through the atmospheric segment and must have the required value at exit, since all exoatmospheric velocity are assumed to be in-plane maneuvers.

3. Boost and reorbit equations

The vehicle exits the atmosphere at $r_f = r_0$, V_f, and γ_f. At this point, the impulse ΔV_2 is applied to raise the apogee of the ascending elliptical orbit to $r_a = r_c$. Conservation of energy and conservation of angular momentum lead to the following expressions for ΔV_2 and V_a (Velocity at apogee) :

$$\Delta V_2 = \left| \frac{2GM(1/r_f - 1/r_c)}{1-(r_f/r_c)^2\cos^2\gamma_f} \right|^{\frac{1}{2}} - V_f \tag{41}$$

$$V_a = \frac{r_f}{r_c}\left| \frac{2GM(1/r_f - 1/r_c)}{1-(r_f/r_c)^2\cos^2\gamma_f} \right|^{\frac{1}{2}} \cos\gamma_f \tag{42}$$

At apogee, the final impulse ΔV_3 is used to increase the velocity to ΔV_c and is given by

$$\Delta V_3 = V_c - V_a \tag{43}$$

The performance index to be minimized is taken as the total velocity impulse

$$J = \Delta V_1 + \Delta V_2 + \Delta V_3 \tag{44}$$

The vehicle used in this study is the MRRV (Maneuverable Reentry Research Vehicle) whose characteristics are presented in references[19] and [20]. We adopt these characteristics to facilitate comparison to his historical solutions. The actual computations are performed in terms of nondimensional variables. These are the effect of scaling all the variables and tends to improve the optimization convergence process. The radius of the

earth, r_s, is used as the reference length ; $(r_s/g_s)^{1/2}$ is the reference time; and m_0 is the reference mass. The non-dimensional variables are

$$\tilde{t} = \frac{t}{(r_s/g_s)^{1/2}} \tag{45}$$

$$\tilde{r} = \frac{r}{r_s} \tag{46}$$

$$\tilde{V} = \frac{V}{(r_s \cdot g_s)^{1/2}} \tag{47}$$

$$\tilde{D} = \frac{D}{m_0 \cdot g_s} \tag{48}$$

$$\tilde{L} = \frac{L}{m_0 \cdot g_s} \tag{49}$$

IV. Results

We have similar trends of control variables and the states of the system in evenly and adaptively spaced RBFs direct optimization algorithms. Thus, we

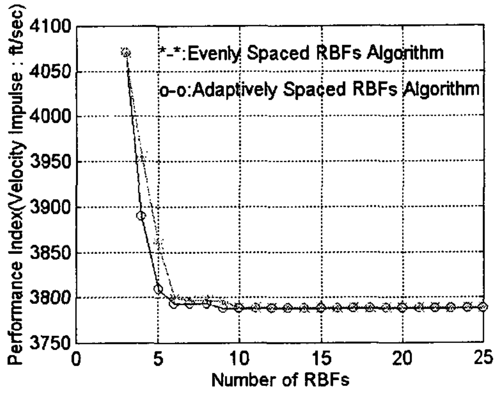

Fig. 6. Comparison of performance index for evenly and adaptively spaced RBFs algorithms.

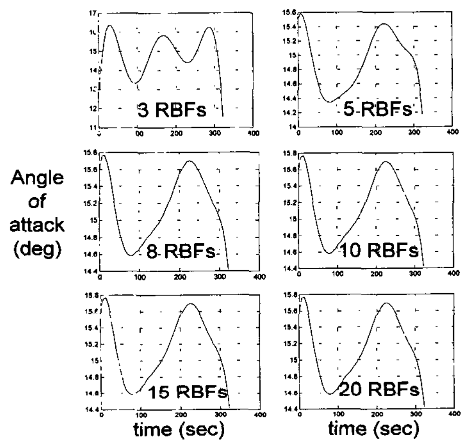

Fig. 7. Control variable(angle of attack).

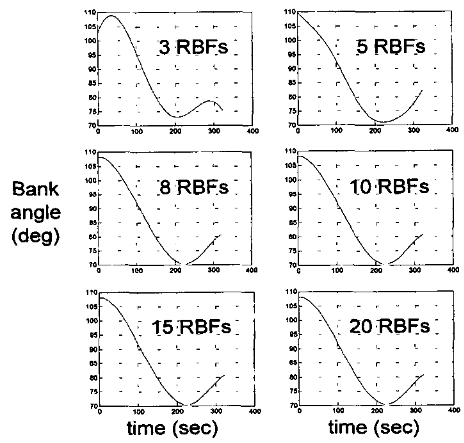

Fig. 8. Control variable(bank angle).

compare those two algorithms with the performance indexes in Fig. 6. The results of control variables and the states of the system are presented in Fig. 7, Fig. 8 and Fig. 9. We notice a clear advantage to adaptively spaced RBF algorithm for $N \leq 9$. For $N=9$, it is an excellent sub-optimal solution in Fig. 6. And we observe that "qualitative convergence" of the optimal aeroassisted planechange occurs early, and for more than five RBFs, there is little visible change in control variables, i.e. angle of attack and bank angle in Fig. 7 and Fig. 8. The states of the system is shown in Fig. 9. Considering from Fig. 6 to Fig. 8 we recognize that an optimal number of radial basis functions may be used to parameterize the control variables of the system while satisfying the terminal conditions.

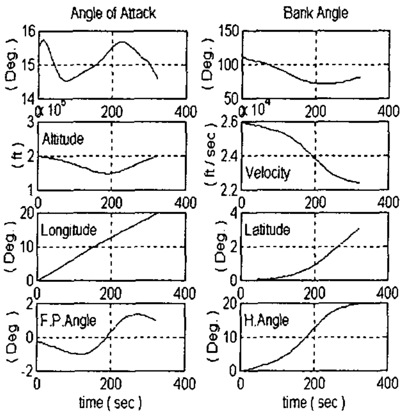

Fig. 9. Control variables and states using adaptive RBF algorithm.

V. Conclusions

The adaptively spaced RBF(Radial Basis Functions) direct optimization algorithm shows significantly more efficient convergence with respect to performance index than the evenly spaced RBF direct optimization algorithm, especially for a small number of basis functions. The adaptively spaced RBFs.

direct optimization algorithm converges more rapidly to the final performance index with small number of RBFs than evenly spaced RBFs direct optimization algorithm. The reason is that our adaptive location of basis functions places the functions where they are needed to best model the control variables. Here, we have the conclusions : the RBF direct optimization algorithms investigated efficiently parameterize function space optimal control problems, two variations (evenly spaced centers and adaptively spaced centers) are studied, a minimum norm nonlinear programming algorithm is used to iteratively adjust RBF weights, these ideas are applied to a multi-control variable optimal control problem.

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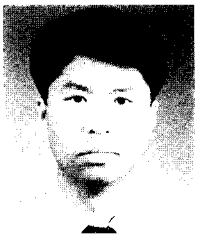

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