Exponential Stability and Feasibility of Receding Horizon Control for Constrained Systems

구속조건이 있는 시스템에 대한 이동구간제어기의 지수 안정성과 실현 가능성

Jae-Won Lee and Wook Hyun Kwon
(이 재 원, 권 욱 현)

요 약 : 본 논문에서는 입력과 상태변수에 hard constraint가 있는 이산시간 시스템에 대한 새로운 이동구간 제어기를 제안한다. 제안된 이동구간제어기가 terminal ellipsoid constraint를 이용하여 지수안정성(exponential stability)을 보장함을 보인다. Feasibility를 증가시키기 위하여 시스템을 안정 모드와 불안정 모드로 바꾸는 방법, constraint를 완화하는 방법을 제안한다. Constraint를 완화할 때 constraint를 만족시키지 못하는 부분의 성능을 개선시키는 방법을 제안하고 예제를 통해서 몇 가지 제안한 방법들을 비교한다.

Keywords: discrete time-varying system, receding horizon control, constraints

I. Introduction

Most practical systems have some constraints on inputs, states, and outputs. For example, actuators, valves, pumps, and compressors which have been widely used as input devices have their limits in the operating region. Drum pressure, temperature, and rpm (revolutions per minute) which may be expressed as outputs or states for boiler-turbine control systems must not exceed their upper bounds for safety. The receding horizon control has been emerged as a powerful strategy for such constrained systems with limitations on inputs, states, and outputs [2,3,5,6,7,8,9,10]. Especially, the stability and feasibility issue of the receding horizon control for constrained systems has been focused on in recent literatures [3,5,6,9,10]. Originally, the terminal equality constraint for full states was first introduced in[2] in order to guarantee stability of the receding horizon control for unconstrained systems. Even if this approach can be directly applied to constrained systems, the terminal equality constraint may decrease feasibility or need much computational burden. In [5,6,10], constrained receding horizon control schemes with the relaxed terminal equality constraint have been proposed. This artificial constraints are satisfied by putting the unstable modes of the states instead of full states to the origin at the finite terminal time. However, this relaxed terminal equality constraint is still restrictive because the equality constraint rather than the inequality constraint should be satisfied, and hence the horizon size may have to be made large so as to make the problem feasible. In order to overcome this drawback, the mixed constraints have been introduced in [10]. However, only attractivity property rather than asymptotic or exponential stability has been shown in these results. It is noted that constrained systems are nonlinear and hence precise statements for stability are required. In [9], the infinite horizon LQR for constrained systems is proposed. While finite horizon problems have been considered in most of results on the constrained receding horizon controller, the infinite horizon problem is also considered in [9]. The idea is to put the states at a finite time step to a ball in which there exists the conventional state feedback LQR satisfying input and state constraints. However, it requires long horizon size to put the states into the ball and hence much computational burden. The motivation of this work is to show the exponential stability of a new receding horizon controller for constrained systems and consider how to increase feasibility while guaranteeing the exponential stability. It is noted that most conventional results on constrained receding horizon controllers showed only their attractivity property.

In order to make optimization problems of the conventional results feasible, long horizon size is needed, since the terminal equality constraint and the terminal LQR constraint are restrictive. It is also noted that as horizon size increases, the computational burden also increases. In this paper, we consider the finite horizon problem with finite terminal weighting matrices. For the artificial constraint to guarantee stability, we introduce the invariant ellipsoid constraint which is less restrictive than the conventional terminal equality constraint. The basic concept of this artificial constraint is to put the terminal state into the ellipsoid which is invariant for the constrained system using a linear state feedback

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이재원 : 삼성종합기술원, systems and control sector
권욱현 : 서울대학교, 전기공학부

gain. It is shown that the proposed receding horizon controller guarantees exponential stability of the closed loop system for all feasible initial states sets. We also propose some implementable versions of the proposed controller, where feasibility is improved by introducing system partition and constraints softening. In these cases, feasibility can be improved since only unstable modes are required to be put into the ellipsoid and only input constraints are considered by softening state constraints. It is also shown that the exponential stability in these cases is guaranteed. An illustrative example is included to compare some constraints softening methods.

This paper is organized as follows : In Section 2, a new receding horizon controller is proposed for the systems with hard constraints on the input and the state. In Section 3, some implementable versions of the proposed controller in Section 2 are proposed. In Section 4, an illustrative example is followed. And Section 5 concludes this paper.

II. Constrained receding horizon control

Throughout this paper, we will consider the following time-invariant discrete linear system with input and state constraints :

$$x_{k+1}=Ax_k+Bu_k \tag{1}$$

subject to

$$\begin{cases} u^-\le u_k\le u^+, & k=0,1,\cdots,\infty \\ g^-\le Gx_k\le g^+, & k=0,1,\cdots,\infty, \end{cases} \tag{2}$$

where $u^-,u^+\in R^m, G\in R^{n_g\times n}$, and $g^-,g^+\in R^{n_g}$. It is assumed that $u_k=0$ and $Gx_k=0$ satisfy the constraint (2), in other words, all elements of u^- are negative and all elements of u^+ are positive. Note that the output constraint $y^-\le y_k\le y^+$ can be expressed as the state constraint in (2), because the output equation is generally expressed as $y_k=Cx_k$. We denote U as the feasible set for the above input constraint and X as the feasible set for the above state constraint. It is noted that since the constraint (2) is linear and include the origin as an interior point, U and X are polyhedra and therefore convex. Denoting that $x_{k+i|k}$ and $u_{k+i|k}$ are predicted variables at the time k with $x_{k|k}=x_k$, we define the following finite horizon cost function which should be optimized at every current time k :

$$J(x_k,k) = \sum_{i=0}^{N-1}(x_{k+i|k}{}'Qx_{k+i|k}+u_{k+i|k}{}'Ru_{k+i|k}) + x_{k+N|k}{}'\Psi x_{k+N|k}, \tag{3}$$

where $Q>0, R>0, \Psi>0$, N is a finite positive integer. We assume that the terminal weighting matrix Ψ satisfies the following inequality condition :

$$\Psi\ge(A+BH)'\Psi(A+BH)+Q+H'RH, \tag{4}$$

where H is a free parameter. First, we introduce a receding horizon controller for unconstrained systems which was proposed in [4] and is defined as the first solution of the following optimization problem :

$$\underset{u_{k|k},\cdots,u_{k+N-1|k}}{Minimize} \quad J(x_k,k). \tag{5}$$

Then, the closed-loop stability of this receding horizon controller is guaranteed by the following theorem.

Theorem 1 [4] : Suppose that the inequality condition (4) is satisfied. Then the receding horizon control $u_k=u^*_{k|k}$ where $u^*_{k+i|k}$ $i=0,\cdots,N-1$ is the optimal solution of the optimization problem (5), exponentially stabilizes the system (1).

Before stating main results, we introduce so-called invariant ellipsoid property which can be interpreted in terms of quadratic stability [1]. Suppose that there exists a $K\in R^{m\times n}$ and a positive definite matrix $P\in R^{n\times n}$ such that

$$(A+BK)'P(A+BK)-P<0. \tag{6}$$

And define an ellipsoid E_P centered at the origin :

$$E_P=\{\xi\in R^n|\xi'P\xi\le1\}. \tag{7}$$

Then, for every initial state $x_0\in E_P$, the state trajectory $x_k>0(\forall k>0)$ with the state feedback control $u_k=Kx_k$ remains in the ellipsoid E_P. Based on this property, we introduce the following lemma for the stability of the system (1) subject to the constraint (2) with a state feedback controller.

Lemma 1 : Suppose that $P>0$ and K satisfying (6) also satisfy the following LMIs for some Z and V with $X=P^{-1}$ and $Y=KX$:

$$\begin{bmatrix} Z & Y \\ Y' & X\end{bmatrix}\ge0,\ Z_{jj}\le \bar{u}_j^{2,}\ j=1,2,\cdots,m \tag{8}$$

$$GXG'\le V,\ V_{jj}\le \bar{g}_j^{2,}\ j=1,2,\cdots,n_g \tag{9}$$

where $\bar{g}_j$ and $\bar{u}_j$ are defined by $\bar{g}_j=\min(-g_j^-,g_j^+)$ and $\bar{u}_j=\min(-u_j^-,u_j^+)$. And g_j^-,g_j^+,u_j^- and u_j^+ are the jth elements of g^-,g^+,u^- and u^+, respectively. Then the state feedback controller $u_k=Kx_k$ exponentially stabilizes the system for all $x_0\in E_P$ while satisfying the constraint (2). And the resultant state trajectory x_k always remains in the region E_P.

Proof : The proof is a simple extension of the result in [1] which is based on continuous time systems. Since P satisfies (6), we can easily show that K is an exponentially stabilizing feedback gain. Now, we should show that the resultant state trajectory remains inside the ellipsoid while satisfying the constraints (2). First, assume that $x_k'Px_k\le1$. Then

$$\begin{aligned} x_{k+1}{}'Px_{k+1} &= x_k'(A+BK)'P(A+BK)x_k \\ &< x_k'Px_k \le 1. \end{aligned}$$

Since $x_0'Px_0\le1$ by hypothesis, we obtain

$$x_1'Px_1 = x_0'(A+BK)'P(A+BK)x_0 < x_0'Px_0 \le 1.$$

Hence, we conclude that the state remains inside the ellipsoid by induction. Now we will investigate if the constraint (2) is satisfied. By hypothesis, u_k can be represented as $u_k = Kx_k = YX^{-1}x_k$. Then, it holds

$$\max_{k\ge 0}|u_k|_i^2 = \max_{k\ge 0}|YX^{-1}x_k|_i^2$$

that

$$\le \max_{x\in E_P}|YX^{-1}x|_i^2 = (YX^{-1}Y')_{ii},$$

where $|u_k|_i$ denotes the absolute value of the ith element of u_k. Therefore, the input constraint $u^- \le u_k \le u^+$ is guaranteed by the LMI (8). The state constraint can be checked with similar procedure. ■

Now using the property of the invariant ellipsoid, we propose a new receding horizon controller which satisfies the constraint on the input and the state, and guarantees exponential stability of the closed loop system. In order to guarantee the closed loop stability, we add an artificial invariant ellipsoid constraint at the terminal time step. Consider the following optimization problem subject to the additional artificial constraint of invariant ellipsoid :

$$\underset{u_{k|k},\cdots,u_{k+N-1|k}}{Minimize} \quad J(x_k, k) \tag{10}$$

$$\text{subject to} \begin{cases} u^- \le u_{k+i|k} \le u^+, & i=0,1,\cdots N-1 \\ g^- \le Gx_{k+i|k} \le g^+, & i=0,1,\cdots,N \\ x'_{k+N|k}\Psi x_{k+N|k} \le 1 \end{cases} \tag{11}$$

where we assume that the terminal weighting matrix Ψ satisfies the following assumption.

Assumption 1 : The finite terminal weighting matrix Ψ satisfies the following LMIs with $X = \Psi^{-1}(>0)$:

$$\begin{bmatrix} X & (AX+BY)' & (Q^{1/2}X)' & (R^{1/2}Y)' \\ (AX+BY) & X & 0 & 0 \\ Q^{1/2}X & 0 & I & 0 \\ R^{1/2}Y & 0 & 0 & I \end{bmatrix} > 0,$$

$$\begin{bmatrix} Z & Y \\ Y' & X \end{bmatrix} \ge 0, Z_{jj} \le \bar{u}_j^2, \quad j=1,2,\cdots,m$$

$$GXG' \le V, V_{jj} \le \bar{g}_j^2, \quad j=1,2,\cdots,n_g$$

where X, Y, Z and V are variables which should be found.

It can be easily verified that Ψ satisfying Assumption 1 also satisfies (4) with $H = YX^{-1}$ and hence $A+BH$ is stable. Then we define a new receding horizon controller as the first optimal solution of the above constrained optimization problem with the terminal weighting matrix satisfying Assumption 1. We also define the feasible initial states set of the optimization problem (10) :

$$F(\Psi, N) = \{x_0 \in R^n \mid \exists\ u_i \in U,\ i=0,\cdots N-1, \text{ such that } x_{i+1} \in X \text{ and } x_N \in E_\Psi\}$$

It is noted that $F(\Psi, N)$ contains an open neighborhood of the origin for sufficiently large N, since X and U include the origin as an interior point. In order to show the exponential stability of the proposed receding horizon controller, we need the following lemma :

Lemma 2 : Suppose that $x_k \in F(\Psi, N)$. Then there exist $\kappa > 0$ and $u_{k+i|k} \in U$, $i=0,\cdots,N-1$ such that $|u_{k+i|k}|^2 \le \kappa |x_k|^2$, $x_{k+i|k} \in X$, $i=0,\cdots,N-1$ and $x_{k+N|k} \in E_\Psi$.

Proof : We consider the case that $x_k \ne 0$ since $x_k = 0$ gives the trivial solution $u_{k+i|k} = 0$. Let $B(\gamma)$ be a closed ball with a radius $\gamma > 0$ such that $B(\gamma) \subset F$. If $x_k \in B(\gamma)$, define $\alpha(x_k) \in [1, \infty)$ such that $\alpha(x_k)x_k \in \partial B(\gamma)$ where $\partial B(\gamma)$ denotes the boundary of $B(\gamma)$. Otherwise, define $\alpha(x_k) \in [1, \infty)$ such that $\alpha(x_k)x_k \in F - B(\gamma)$ Then there exists a control sequence $\hat{u}_{k+i|k} \in U$ which drives $\alpha(x_k)x_k$ into the ellipsoid E_Ψ in N steps while satisfying the state constraint. Since the system is linear $\frac{1}{\alpha(x_k)}\hat{u}_{k+i|k} \in U$ drives x_k into E_Ψ while satisfying the state constraint. Denoting $\bar{u} = \max\{|u^-|_\infty, |u^+|_\infty\}$, we obtain $|\hat{u}_{k+i|k}|^2 \le m\bar{u}^2$. Hence, the following inequality holds

$$\left|\frac{1}{\alpha(x_k)}\hat{u}_{k+i|k}\right|^2 \le \frac{1}{\alpha(x_k)^2} m\bar{u}^2 \le \frac{m\bar{u}^2}{\gamma^2}|x_k|^2,$$

which completes the proof. ■

Now, we are ready to state our main results on the exponential stability.

Theorem 2 : Suppose that Assumption 1 holds. Then the optimization problem minimizing $J(x_k, k)$ subject to the constraint (11), is always feasible for all $k \ge 0$ and for all initial states $x_0 \in F(\Psi, N)$. Also, $x_k = 0$ is the exponential stable equilibrium of the closed-loop system with the receding horizon controller stemming from this optimization problem, for all initial states $x_0 \in F(\Psi, N)$.

Proof : Suppose that there exists the optimal solution $u^*_{k+i|k}$ at the current time k and let $H = YX^{-1}$. Then, at the next time step $k+1$, consider the following control sequence :

$$\left.\begin{aligned} u_{k+i|k+1} &= u^*_{k+i|k} \quad i=1,2,\cdots,N-1, \\ u_{k+N|k+1} &= Hx_{k+N|k+1} \end{aligned}\right\}. \tag{12}$$

Then the above control sequence gives a feasible solution for the optimization problem (10) subject to (11) at the next time step $k+1$, because $x^*_{k+N|k}(= x_{k+N|k+1})$ remains in E_Ψ and the control input $u_{k+N|k+1} = Hx_{k+N|k+1}$ satisfies the invariant ellipsoid property. Hence, by induction, we observe that the optimization problem is always feasible for all

k and every initial state $x_0 \in F(\Psi, N)$. In order to show the exponential stability of the closed loop system, we will show that there exist $a, b, c > 0$ such that

$$|x_k|^2 \le J^*(x_k, k) \le b|x_k|^2, \quad \Delta J^* < -c|x_k|^2, \tag{13}$$

and hence $J^*(x_k, k)$ serves as a Lyapunov functional for the exponential stability. We can easily show that the following inequalities holds :

$$J^*(x_k, k) \ge x_k' Q x_k \ge \lambda_{\min}(Q)|x_k|^2. \tag{14}$$

From Lemma 2, there exists a feasible $\tilde{u}_{k+i|k}$, $i = 0, \cdots N-1$ and $x > 0$ such that $|u_{k+i|k}|^2 \le x|x_k|^2$. Denoting $\tilde{x}_{k+i|k}$ as the resultant state trajectory with this control sequence, we obtain

$$\begin{aligned} J^* &\le \sum_{i=0}^{N-1} (\tilde{x}'_{k+i|k} Q \tilde{x}_{k+i|k} + \tilde{u}'_{k+i|k} R \tilde{u}_{k+i|k}) \\ &\quad + \tilde{x}'_{k+N|k} \Psi \tilde{x}_{k+N|k} \\ &\le [(N+1)\hat{A}^2 (1 + N\|B\|\sqrt{x})^2 \max\{\lambda_{\max}(Q), \\ &\quad \lambda_{\max}(\Psi)\} + xN\lambda_{\max}(R)]|x_k|^2, \end{aligned} \tag{15}$$

where $\hat{A} := \max_{i=1,\cdots,N}\|A^i\|$. Next, let $\bar{J}$ be the cost value when control sequence (12) is implemented at the next time step $k+1$. Then the following inequality is easily derived :

$$\begin{aligned} J^*(x_k, k) &\ge x_k' Q x_k + u_k' R u_k + \bar{J} \\ &\ge x_k' Q x_k + u_k' R u_k + J^*(x^*_{k+1|k}, k+1), \end{aligned}$$

which can be represented as

$$\Delta J^*(x_k) \le -x_k' Q x_k - u_k' R u_k \le -\lambda_{\min}(Q)|x_k|^2. \tag{16}$$

Hence the inequalities (14), (15), and (16) show that there exist $a, b, c > 0$ such that the inequality (13) is satisfied. ■

III. Feasibility and implementation

Generally, input constraints are imposed by physical limitations of actuators, valves, pumps, and etc., while state constraints are often desirable. There are often cases that state constraints cannot be satisfied all the time and hence some violations of state constraints are allowable. Especially, even if state constraints are satisfied in nominal operations, unexpected disturbances may put the states aside from the feasible region where state constraints are satisfied. In this case, it may happen that some violations of state constraints are unavoidable, while input constraints can be still satisfied [6,10]. Hence, if some violations of state constraints are allowed, we can guess larger feasible initial sets. Moreover, if the terminal ellipsoid constraints are relaxed, we can guess much larger feasible initial sets. In this section, we propose some methods to increase feasibility of the proposed receding horizon controller by introducing mixed constraints and system partition. The mixed constraints (We will represent the hard state constraint $g^- \le Gx_k \le g^+$ as $Gx_k \le g$, which can be easily derived by modifying G) are given by

$$\begin{cases} u^- \le u_k \le u^+, & k = 0, 1, \cdots, \infty \\ Gx_k \le g + \varepsilon_k, & k = 0, 1, \cdots, \infty \end{cases} \tag{17}$$

and $\varepsilon_k \ge 0$ denotes tolerance for violation of state constraints. In order to enlarge feasibility, we partition the system matrix A into stable and unstable modes as follows :

$$A = VJV^{-1} = [V_u \ V_s] \begin{bmatrix} J_u & 0 \\ 0 & J_s \end{bmatrix} \begin{bmatrix} \hat{V}_u \\ \hat{V}_s \end{bmatrix} \tag{18}$$

where J_u's eigenvalues are unstable and J_s's eigenvalues are stable. Then the unstable mode $z^u = \hat{V}_{ux}$ satisfies

$$z^u_{k+1} = J_u z^u_k + \hat{V}_u B u_k. \tag{19}$$

Now, the idea is to put only unstable mode into the ellipsoid instead of full states. We assume that there exist $X_u > 0$ and Y_u which satisfy the following couple of LMIs and denote that $\Psi_u = X_u^{-1}$ and $H_u = Y_u \Psi_u$.

Assuption 2 : There exists $\Psi_u (> 0)$ which satisfies the following LMIs :

$$\begin{bmatrix} X_u & (J_u X_u + B_u Y_u)' & (Q_u^{1/2} X)' & (R^{1/2} Y_u)' \\ (J_u X_u + B_u Y_u) & X_u & 0 & 0 \\ Q_u^{1/2} X_u & 0 & I & 0 \\ R^{1/2} Y_u & 0 & 0 & I \end{bmatrix} > 0, \tag{20}$$

$$\begin{bmatrix} Z & Y_u \\ Y_u' & X_u \end{bmatrix} \ge 0 \quad Z_{ii} \le u_j^2, \quad j = 1, 2, \cdots, m \tag{21}$$

where $X_u = \Psi_u^{-1}, B_u = V^{-1}B$, and $Q_u = V_u' Q V_u$. Since J_s is stable, there exists Ψ_s satisfying the following inequality :

$$\Psi_s \ge J_s' \Psi J_s + V_s' Q V_s.$$

Now denoting

$$\begin{aligned} \Psi &= V \begin{bmatrix} \Psi_u & 0 \\ 0 & \Psi_s \end{bmatrix} V^{-1} \\ H &= [H_u \ 0] V^{-1}, \end{aligned} \tag{22}$$

it can be easily verified that Ψ and H denoted above satisfy the inequality condition (4). Using Ψ in (22) as the terminal weighting matrix and introducing a cost function $S(\varepsilon(k))$ for violations of state constraints, we modify the cost function and the corresponding optimization problem as

$$J_\varepsilon(x_k, k) = J(x_k, k) + S(\|\varepsilon(k)\|) \tag{23}$$

$$\underset{u_{k|k}, \cdots, u_{k+N-1|k}, \varepsilon(k)}{Minimize} \quad J_\varepsilon(x_k, k) \tag{24}$$

subject to

$$\begin{cases} u^- \le u_{k+i|k} \le u^+, & i = 0, 1, \cdots N-1 \\ Gx_{k+i|k} \le g + \varepsilon_{k+i|k}, & i = 1, \cdots, N \\ G(A+BH)^j x_{k+N|k} \le g + \varepsilon_{k+N+j|k}, & j = 1, \cdots, \infty \\ \varepsilon_{i|k} \ge 0 & i \ge k \\ x_{k+N|k}' \Psi x_{k+N|k} \le 1 \end{cases} \tag{25}$$

where $\tilde{\Psi}=\tilde{V}'_u\Psi_u\tilde{V}_u$ $\varepsilon(k)=\{\varepsilon_{k+i|k}\geq 0,\ i=1,\cdots N_\varepsilon\}$, N_ε is chosen to satisfy $\varepsilon_{k+j|k}=0$ for $j>N_\varepsilon$. It is well known that such an N_ε indeed exists because $A+BH$ is stable [6]. We can observe the terminal ellipsoid constraint in this optimization is relaxed from the optimization problem proposed in the optimization in the previous section, because we only have to put the unstable modes instead of full states into the ellipsoid. We also define the feasible initial states set of the optimization problem (24) :

$$F_u(\tilde{\Psi},N)=\{x_0\in R^n \mid \exists\, u_i\in U,\ i=0,\cdots,N-1, \text{ such that } x_N\in E_{\tilde{\Psi}}\}$$

It is noted that $F_u(\tilde{\Psi},N)\supseteq F(\Psi,N)$ because there is only input constraint and the terminal ellipsoid constraint is much relaxed. Before stating the theorem on the exponential stability, we made an assumption on the cost function $S(\varepsilon(k))$.

Assumption 3 : $S(\varepsilon)$ satisfies the following conditions:
-Define $\varepsilon^\infty(k)$ such that the jth element of $\varepsilon^\infty(k)$ denotes $\max_{i=1,\cdots,N_\varepsilon} e_j'\varepsilon_{k+i|k}$, where $e_j=[0\cdots1\cdots0]'$ is the unit vector with nonzero jth element. Then there exists $b_\varepsilon>0$ such that $S(\varepsilon(k))\leq b_\varepsilon|\varepsilon^{\infty(k)}|^2$.
- $S(\varepsilon_1(k))\leq S(\varepsilon_2(k))$, if $\|\varepsilon_1(k)\|\leq\|\varepsilon_2(k)\|$.

Theorem 3 : Suppose that Assumption 2, 3 are satisfied and the terminal weighting matrix Ψ is defined by (22). Then the optimization problem minimizing $J_\varepsilon(x_k,k)$ subject to the constraint (25), is always feasible for all $k\geq 0$ and for all initial states $x_0\in F_u(\tilde{\Psi},N)$. Also, $x_k=0$ is the exponential stable equilibrium of the closed loop system with the receding horizon controller stemming from this optimization problem, for all initial states $x_0\in F_u(\tilde{\Psi},N)$.

Proof : Let $\varepsilon^*_{k+i|k}$ $i=1,\cdots,N_\varepsilon$ be the optimal solution Then the following sequence gives a feasible solutions at the next time step $k+1$:

$$\begin{cases} u_{k+i|k+1} = u^*_{k+i|k} & i=1,\cdots,N-1 \\ u_{k+N|k+1} = Hx_{k+N|k+1}, & \\ \varepsilon_{k+i|k+1} = \varepsilon^*_{k+i|k} & i=1,\cdots,N_\varepsilon \\ \varepsilon_{k+N_\infty+1|k+1} = 0 & \end{cases} \tag{26}$$

Now we will show the exponential stability using the optimal cost value $J^*_\varepsilon(x_k,k)$ as a Lyapunov candidate. Since $S(\varepsilon^*(k))\geq S(\varepsilon(k+1))$ with the sequence (26), we can easily obtain the following inequality :

$$J_\varepsilon^*(x_k,k)\geq x_k'Qx_k+u'_kRu_k+J_m^*(x_{k+1},k+1)$$

which shows that there exists $c>0$ such that $\Delta J^*_\varepsilon(x_k,k)\leq -c|x_k|^2$. And we can easily show that there exists $a>0$ such that $J^*_\varepsilon(x_k,k)\geq a|x_k|^2$. Now we will show that there exists $b>0$ such that

$$J_\varepsilon^*(x_k,k)\leq b|x_k|^2. \tag{27}$$

As in Lemma 2, we can easily show that there exists a feasible control sequence $\tilde{u}_{k+i|k}$ for $x_k\in F_u(\tilde{\Psi},N)$ such that $|\tilde{u}_{k+i|k}|^2\leq\varkappa|x_k|^2$. The following $\tilde{\varepsilon}$ also gives a feasible solution :

$$\tilde{\varepsilon}_{k+i|k}=\max\{1,\overline{A}\}\hat{A}(1+N\|B\|\sqrt{\varkappa})|x_k|[1\ \cdots\ 1]',$$

where $\overline{A}:=\max_{i=1,\cdots,N_\varepsilon}\|(A+BH)^i\|$. Since S satisfies Assumption 3, we can show that there exists $b>0$ such that (27) is satisfied. ■

Now we consider how to choose $S(\varepsilon)$. Note that the additional cost $S(\varepsilon)$ is included so as to penalize a measure of violation of the state constraint. First, we consider an ∞ norm of $\varepsilon(k)$. In this case, $S(\varepsilon)$ is represented as

$$S(\varepsilon)=\varepsilon^\infty(k)'S\varepsilon^\infty(k), \quad S>0. \tag{28}$$

And then the second and third constraint in (25) can be represented as

$$Gx_{k+i|k}\leq g+\varepsilon^\infty(k),\ i=0,1,\cdots,N$$
$$G(A+BH)^jx_{k+N|k}\leq g+\varepsilon^\infty(k),\ j=1,\cdots,\infty.$$

In view of the optimization problem (24), the decision variables are $u_{k|k},\cdots,u_{k+N-1|k},\varepsilon^\infty(k)$. Hence the size of decision variables are $m\times N+n_g$, which doesn't require much additional computation burden. However, this approach has some drawbacks such as mismatch between open-loop predictions and closed-loop behavior, which can lead to poor performance and tuning difficulties. One of alternatives to overcome these drawbacks is to introduce 2-norm cost for violations.A 2-norm cost for violations can be chosen

$$S(\varepsilon)=\sum_{i=0}^{N}\varepsilon_{k+i|k}'S\varepsilon_{k+i|k}, \quad S>0. \tag{29}$$

In this approach, there is no such a big mismatch between open-loop predictions and closed-loop behavior. Therefore, it is somewhat easy to tune intuitively. However, in this case, the decision variables are $u_{k|k},\cdots,u_{k+N-1|k},\varepsilon_{k|k},\cdots,\varepsilon_{k+N_\infty-1|k}$, and therefore the size of decision variables increases to $mN+n_gN_\varepsilon$, which requires much computational burden. Hence, we conclude that there is a trade-off between two approaches. Some comparisons between two approaches will be presented through an example in the following section.

IV. Illustrative example

In this example, we present the effects of constraints softening which was introduced in the previous section. Consider the output regulation problem of the following discrete linear system :

$$x_{k+1}=\begin{bmatrix}2 & -1.45 & 0.35\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}x_k+\begin{bmatrix}1\\0\\0\end{bmatrix}u_k$$
$$y_k=[-1\ 0\ 2]x_k,$$

where we assume that there is no input constraints and the output constraint is given by

$$-1 \le y_k \le 1, \quad i=1,2,\cdots,\infty.$$

We also assume that $Q=C'C, R=I$ and the initial state is given by $[1.5\ 1.5\ 1.5]'$. We construct two constrained receding horizon controller with the ∞ norm cost and the 2-norm cost for constraint violations, respectively. First, we construct the constrained receding horizon controller with the ∞ norm cost for violation defined by (28). Figure 1 shows the corresponding output responses according to several violation weightings ($S=1,20,50,100$). In this figure, we observe that the peak of violation reduces as violation weighting increases, while duration of violations increases. In other words, increased weighting makes an effect of hardening the state constraint. Hence, we conclude that there is a trade-off between peak and duration of violation. In this figure, we also observe that the responses are not so good when violations appear. Figure 2 shows the mismatch between finite horizon (open-loop) predictions and the receding horizon (closed-loop) responses. We observe that the result is not so satisfactory, because the mismatch is so big. Second, we construct the receding horizon control with the 2-norm cost for violations defined by (29). Figure 3 shows the corresponding output responses according to several violation weightings ($S=1,20,50,100$). In this figure, we also observe that the peak of violation reduces as violation weighting increases as in the case of ∞ norm cost for violation. However, the responses in this case are better than those in the first case. Especially, when the weighting is large ($S=50,100$), the responses are much better than those of Figure 1. Figure 4 shows the mismatch between finite horizon (open-loop) predictions and the receding horizon (closed-loop) responses. In this figure, we observe that the mismatch is much smaller than that of Figure 2.

From these figures, we observe that the violation peak is small in the case of ∞ norm, while duration of violation is small in the case of 2-norm. We can also observe that overall performances including mismatch between finite horizon and receding horizon, are better when the 2-norm cost for violations is adopted. However, the case of 2-norm cost needs much computational burden, because the size of the optimization problems becomes large.

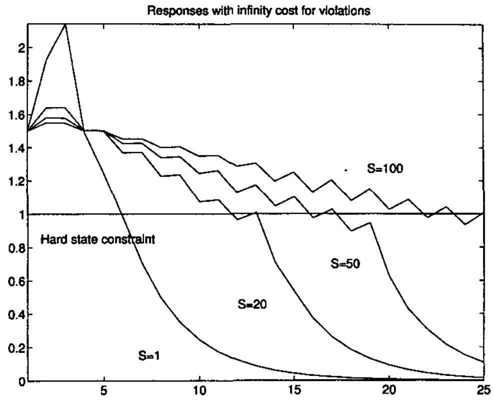

Fig. 1. The case of ∞ -norm cost for violations.

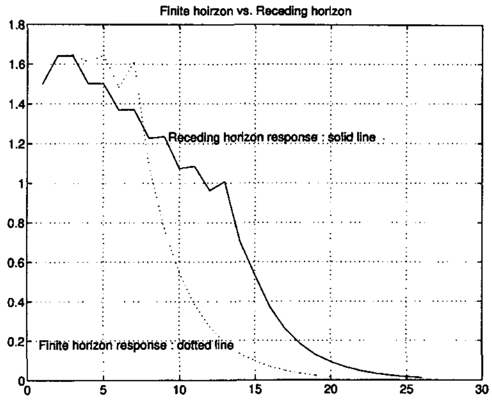

Fig. 2. Mismatch between finite horizon prediction and receding horizon response (the case of ∞ -norm).

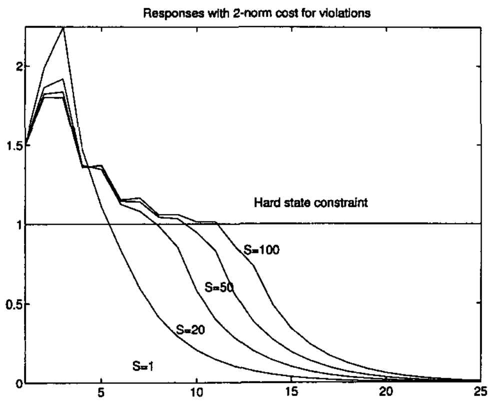

Fig. 3. The case of 2-norm cost of violations.

V. Conclusion

In this paper, a new receding horizon controller for linear systems with input and state constraints, is proposed. It is based on the finite horizon optimization problem with an artificial terminal ellipsoid constraint. It is shown that the proposed controller guarantees exponential stability of the closed loop system for all feasible initial states. It is noted that most of conventional results for constrained receding horizon controller guarantees only attractivity of the closed loop system. Some implementable versions of the proposed controller are proposed so as to improve feasibility and performance by introducing system partition and constraints softening. It is also shown that even these cases guarantees exponential stability. The future work will be to extend the proposed results to the output feedback case and the robust control case. R.

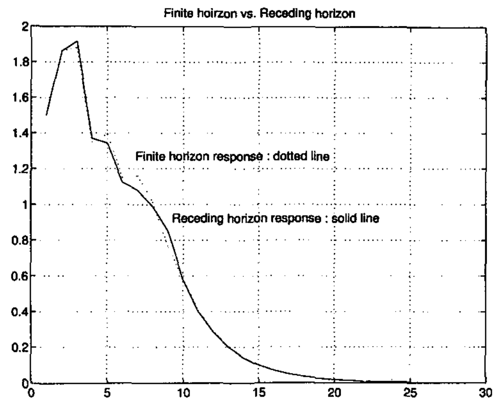

Fig. 4. Mismatch (the case of 2 ∞-norm).

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이 재 원

1991년 서울대 제어계측공학과 졸업. 동대학원 석사,박사 (1993, 1997). Stanford 대학 박사후 과정(1997). 1997.11 -현재 삼성종합기술원. 1996년 휴먼테크 논문대상 은상 수상. 1997년 학술진흥재단 신진연구인력 선정. 1977년 아시아 자동 제어 학술대회 Young Author Prize 수상. 관심분야는 예측 제어, 자동화 및 지능형 차량 시스템.

권 욱 현

제어 · 자동화 · 시스템공학 논문지 제1권, 제1호, 19쪽 참조.