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# Dynamic Response Analysis of Slender Marine Structures under Vessel Motion and Regular Waves
# 파랑 및 부유체 운동을 고려한 세장해양구조물의 동적 거동 해석

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**Abstract** □ Dynamic response analysis is carried out for slender marine structures such as tensioned risers and tethers of tension leg platform, which are subjected to floating vessel motions as well as environmental forces arising from ocean waves. A numerical analysis procedure is developed by using finite element model of the structural member. Dynamic analyses are performed in the time domain for regular waves. Parameter studies are carried out to highlight the effects of surface vessel motions on the lateral dynamics of the structures. Example results of displacements, bending stresses are compared for various cases in water depth, environmental condition and vessel motion. Some instability conditions of the structures due to time-varying tension by vessel heave motion are discussed through the example analyses. As the results, the interaction between vessel surge and heave motions amplifies the total structural response of a riser. In the case of a tether, the effect of vessel heave motion during heavy storm is seemed to be quite significant to lateral response of the structure.

**Keywords :** slender marine structure, vessel motions, regular wave, finite element method

**要 旨** : 해저자원개발에 사용되는 riser나 TLP의 인장각과 같은 세장해양구조물의 파랑 및 상단부유체의 운동에 대한 동적거동해석을 수행하였다. 구조부재의 유한요소모델을 사용한 수치해석기법을 개발하고 규칙파에 대한 시간영역해석을 수행하였다. 본 연구는 상단부유체의 수평 및 수직운동이 구조물의 횡방향거동에 미치는 영향을 분석하였으며, 특히 부유체 수직운동의 영향을 주로하여 패러미터연구를 수행하였다. 수심, 파랑조건 그리고 부유체운동 등 여러경우에 대한 구조물의 변위, 휨응력을 비교검토하였고, 이 해석을 통하여 부유체의 수직운동에 의한 시간변화 인장력으로 야기되는 불안정조건들을 검토하였다. 예제해석결과, 부유체의 수평 및 수직운동의 상호작용으로 riser의 동적응답이 증폭되었다. TLP 인장각의 경우 부유체의 수직운동효과가 구조물의 거동에 상당히 크게 작용하는 것으로 나타났다.

**핵심용어** : 세장해양구조물, 부유체운동, 규칙파, 유한요소법

## 1. INTRODUCTION

Many numerical analyses have been carried out on the dynamics of long-cylindrical pipe, and various experiments were performed to verify the numerical results. Chung *et al.* (1994) reported 3-D coupled response of deep-ocean pipe system to axial bottom vibration and ship motion. This research is highlighted on axial vibration and torsion using 3-D nonlinear finite element model considering large deformation of structural member. Patel and Sorahia (1984) reported finite element analysis of riser in frequency domain and

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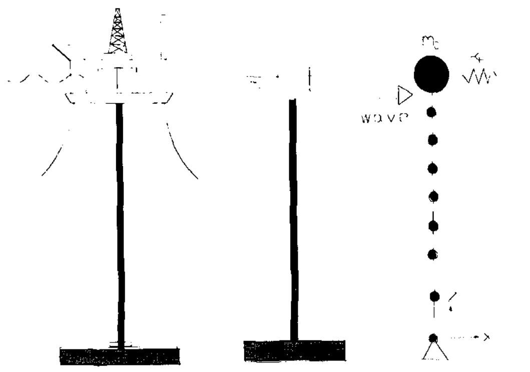

(a) riser (b) tether (c ) finite element model

**Fig. 1.** Slender marine structures and its finite element model.

time domain. This study considered vessel surge motion as well as wave and current loading. On the other hand, the problem of parametric excitation, i.e. time-varying tension variation on the top of structure, has been reported in many articles (Hsu, 1975; HaQuang and Mook, 1987; Thampi and Niedzwecki, 1992; Park, 1994). Park (1994) reported the effect of vessel surge and heave motions on lateral dynamics of TLP tether by using analytic approach of continuous model. Yun *et al.* (1996) discussed the effect of vessel motions as a part of analysis and design methodologies for TLP.

The long-cylindrical marine structures such as risers, TLP tendons as shown in Fig. 1 are subjected to excitations due to vessel surge and heave motions as well as environmental forces. When vessel surge motion is considered together with lateral wave forces in the dynamic analysis of the slender structure, it becomes forced vibrations or a forcing excitation problem. On the other hand, vessel heave motion causes tension variation on the slender structure. When the heave motion, which causes time-varying axial force on the structure, is considered together with lateral wave forces, the resulting motion becomes parametrically excited vibration (the Mathieu stability problem; Stoker, 1950), i.e. structural properties such as stiffness vary with time (Hsu, 1975; Park, 1994). When surge and heave motions of the floating vessel are considered simultaneously, which is more practical, the dynamic behavior of slender structure becomes a combined forcing and parametric excitation problem. The responses of the structures due to combined excitation would be most dominant.

In this study, a numerical technique which may account efficiently for the combined excitation in dynamic analysis of tensioned slender marine structures is developed by using finite element model, and relying on parameter studies, i.e. for the various cases in water depth, environmental condition and vessel surge and heave motions the dynamic behaviors of riser and tether of tension leg platform are analyzed. The dynamic analysis is carried out in the time domain for regular waves in this study, and the analysis has an emphasis on the effect of vessel heave motion to the lateral dynamics of riser and tether. It is noticed, however, that in some cases heave motions of a vessel do not influence the riser boundary condition and tensions because of the use of heave compensating devices such as tensioner and slip joint (Kozik *et al.*, 1990). The present study is an extension of the previous research works by Patel and Sorahia (1984) and Park (1994) in the analysis methods by considering the effect of tension variation due to vessel heave motion and by using discrete model, respectively. The discrete model is more available in considering spatial variation of axial tension, lateral load and structural properties along the structural length.

## 2. BASIC EQUATIONS OF MOTION

A slender structure has the characteristics of beam-column which has not only bending stiffness but also geometric stiffness which is a function of deflected geometry and the axial force on the member. The governing equation of motion of the structure has been derived in previous researches as:

$$\bar{m}\frac{\partial^2 x}{\partial t^2}+c\frac{\partial x}{\partial t}+\frac{\partial^2}{\partial z^2}\left[EI(z)\frac{\partial^2 x}{\partial z^2}\right]-\frac{\partial}{\partial z}\left[T(z,t)\frac{\partial x}{\partial z}\right]$$
$$=f(x,z,t) \qquad (1)$$

where $x$ = horizontal deflection; $z$ = vertical distance from bottom; $\overline{m}$ = mass per unit length of the member, including inner contents; $c$ = damping coefficient of the member; $EI$ = flexural rigidity; $T$ = local effective tension force; $f$ = lateral wave force per unit length of the member. In this study, steady deflection due to current, wind, etc. is not accounted for.

The local effective axial tension, $T(z, t)$ is calculated by accounting for the modification due to hydrostatic pressures in the surrounding fluid as follow.

$$T(z,t)=T_0+T(t)-\int_z^L w(z)dz+p_o(z)A_o(z)-p_i(z)A_i(z) \quad (2)$$

where $T_0$ represents mean top tension, i.e. top tension at still water; $T(t)$ is tension variation in time, which results directly from heaving motion of floating structure; $w$ is weight in air per unit length of the structure, excepting inner fluid weight if any; $p_o$, $p_i$ are external and internal local hydrostatic pressures, respectively; $A_o$, $A_i$ are outer and inner cross-sectional areas of the structure, respectively; $L$ is total length of structure.

Since the diameter of structure is much smaller than wave length, the hydrodynamic loading per unit length of a cylindrical member due to ocean wave is evaluated using the modified Morison equation (Sarpkaya and Isaacson, 1981).

$$f(z,t)=0.5\rho C_d D_e\left(v-\frac{\partial x}{\partial t}\right)\left|v-\frac{\partial x}{\partial t}\right| + \rho A_e\left(C_m\frac{\partial v}{\partial t}-C_a\frac{\partial^2 x}{\partial t^2}\right) \quad (3)$$

where $D_e$, $A_e$ are effective external diameter and effective area of the cross section; $v$ is the horizontal local component of water particle velocity; $\rho$ is mass density of sea water; $C_d$, $C_m$ and $C_a$ are drag coefficient, inertia coefficient and added mass coefficient of fluid dynamics, respectively.

## 3. FINITE ELEMENT ANALYSIS

The two dimentional beam element model is used to describe the structural properties in this study. In the formulation of beam element mass matrix, the lumped mass or the consistent mass approach may be used. In this study the lumped mass formulation is chosen for simpler definition of element properties. The vertical displacement of the structure is assumed sufficiently small to neglect the vertical dynamics of structural mass, but the tension varition in the structure due to vessel heave motion is considered in the horizontal force equilibrium of the structure. It is assumed that the time-varying axial force is sinusoidal and constant along the structural length.

Figs. 1(a) and 1(b) show typical slender structures incorporating the boundary condition at top end which must follow the motions of surface vessel. The motions of vessel may be simulated by using previous reports (Yeo and Pyun, 1985), and these are assumed to be known by pre-process in this study. The known horizontal nodal translation at the surface may be separated from all the unknown degree of freedom, or instead a virtual force required to cause the specified motions at the surface can be applied. In this study, this virtual force, $F_H$, is used together with a virtual mass-spring system of which the mass $m_0$, and the stiffness $k_H$ are sufficiently large as shown in Fig. 1(c). If the horizontal motion of surface vessel, $x_H$, is assumed to be harmonic with amplitude $a$ and circular frequency $\overline{\omega}$, it is represented as:

$$x_H(t)=a\sin(\overline{\omega}t+\varphi_h) \quad (4)$$

where $\varphi_h$ is the phase angle between wave and vessel horizontal motion. It is noted that $\overline{\omega}$ is same as wave frequency. For the horizontal virtual mass-spring system responding in Eq. (4), adopting a spring constant sufficiently large, the horizontal virtual force can be obtained from the vessel motion as:

$$F_H(t)=F_S(t)+F_D(t)+F_M(t) \quad (5)$$

where $F_S$ is the spring force, $F_D$ is the damping force and $F_M$ is the inertia force of the virtual mass-spring system, which are obtained using Eq. (4).

The effect of vessel heave motion is represented as tension variation in structural member. If the heave motion is assumed to be harmonic with amplitude $b$ and circular frequency $\bar{\omega}$, the tension variation, $T(t)$, is obtained as:

$$T(t) = \kappa_{Veq}\{b \sin(\bar{\omega}t + \varphi_v)\} = s \sin(\bar{\omega}t + \varphi_v) \quad (6)$$

where, $s$ is amplitude of tension variation; $\varphi_v$ is phase angle between wave and vessel heave motion, and $\kappa_{Veq}$ represents equivalent axial stiffness for total structural length. Then, the ratio of amplitude of tension variation to mean top tension is defined as:

$$R = s/T_0 \quad (7)$$

The ratio of tensions may be used in parameter study for the effect of tension variation on the lateral dynamics of slender structures. In the case of TLP, there is additional tension variation due to vessel pitching motion, which may increases the ratio of tensions in Eq. (7).

The phase angle between horizontal and vertical motion of wave particle is 90 degree. In general, the horizontal period of floating structure such as TLP, semi-submersible or floating production system is larger than wave period. On the other hand, the vertical period of floating structure may be larger or less than wave period. On this basis, the phase angle between vessel horizontal motion and vertical motion is assumed to be $\pm 90$ degree approximately in this study. It is noted, however, that in some cases these assumptions may not be valid due to the slip condition at top of riser.

Based on Eq. (1), the equation of motion of a multiple degree of freedom system may be written in global description as:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K(t)]\{X\} = \{F_w(t)\} + \{1\}F_H(t) \quad (8)$$

where, $[M]$=mass matrix, including added mass; $[C]$= structural damping matrix; $[K]$=stiffness matrix; $X, \dot{X}, \ddot{X}$ = horizontal displacement, velocity and acceleration of the structure, respectively; $\{1\}$ is a constant vector which has 1 for the lateral degree of freedom at top node, and 0 for others; $\{F_w\}$=nodal wave force vector; $F_H$ is virtual horizontal force defined by Eq. (5).

The stiffness matrix in Eq. (8) is the sum of elastic stiffness and geometric stiffness due to axial tension as below.

$$[K(t)] = [K_b] + [K_{T0}] + [K_{T(t)}] \quad (9)$$

where $[K_b]$ denotes elastic stiffness; $[K_{T0}]$ and $[K_{T(t)}]$ are geometric stiffness contributed by $T_0$ and $T(t)$, respectively. It is noted that since axial deformation of element mass is not considered in this study, the time varying tension is consistent in all elements depending on the time varying top tension, but the mean tension varies in space due to buoyancy and gravity weight of upper elements as indicated in Eq. (2).

## 4. SOLUTION OF EQUATION

The governing equation of motion (8) is nonlinear in hydrodynamic loading, and the nonlinearity is concerned with hydrodynamic damping. Although the stiffness matrix is time varying, it does not depend on structural response. On this basis, Eq. (8) is rewritten as follows for use of modal analysis relying on average stiffness.

$$\begin{aligned}&[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K_b + K_{T0}]\{X\} \\ &= -[K_{T(t)}]\{X\} + \{F_w(t)\} + \{1\}F_H(t) = F(X, t) \quad (10)\end{aligned}$$

Using undamped free vibration modes which may be obtained from $[M]$ and $[K_b+K_{T0}]$, the responses are evaluated approximately in terms of generalized coordinates, as follows.

$$\{X(t)\} = [\Phi]\{q(t)\} \quad (11)$$

where, $[\Phi]$ is $(n \times l)$ matrix of free vibration modes for n-degree of freedom system, which is normalized to have all generalized masses get unity i.e. $[\Phi]^T[M][\Phi]= [I]$; $q$ represents generalized coordinate; $l$ is total numbers of modes used. It is noted that the lowest a few natural frequencies and corresponding modes are used, and thereby reducing unknowns to carry out the analyses effectively. Substituting Eq. (11) into Eq. (10), and pre-multiplying $[\Phi]^T$, and assuming uncoupled structural damping with damping ratio $\xi$, the following equation which is coupled in loading term only is obtained.

$$\{\ddot{q}\}+[2\xi_k\omega_k]\{\dot{q}\}+[\omega_k^2]\{q\}=[\Phi]^T\{F(X,t)\} \qquad (12)$$

where, $\omega_k$ is k-th natural frequency. It is noted that since structural damping is generally small with respect to hydrodynamic damping, the second term in Eq. (12) may be effectively represented by using small structural damping ratios. Solution of the nonlinear Eq. (12) can be obtained by Newmark-β method, or by Runge-Kutta's numerical integration method. In this study, a numerical integration program HPCG is used to solve the nonlinear equation, which is based on Runge-Kutta's method and is one of IBM application programs (IBM, 1970). From the solution of Eq. (12), the displacements are obtained by Eq. (11), and member forces and stresses by nodal displacements.

## 5. EFFECTS OF VESSEL MOTIONS

To carry out parameter studies efficiently, it is necessary to have an insight into the problems involved in vessel motions by using analytic solutions for a quite simple model. Here is considered the response of a slender structure of uniform cross section, which has hinged boundary conditions at both ends. Neglecting structural damping and variation of mean tension along structural length, the governing differential Eq. (1) is rewritten as:

$$m\frac{\partial^2 x}{\partial t^2}+EI\frac{\partial^4 x}{\partial z^4}-(T_0+T(t))\frac{\partial^2 x}{\partial z^2}=f_w(z,t) \qquad (13)$$

The partial differential Eq. (13) may be reduced to an ordinary nonlinear differential equation by using the method of separation of variables (Clough and Penzien, 1975). The modes of motion are readily reduced to a rigid body mode and sinusoidal elastic response modes. An approximate solution to Eq. (13) is written in the form as (Park, 1994):

$$X(z,t)=x_H(t)\frac{z}{L}+\sum_{n=1}^{l}q_n(t)\sin\frac{n\pi z}{L} \qquad (14)$$

The horizontal motion of vessel is defined as Eq. (4), and axial tension variation in time as Eq. (6), and considering 90 degree phase angle between horizontal motion and vertical motion, the prescribed motion and tension are

$$x_H(t)=a\sin(\bar{\omega}t),\quad T(t)=s\cos(\bar{\omega}t) \qquad (15)$$

Substituting Eqs. (14)-(15) into governing Eq. (13), and multiplying m-th elastic mode to this and integrating over structural length leads to the following non-dimensional modal equations.

$$\frac{d^2q_n}{d\tau^2}+\alpha_n(1+\beta_n\cos\tau)q_n=-(-1)^n\frac{2a}{n\pi}\sin\tau$$

$$+2\int_0^L f_W(z,\tau)\sin\frac{n\pi z}{L}dz \qquad (16)$$

where, $\tau=\bar{\omega}t$, and

$$\alpha_n=(\omega_n/\omega)^2,\quad \beta_n=\alpha_n s/[T_0+EI(n\pi/L)^2] \qquad (17)$$

$\omega_n$ in Eq. (17) is natural frequency of n-th elastic mode. If the right hand side of Eq. (16) is not considered, it becomes well-known Mathieu equation (Stoker, 1950). When certain conditions on $\alpha_n$ and $\beta_n$ are met, in this case, the trivial solution $q_n \equiv 0$ is unstable and the response $q_n$ becomes unbounded. However, since the quadratic nonlinear hydrodynamic damping term exists, even unstable solutions are limited in amplitude. The solution of Eq. (16) depends on frequency ratio $\alpha_n$, strength of parametric excitation $\beta_n$ and strength of forcing excitation $a$. On this basis, parameter studies on $\bar{\omega}$, $L$, $s/T_0$ and $a$ will be carried out through example analyses on a typical riser and TLP tethers.

## 6. EXAMPLE ANALYSIS AND DISCUSSION

Time domain dynamic analysis is carried out using a regular wave with period of 8.5-13.7 s and wave height of 7-18 m. It is noted that the current effect is not considered in this study in order to analyze the dynamic response of structure to wave and vessel motions more efficiently. For an example analysis in this study, a hypothetical riser with a virtual mass-

**Table 1.** Riser model data for example analysis.

water depth: 300 m
riser length: 300 m
-outer diameter: 0.5 m
-wall thickness: 0.017 m
-Young's modulus: $2.1 \times 10^7$ $t/m^2$
-weight density: 7.85 $t/m^3$
density of sea water: 1.025 $t/m^3$
effective diameter for wave load: 0.8 m
drag coefficient: 1.0
inertia coefficient: 2.0
top tension: 180 tons
top virtual mass: $1.8 \times 10^7$ $t.s^2/m$
-virtual stiffness: $9 \times 10^5$ $t/m$
-virtual damping: 10%
vessel surge amplitude: 5 m
vessel surge phase angle: 15°

spring system in water depth of 300 m is adopted. Table 1 gives the input data for parameter studies of this example. It is noted that the stiffness of virtual mass-spring system should be sufficiently large to neglect the lateral stiffness of structural elements. Also, the virtual mass should be sufficiently large to make sure that the natural frequency of single-degree virtual mass is identical to the smallest one of multi-degree structural system. By undamped free vibration analysis, first five natural frequencies are calculated as $\omega_1 = 0.223$ rad/s, $\omega_2 = 0.5027$ rad/s, $\omega_3 = 1.022$ rad/s, $\omega_4 = 1.570$ rad/s, $\omega_5 = 2.162$ rad/s. It is noted that $\omega_1$ is in sway mode which responds to vessel surge motion and contributes significantly to lateral displacement of riser. Meanwhile, $\omega_2$ and higher frequencies are in flexural mode which contribute significantly to bending stress of the structure. Table 2 displays the three wave conditions; those are moderate, severe and extreme wave conditions with corresponding maximum wave heights and periods chosen empirically. It is noted that the basis of these analyses is the severe limited condition with wave height 10 m, wave period 10.5 s.

Park(1994) reported a numerical result on long-

**Table 2.** Wave conditions for parameter study.

| | moderate | severe | extreme |
|---|---|---|---|
| wave height | 7 m | 10 m | 18 m |
| wave period | 8.5 s | 10.5 s | 13.7 s |

cylindrical marine structures subjected to forcing, parametric and combined excitations, in which forcing excitation means vessel surge motion input; parametric excitation is vessel heave motion input; and combined excitation is surge and heave motion inputs. On the other hand, instability regions are decided by $\alpha$ and $\beta$ values which depend on input period and strength and structural physical data as can be seen from Eq. (17). As a bench mark for the result of the riser example, here introduced an artificial instability condition at $\alpha = 1$ and $\beta = 1$ in the second vibration mode using appropriate input frequency and tension variation, which falls in the second instability region of the Mathieu stability chart. It is noted that the case of $\beta = 1$ is not realistic considering the working of tensioner at the top of riser. Fig. 2 shows comparisons between forcing, parametric and combined excitations for the displacement at mid point of structure. These comparisons give an interesting point, as can be seen also at Park (1994), that even though the responses to forcing and parametric excitations are small, the response to combined excitation is much larger than that to the separate excitation. This result means that the interaction between forcing and parameter excitation is significant in increasing the total response. It is noted that since the parametric excitation in the forcing term at Eq. (10), $[K_{T(t)}]\{X\}$, is a function of response $\{X\}$, the effect of this interaction would be significant.

Fig. 3 shows response configurations according to the three top end boundary conditions: one is no vessel motion, another is vessel surge input, the other is

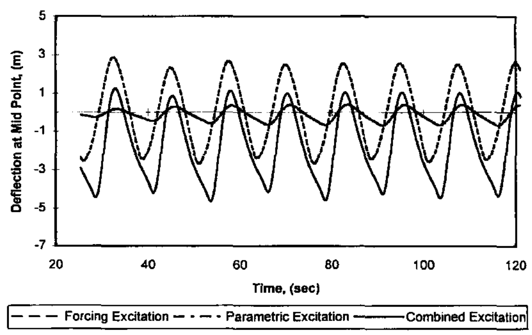


**Fig. 2.** Displacememt time histories as to different excitations to a riser.

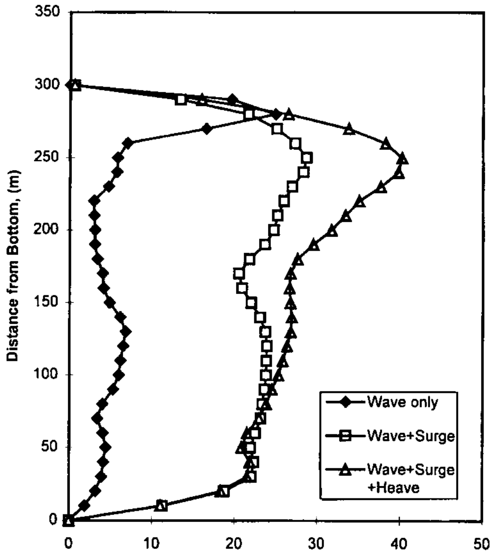


**Fig. 3.** Bending stresses of a riser as to various excitations.

vessel surge and heave inputs. The wave condition is severe wave. The amplitude of sinusoidal tension variation is assumed to be 40% of mean top tension. It can be seen that vessel motion inputs at the top end give severe changes in the distribution of bending stress, and it seems that most of the changes result from vessel surge motion. It can be seen also that about 40% of bending stresses are additionally superposed in the upper part of the structure by vessel heave motion. These imply that time varying vessel heaves as well as vessel surge motions should be considered in the same time for the effective dynamic analysis of marine risers. However, it is noticed that in some cases heave motions of a vessel do not influence the tensions due to the use of heave compensating devices (Kozik *et al.*, 1990).

Another example is analyzed by using a typical tether system of a TLP. Table 3 gives physical data for the tether. TLP tether is a kind of long-cylindrical marine structures, but has some special characteristics. The worst sea state is an important environmental condition for design of a tether. Such heavy storm essentially creates large heave and pitching motions of a TLP, and then high time- varying tension. TLP gets

**Table 3.** Tether model data for example analysis.

tether length: 510 m
-outer diameter: 0.6 m
-wall thickness: 0.02 m
top tension: 380 tons
virtual mass: $1.8\times10^{8}$ $t.s^2/m$
virtual stiffness: $9\times10^{5}$ $t/m$

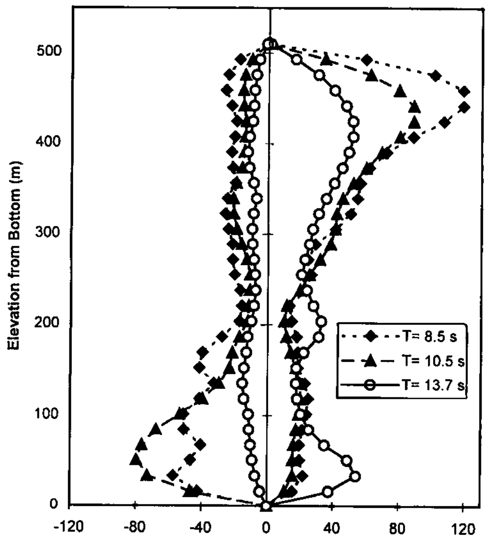


**Fig. 4.** Bending stresses in a tether as to wave periods.

interests in deeper water exploration due to relative low cost. On these view points, this example of a typical tether system is highlighted on large tension variations and deep water cases. It is noted that for a given structure the stability parameters α and β result eventually from wave period and height. For slender deep water structures β/α can be approximated to $s/T_0$ which is the ratio of tension variation to mean top tension. As seen in Mathieu stability chart (Stoker, 1950), when becomes small, i.e. α approaches to zero the possibility for a slender structure to meet instability condition is high. However, is increased in heavy storm condition. A parameter study is carried out in wave periods of T = 8.5 s, 10.5 s and 13.7 s typically. Fig. 4 shows bending stress configurations of maximum positive and negative peaks, which is intended to show the sensitivity of structural response to α. For all the wave period conditions, wave height of 10 m, vessel

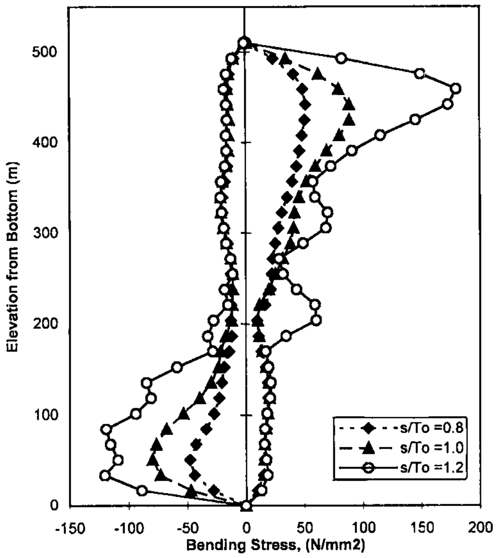


**Fig. 5.** Bending stresses in a tether as to tension variations.

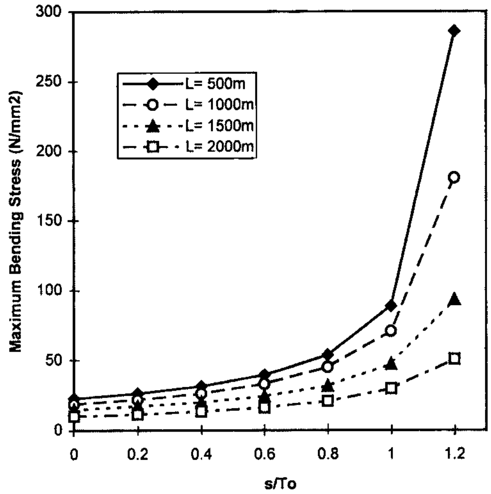


**Fig. 6.** Maximum bending stresses as to tether lengths and tension variations.

surge amplitude of 5 m and $s/T_0$ are applied. It can be seen that although the three input period cases are quite different in the expression of response configurations the differences of maximum values are not so significant, considering appropriate wave heights. Fig. 5 shows response configurations according to three cases of slight different tension variations. It is noted that wave height of 10 m, period 10.5 s and vessel surge amplitude of 5 m are assumed for all the three cases in convenience. It can be seen that the differences of maximum responses are very significant, and the effect of parametric excitation is very sensitive to the strength of parametric excitation around $s/T_0 = 1.0$ which may be considered in heavy storm condition. So, the worst sea state is an important environmental condition for the investigation of the effect of parametric excitation.

Another parameter study for the effect of parametric excitation is made using four different tether lengths according to water depth. All the physical data are same as those in Table 3, except the length of tether and mean top tension. The tether lengths are varied from 500 m to 2000 m, but mean top tensions are assumed to be same by 500 tons for all cases and all the members in a tether. Extreme wave condition given in Table 2 is applied. Fig. 6 shows maximum bending stress according to tether length , and as to strength of parametric excitation $s/T_0$ which can be approximated to $\beta/\alpha$ due to deep water condition. The response amplitudes are increased as a whole as the smaller length of tether. These results are due to the characteristics of dynamic response to combined excitation in individual instability regions according to each $L$ and $s/T_0$. Another interesting point that can be seen from the Figure is that when $L$ is 1000 m or less, the maximum responses are catastrophically amplified on high $s/T_0$. With $s/T_0 = 1.0$, the case of $L = 500$ m falls in the second instability region and the case of $L = 1000$ m falls in the first instability region. But, in the higher water depth the natural frequencies are reduced and the point in Mathieu stability chart moves toward zero from the center of first instability region. Consequently, the effect of parametric excitation, i.e. time-varying axial force could be neglected for higher water depth than about 1000 m and vice versa, although the stability condition should be checked together with the strength of parametric excitation.

## 7. CONCLUSION

A finite element model is developed for efficient dynamic response analysis of slender marine structures which are subjected to excitations by vessel surge and heave motions as well as by waves. Dynamic analyses are performed in time domain using regular waves. Parameter studies are carried out to highlight the effects of vessel motions on the lateral dynamics of slender marine structures. Example results are compared for various cases in water depth, wave condition and vessel motion. In case of riser, bending stresses become severe in overall height by vessel surge motion. By the effect of vessel heave motion, bending stresses are amplified in the upper part. As the results, the interaction between forcing and parametric excitation is significant, and then the total response is quite increased. In case of tether of TLP, the structural response is more sensitive to wave height rather than wave period, even though the possibility to fall in unstable region is higher under smaller period of wave. TLP is competent in deeper water in general, and the natural frequencies are very small. From a parameter research in this study, the effect of parametric excitation is seemed to be quite significant to lateral response of a tether.

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## REFERENCES

Chung, J.S., Huttelmaier, H.P. and Cheng, B., 1994. 3-D coupled response of deep-ocean crust mining pipe systems to axial bottom vibration and ship motion, *Proc. 4th Int. Offshore and Polar Engrg. Conf.*, Osaka, pp. 275-285.

Clough, R.W. and Penzien, J., 1975. *Dynamics of structures*, McGraw-Hill.

HaQuang, N. and Mook, D.T., 1987. Non-linear structural vibrations under combined parametric and external excitations, *J. Sound and Vibration*, **118**, pp. 291-306.

Hsu, C.S., 1975. The response of a parametrically excited hanging string in fluid, *J. Sound and Vibration*, **39**, pp. 305-316.

IBM, 1970. *Scientific subroutine package version III*, IBM Application Program.

Kozik, J.J., Lowell, J.E. and Ertas, A., 1990. Upper ball joint force variations due to riser tensioner and vessel motions- Part I: Derivation of general equations, *J. Energy Resources Tech.*, Trans. ASME, **112**, pp. 188-197.

Park, H.I., 1994. The response characteristics of long cylindrical marine structures under different excitations, *Proc. 4th Int. Offshore and Polar Engrg. Conf.*, Osaka, pp. 231-237.

Patel, M.H. and Sorohia, S., 1984. Finite-element analysis of the marine riser, *Engineering Structures*, **6**, pp. 175-184.

Sarpkaya, T. and Isaacson, M., 1981. *Mechanics of wave forces on offshore structures*, Van Nostrand Reinhold, New York.

Stoker, J.J., 1950. *Nonlinear vibrations*, Interscience Publishers, New York.

Thampi, S.K. and Niedzwecki, J.M., 1992. Parametric and external excitation of marine risers, *J. Engineering Mechanics*, ASCE, **118**(5), pp. 942-960.

Yeo, W.K. and Pyun, C.K., 1985. Dynamic response analysis of tension leg platform, *J. Korean Soc. of Civil Engrs.*, **5**(1), pp. 21-33.

Yun, C.B., Ryu, C.S. and Kim, D.S., 1996. Development of analysis/design methodologies for tension leg platforms, *Research Report*, Korea Institute of Science and Technology.