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Nonlinear Flexural Modelling of Prestressed Concrete Beams with Composite Materials


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ABSTRACT

Recently, application of composite materials such as fiber reinforced concretes(FRCs) and fiber reinforced plastics(FRPs) in conjunction with conventional structural components has become one of the main research areas. A proper use of advanced composite materials requires understanding their resistance mechanism and failure mode when they are applied to structures or their components.

Particular considerations are given in this research to develop an analytical model which can predict the nonlinear flexural responses of bonded and unbonded prestressed concrete beams possibly having layers of different cementitious composite matrices in a section and/or FRP tendons. The block concept is used, which can be regarded as an intermediate modeling method between the couple method with one block and the layered method with multiply sliced layers in a section. In order to find a particular deflection point of a beam under load, solutions to the 2N-variables are found numerically by using approximate N-force equilibrium equations and N-moment equilibirum equations. The model is shown to successfully predict the flexural behavior of variously reinforced bonded and unbonded prestressed concrete beams. The model is also successful in simulating a gradually increasing load after sudden drop in load resistance due to fracture of one or more FRP tendons. This feature is useful in tracing the overall load-deflection response of a beam prestressed with brittle FRP tendons.

keyword: Composite Materials, Nonlinear Flexural Modeling, Block Concept, Prestressed, Bonded, Unbonded

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1. INTRODUCTION

Recently, along with the fiber reinforced concretes, fiber reinforced plastics(FRPs) are emerging as potential new construction materials. FRPs form a group of advanced composite materials[1,2]. FRPs consist of strong and continuous long fibers of different chemical origin embedded in a plastic matrix. There is a wide range of potential applications of FRP reinforcements, that covers new construction and strengthening and/or rehabilitation, prestressed and non prestressed members, and prefabricated and cast in-place construction. Currently, FRP products are being used in form of reinforcement rods, prestressing tendons, grid type reinforcement, and external reinforcing sheets. The main advantage of reinforcing structural components by FRPs can be summarized as follows[1]: 1) FRP reinforcements are corrosion-free; 2) they are non magnetic, non conductive to electricity, and transparent to radio waves; 3) they have lighter unit weight; and 4) FRP tendons have almost the same or higher tensile strength than steel tendons. Although FRPs have potential advantages, they have some drawbacks compared to steel rebars or tendons. FRPs usually exhibit lower ductility, lower shear strength, and stress rupture phenomena. They are more expensive than steel.

A proper use of advanced composite cementitious materials(SFRC, SIMCON, and SIFCON etc.) and FRPs(sheet or rods) requires understanding their resistance mechanism and failure mode when they are applied to structures or their components. The main objective of this research is to develop an analytical model which can comprise various characteristics of composite materials, different placement, and casting of composite materials in combination with conventional cementitious matrices in model. A beam section is divided into different rectangular blocks characterized by its composite material property and location in a beam section. The analytical expressions are then formulated for these individual blocks. Complete flexural load deflections of simply supported bonded or unbonded beam are generated by the model.

2.DEVELOPMENT OF THE MODEL

2.1 Assumptions and Limitations of the Model

The following assumptions are made in developing the current model: 1) plane section remains plane after and before bending(Euler Bernoulli's hypothesis); 2) the internal moment resistance at midspan is equal to the externally applied midspan moment; 3) constitutive relationships for the steel, composite reinforcing bars(i.e., FRP tendons), concrete or composite matrices are known for compression or tension; 4) post peak tensile behavior of ductile composite matrix and its crack opening are known; and 5) the eccentricities of the bonded and/or unbonded prestressing tendons and the external moment varying along the span are given in accordance with the pre selected tendon profile and loading type geometries.

There are limitations to the developed model: 1) no shear failure, nor the shear effect on the flexural deformation is considered; and 2) beams with symmetric sections both in geometry and in constituent matrix materials in a section are considered only.

2.2 Development of Model : Block Modeling

2.2.1 Definitions on the Blocks and Overall Numerical Scheme

In order to take into account partial placement of composite matrices in a section, a beam section is modeled by using block concept(Fig.1).

A beam may be composed of different blocks where each block is identified by its own material property and its geometry. The blocks having the same dimension (width and height) and material, being separated horizontally at the same level are defined identical blocks. Unlikely, sections having different block characteristics(i.e., in their dimensions, their materials, and/or both) placed within the same level can not be considered by the model (see Fig.1).

In order to find the overall load-deflection of the beam, the step-by-step method of increasing the bottom strain at midspan is used.

Increment of the bottom strain is preferred to increment of the top strain in this study since it has a few advantages especially in the analysis of prestressed concrete beam. By increasing the bottom strain, decompression moment, cracking moment, and yield moment at which yielding of steel rebar or tendon can be predefined and easily found.

2.2.2 Section Force

For a given bottom strain at section j and a numerically chosen top trial strain, sum on axial forces and moments from each block i at section j can be calculated as follows(see Fig.2):

$$F^k_{i,j} = b_i \cdot \left(\frac{h_i}{\varepsilon^k_{i,j} - \varepsilon^k_{i-1,j}} \right) \times \int_{\varepsilon^k_{i-1,j}}^{\varepsilon^k_{i,j}} \sigma_{m(i)} d\varepsilon \qquad (1)$$

where :

$F^k_{i,j}$ = i-th block axial force at section j for loading step k;

$\varepsilon^k_{i,j}$ = matrix strain at top of the i-th block at section j for loading step k;

$\varepsilon^k_{i-1,j}$ = matrix strain at bottom of the i-th block at section j for loading step k;

$\sigma_{m(i)}$= matrix stress for i-th block material, $m(i)$.

Strain distributions are obtained using similar triangular shape and contributions from rebars and tendons to sectional resistance are obtained accordingly.

For all sections (j=0,1,2, . . . , N) along the beam span(Fig.3), total sum on the sectional forces from each block must vanish at loading step k:

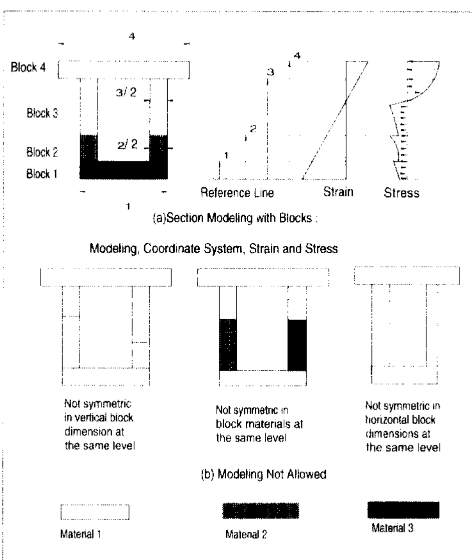

Fig.1 Concept of Block Modeling

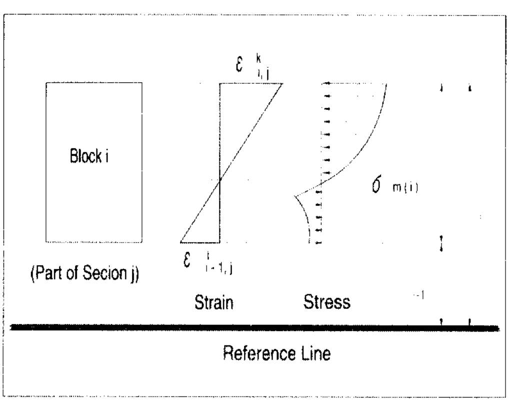

Fig.2 Typical i-th Block: Possible Strain and Stress Distributions at Loading Step k.

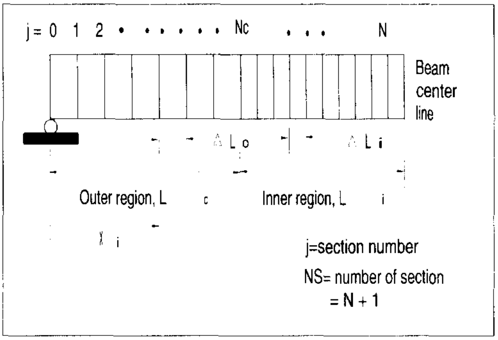

Fig.3 Beam Sections along the Beam Span

$$F_j^k = \sum_{i=1}^{NDB} F_{i,j}^k = 0 \quad (j=0,1,2,\ldots,N) \quad (2)$$

where:

$F_j{}^k$ =sum of the axial forces at section j for loading step k;

$F_{i,j}^k$ =i-th block axial force at section j for loading step k. ; and.

NDB =last block number at section.

2.2.3 Section Moment

Once sectional equilibrium is reached at section j, sectional moment resistance at this section is obtained by adding all the moment resistances from each block :

$$M_j^k = \sum_{i=1}^{NDB} M_{i,j}^k \quad (3.a)$$

where:

M_j^k = moment resistance at section j for loading step k; (3.b)

$$M_{i,j}^k = -\alpha_{i,j}^k \cdot \left[\int_{\varepsilon_{i-1,j}^k}^{\varepsilon_{i,j}^k} \sigma_{m(i)} \cdot \varepsilon \cdot d\varepsilon + \beta_{i,j}^k \int_{\varepsilon_{i-1,j}^k}^{\varepsilon_{i,j}^k} \sigma_{m(i)} d\varepsilon \right] \quad (3.c)$$

= i-th block moment at section j;

$$\alpha_{i,j}^k = \frac{b_i \cdot h_i^2}{\left(\varepsilon_{i,j}^k - \varepsilon_{i-1,j}^k \right)^2} \quad (3.d)$$

$$\beta_{i,j}^k = \frac{y_{i-1} \cdot \varepsilon_{i,j}^k - y_i \cdot \varepsilon_{i-1,j}^k}{h_i} \quad (3.e)$$

$\varepsilon_{i-1,j}^k$ =bottom strain of block i at section j for loading step k;

$\varepsilon_{i,j}^k$ =top strain of block i at section j for loading step k;

y_i , y_{i-1} =y coordinates of top and bottom of i-th block,respectively;

$h_i = y_i - y_{i-1}$ =height of i-th block;

$\sigma_{m(i)}$= stress function corresponding to i-th block matrix.

Intergrations appearing at the above equations are performed using 5-point Gauss-Legendre quadrature.

2.2.4 Force Equilibrium and Moment Equilibrium

For a given increased bottom strain at k-th loading step at section j ($\varepsilon_{1,j}^{b,k} = \varepsilon_{1,j}^{b,k-1} + \delta\varepsilon$), the desired extreme top strain in that section (= $\varepsilon_{NDB,j}^{t,k}$) for sectional equilibrium can be obtained using the Modified Regula-Falsi method where subscript 1 and NDB stand for the first block and the last block in a section, repectively at section j[3]. At the initiation of each iteration, two extreme top fiber strains are determined to enclose correct extreme top fiber strain: the one from previous loading step($\varepsilon_{NDB,j}^{t,k-1}$) and the one from user-defined ultimate compressive strain at failure($\varepsilon_{ultimate}^t$). Assigning $\varepsilon_{1,j}^{b,k}$ and $\varepsilon_{NDB,j}^{t,k-1}$ as initial guesses would result in positive sum on axial forces for F_j^k at section j. Note that $F_j^k = \sum_{i=1}^{NDB} F_{i,j}^k$ With the increased bottom strain and ultimate extreme top fiber strain being equal to the ultimate compressive strain, negative sum on axial forces can, in general, be obtained. If these initial guesses correctly contain the case of $F_j^k = 0$, then iterative searching starts to find extreme top fiber strain for a given extreme bottom fiber strain. When there are softening behavior in matrix either in tension or compression, the initial guesses may not yield desired bracket. This was observed especially when the top extreme fiber strain approaches the ultimate strain of the matrix. If such case is encountered, extreme top fiber strain is incrementally adjusted between two given extreme top strain values until the sum on forces resulted from a selected extreme top fiber strain becomes negative. The procedure outlined above is called "Force Equilibrium."

Once the extreme top fiber strain satisfying sectional equilibrium is found, corresponding internal moment at section is calculated using eq.(3.a). If the difference between the numerically found internal moment and the external moment given at a section fall within a tolerable difference, it is assumed that correct extreme top fiber strain and extreme bottom fiber strain are found. Otherwise, another trial extreme bottom fiber strain is assumed and searching procedure is repeated until convergence criterion is satisfied. This procedure is called "Moment-Equilibrium."

Basic assumptions are made on the selection of bottom strain at extreme bottom fiber at section j: 1) increasing bottom strain increases the internal moment resistance at any loading stage; and 2) the extreme bottom fiber strain at loading step k ($\varepsilon_{i,j}^{b,k}$) is greater than or equal to the one at previous loading step k-1 ($\varepsilon_{i,j}^{b,k-1}$) and less than or equal to the one at the midspan at the loading step k ($\varepsilon_{i,j}^{b,k}$) (see Fig.4)

$$M_j^{k-1} \le M_j^k \le M_N^k ,\quad j=0,1,2,\ldots\ldots, N-1 \qquad (4)$$

Let, $\triangle M_j^{k,L} = M_j^{k-1} - M_j^k$ and

$$\triangle M_j^{k,U} = M_N^k - M_j^k .$$

By the assumptions made above, it can be seen that $\triangle M_j^{k,L} \le 0$ and $\triangle M_j^{k,U} \ge 0$.

The extreme bottom fiber strains corresponding to the lower bound, ($\varepsilon_{1,j}^{b,k,L}$) and the upper bound($\varepsilon_{1,j}^{b,k,u}$) for the given external moment at section j (M_j^k) can be assigned with the previously

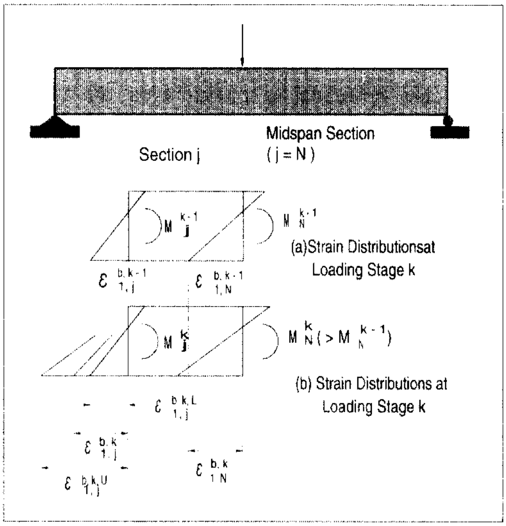

Fig. 4 Determination of Lower and Upper Bound for the Bottom Strain at Loading Stage k

obtained strains at section j for loading step k-1,

$(\varepsilon_{1,j}^{b,k-1})$, and at midspan for loading step k, $(\varepsilon_{1,N}^{b,k,})$:

$$\varepsilon_{1,j}^{b,k,L} = \varepsilon_{1,j}^{b,k-1} \quad \text{and}$$

$$\varepsilon_{1,j}^{b,k,u} = \varepsilon_{1,N}^{b,k,} \tag{5}$$

The correct bottom strain at extreme bottom fiber corresponding to the given external moment is then enclosed in the interval $[\varepsilon_{1,j}^{b,k,L}, \varepsilon_{1,j}^{b,k,u}]$. Let $\triangle M_j^k = \overline{M}_j^k - M_j^k$, where M_j^k is the moment obtained by the assumed extreme bottom fiber strain at section j and loading stage k and $\overline{M}_j^k$ by external load. Note that at the correct extreme bottom fiber strain $(\varepsilon_{1,j}^{b,k})$, the difference between internal moment resistance and external moment vanishes zero (i.e., $\triangle M_j^k = 0$). Midspan deflection is then calculated using Simpson's Rule and the Moment Area theorems by integrating curvatures at each section along the span..

3. PRESTRESSED CONCRETE BEAM

For the bonded prestressed concrete beam, strain compatibility is assumed. This implies that member behavior can be approximated by sectional behavior of the beam. Emphasis is given in this paper for the modeling technique of unbonded prestressed concrete beam whose flexural behavior is governed by member behavior rather than sectional behavior.

The prestrains at the initiation stage of iteration can be obtained by:

$$\varepsilon_{psb,l} = \varepsilon_{peb,l} + \varepsilon_{ceb,l} \; ; \text{ and}$$

$$\varepsilon_{peub,l} = \varepsilon_{psub,l} + \Omega\,\varepsilon_{ceub,l} \tag{6}$$

where:

$\varepsilon_{psb,l}$, $\varepsilon_{psub,l}$ = l-th bonded and l-th unbonded tendon prestrains at the reference stage, respectively;

$\varepsilon_{peb,l}$, $\varepsilon_{pceub,l}$ = l-th bonded tendon effective strain and l-th unbonded tendon effective strains, respectively;

$\varepsilon_{ceb,l}$, $\varepsilon_{ceub,l}$ =strain increments in the matrix at the level of l-th bonded and l-th unbonded tendon for the load beyond reference stage, respectively.

In calculating unbonded prestrains ($\varepsilon_{pceub,l}$) above, the bond reduction coefficient(Ω) is used to take into account their member dependent (not section dependent) quantities[4]:

$$\Omega = \frac{(\varepsilon_{psub})_m}{(\varepsilon_{psb})_m} = \frac{(\varepsilon_{psub})_{av}}{(\varepsilon_{pcsub})_m} \tag{7}$$

where :

ε_{psub}=strain increase in the unbonded tendon taken at the section of maximum moment;

ε_{psb} =strain increase in the equivalent bonded tendon taken at the section of maximum moment; and

ε_{cpsub} = strain increase in the concrete at the level of the unbonded tendon taken at the section of maximum moment.

m and av = maximum and average, respectively

The above strains are all taken at the section of maximum moment. From this, the average strain increase of unbonded tendon can be obtained in relation to the matrix strain increase at the maximum moment at the level of unbonded tendon:

$$(\varepsilon_{psub})_{av} = \Omega \cdot (\varepsilon_{cpsub})_m \quad (8)$$

Using the Hooke's law and classical elastic theory, the expression for Ω can be formulated as[4,5]:

$$\Omega = \frac{2}{L \cdot M_{\max} \cdot (e_{oub})} \times \int_0^{l/2} M(x) \cdot e_{ub}(x) \cdot dx \quad (9)$$

where:

L = beam span;

$M_{\max}$ = changes in bending moment at the critical section (or midspan in our study);

e_{oub} =eccentricity of unbonded tendon at the critical section; and

$e_{ub}(x)$ =eccentricity of unbonded tendon at the distance x along the beam axis from the support.

In the current nonlinear study, the concept of bond reduction coefficient (Ω) is adopted numerically rather than analytically to find strains in unbonded tendons. Concrete strains at the level of these tendons are found for each section and then these strain values along the beam axis are integrated numerically in order to evaluate the corresponding elongation of the unbonded tendon. The additional strain is obtained by dividing this elongation of the tendon by corresponding tendon length. This strain is then added to the effective prestrain given at the reference stage.

Since average concrete strain at the level of tendon depends on beam deformation which again depends on the tendon force, an additional iteration is necessary to find proper average concrete strain at the level of unbonded tendons. At each iteration for loading step k, beam elongation is calculated by numerically integrating the concrete strains at the level of corresponding tendons along the beam. This elongation is then divided by tendon length to find average concrete strain. Added average concrete strains from each level of unbonded tendons are compared with the values obtained from the previous step and if this absolute difference is less than 1.0% of the previous value, then the calculated average strains for each unbonded tendon are accepted. Otherwise the iteration continues until this difference reduces to 1.0% or less. Simpson's rule is used to integrate the strains.

4. CONSTITUTIVE MODELS

The model incorporates different types of constitutive models for cementitious composite matrices, rebers and prestressed steel tendons or FRP tendons. The mathematical expressions for these models are briefly presented below. More detailed descriptions on each notation for each model can be found elsewhere[5-9].

4.1 Matrix in Compression

The model can represent constitutive behavior ranging from brittle plain concrete to ductile composite materials like SIFCON[5]:

$$\sigma = -\frac{\sigma^c_{\max}}{\varepsilon^{c\,2}_{\max}} \cdot (\varepsilon - \varepsilon^c_{\max}) + \sigma^c_{\max}$$

$$\text{for } \varepsilon \leq \varepsilon^c_{\max} \quad (10)$$

$$\sigma = a \cdot e^{-b \cdot \varepsilon^m_{\max} \cdot (\frac{\varepsilon}{\varepsilon_{\max}} - 1)^m} + \sigma^c_a$$

$$\text{for } \varepsilon > \varepsilon^c_{\max} \text{ for :}$$

$\varepsilon^c_{max} = 0.001648 + 1.62\times10^{-6}\cdot\sigma^c_{max}\ [kg/cm^2]$

$a = E_0(\varepsilon^c_{max}\ /\ \sigma^c_a)$

σ_{asym} = plateau stress as ε becomes infinity;

$m = 1/[1 + l_n(\sigma_i - c)/(f_c - c)]$

$b = (m-1)/m/\ (\varepsilon_i - \varepsilon^c_{max})^m$

σ_i, ε_i = stressed strain at inflection point

4.2 Matrix in Tension

Three different models have been adopted: 1) model identical to the compressive constitutive model in pre-peak and post-peak; 2) model derived from the tension member[9]; and 3) model for fiber reinforced concrete like SIFCON[6,7]:

4.2.1 Model by Virlogeux[9]

The strain represents both the average strain of steel and the average strain of concrete including crack width:

$$\sigma = E_c\cdot\varepsilon \quad \text{for} \quad \varepsilon \le \varepsilon^t_{max} \qquad (11)$$

$$\sigma = \sigma^t_{max}\cdot\left(\frac{\varepsilon^t_{max}}{\varepsilon}\right)^{0.4} \quad \text{for} \quad \varepsilon > \varepsilon^t_{max}$$

where;

$\sigma^t_{max} = 1.0\cdot\sqrt{f_c}\quad (kg/cm^2)$

$\varepsilon^t_{max} = 0.00008$, and

$E_c = 12460\cdot\sqrt{f_c}$. (kg/cm^2)

4.2.2 Model by Naaman[6,7]

The model is included for the material type of SIFCON. In order to model the descending branch of the constitutive model in terms of strain rather than crack opening, a correlation between crack width and strain is made by dividing the crack opening by the smeared length of the strain which is assumed to be the depth of the member[6,7]:

$$\sigma = \frac{-\sigma^t_{max}}{(\varepsilon^t_{max})^2}\cdot(\varepsilon-\varepsilon^t_{max})^2 + \sigma^t_{max}$$

$$\text{for} \quad \varepsilon \le \varepsilon^t_{max} \qquad (12)$$

$$\sigma = \sigma^t_{max}\cdot\left(1-k\frac{2\delta}{l_f}\right)^2 \quad \text{for} \quad \varepsilon > \varepsilon^t_{max}$$

In the above expressions, the following values are adopted from the literature[2,6] for the SIFCON with V_f = 4.1% and aspect ratio of 100: $k = 1.0$, l_f = fiber length, δ = crack opening $\simeq \varepsilon_j\cdot h$., ε_j = strain at section j, h = beam depth, $\sigma^t_{max} = 98.4\ kg/cm^2$, and $\varepsilon^t_{max} = 0.0078$.

4.3 Reinforcing Bar in Tension and Compression

Two different types of constitutive models are used: 1) linear-elastic (for FRP type bar); and 2) strain-hardening type. Given below is only the second type of reinforcement materials[10]:

$$\sigma_s = E_s\cdot\varepsilon_s \quad \text{for} \quad 0 \le \varepsilon_s \le \varepsilon_y \qquad (13)$$

$$\sigma_s = f_y \quad \text{for} \quad \varepsilon_y < \varepsilon_s \le \varepsilon_{sh}$$

$$\sigma_s = f_y + E_{sh}\cdot(\varepsilon_s-\varepsilon_{sh})\cdot\left(1-E_{sh}\cdot\frac{\varepsilon_s-\varepsilon_{sh}}{4(\sigma_{su}-f_y)}\right) \quad \text{for} \quad \varepsilon_{sh} < \varepsilon_s \le \varepsilon_{su}$$

4.4 Prestressing Tendon in Tension

Two different types of constitutive models are incorporated: 1) linear elastic (for FRP tendons); and 2) strain-hardening (for steel tendons). The second type of the model is presented below:

$$\sigma_{ps} = E_{ps}\cdot\varepsilon_{ps}\cdot\left[Q + \frac{1-Q}{\left(1+\left(\frac{E_{ps}\cdot\varepsilon_{ps}}{K\cdot f_{py}}\right)^N\right)^{\frac{1}{N}}}\right]$$

where :

$$Q = (\sigma_{pu} - K \cdot f_{py}) / (E_{ps} \cdot \varepsilon_{ps} - K \cdot f_{py})$$

For G270 tendons, N=7.344, K=1.0618 and Q=0.01174[11] are used.

5. COMPARISON WITH TEST RESULTS

The developed model is compared with test results for bonded and unbonded prestressed concrete beams. Some of these beams are fully or with steel tendons, or with both. Also some or partially prestressed either with FRP tendons beams are cast with different layers of different cementitious composite matrices in a section.

5.1 Bonded Prestressed Concrete Beam with Steel Tendons.

Fully or partially prestressed concrete beams are used for the comparison(Fig.5). Loading conditions and typical cross sections as well as their properties are given in[11].

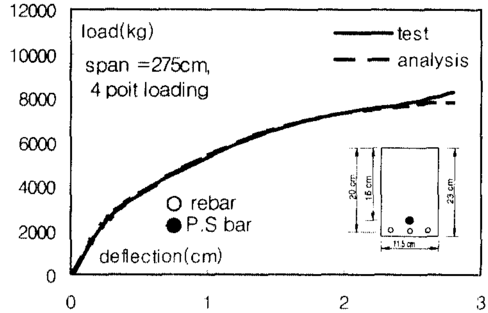

Fig.5(b) Beam PP2S2

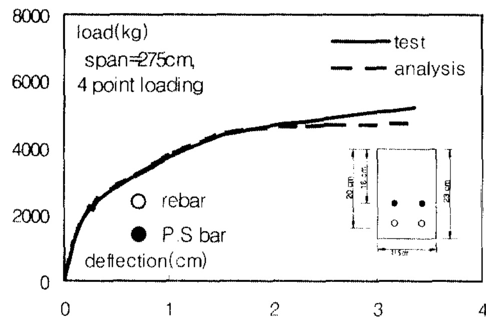

Fig.5(c) Beam PP2SE

Fig.5 Comparison between the Model and the Test Results for Bonded Prestressed Concrete Beams[11]

Table 1 Summary of reinforcing parameters[11]

Beam	Tensile Steel		Comp. Steel		f'c (kg/cm²)	fy (kg/cm²)	Tendon						
	Area (cm²)	ds (cm)	Area (cm²)	ds' (cm)			Area (cm²)	fpu (kg/cm²)	Eps (kg/cm²)	ε_{pu}	fpy (kg/cm²)	dps (cm)	Pe/bar (kg)
PS3	-	-	0.11	2.54	403	4921	1.65	19000	196067	0.069	17120	12.1 15.9 19.7	5244
PP2S2	0.97	20	0.11	2.54	372	4921	0.55	19000	196067	0.069	17120	15.9	5130
PP2S3	1.42	20	0.11	2.54	436	4921	1.10	19000	196067	0.069	17120	15.9	5130

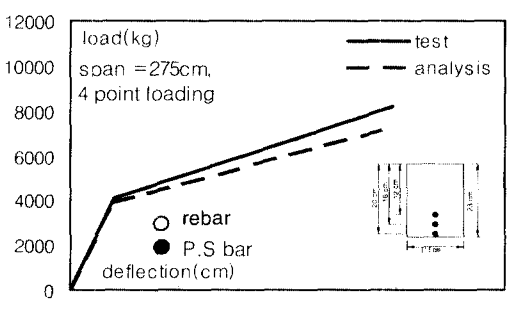

Fig.5(a) Beam PS3

5.2 Bonded Prestressed Concrete Beam with CFRP Tendon and Composite Materials

Test results of bonded prestressed concrete beam with CFRP tendons and SIFCON [2] are compared with model predictions(see Fig.6). It is worth mentioning that the model is able to simulate residual resistance after brittle failure of beam due to fracture of one or more FRP tendons.

Table 2 Summary of reinforcing parameters[2]

Beam	Tensile steel			f'c (kg/cm²)	CFRP Tendon					
	Steel Area(cm²)	ds(cm)	fy (kg/cm²)		Area (cm²)	fpu (kg/cm²)	E (kg/cm²)	ε_u	dps(cm)	Pe/bar(kg)
TC6	2#D13 (2.54)	28	4218	429 (plain)	2CFCC(0.61) 2CFCC(0.61)	14060 14060	1406000 1406000	0.015 0.015	17.2 23.5	4050 4050
RFC	2G270 (1.97)	28	17118	429 (plain) 436 (SIFCON)	1CFCC(0.30) 2CFCC(0.61) 1CFCC(0.30)	14060 14060 14060	1406000 1406000 1406000	0.015 0.015 0.015	19.1 23.5 27.3	4050 4050 4050
RFCa 1	2G160 (10.13)	28	10000	436 (SIFCON)	1CFCC(0.30) 2CFCC(0.61) 1CFCC(0.30)	14060 14060 14060	1406000 1406000 1406000	0.015 0.015 0.015	19.1 23.5 27.3	4050 4050 4050

SIFCON						
Compression					Tension	
ε^c_{max}	Eo (kg/cm²)	σ_i(kg/cm²)	ε_i	σ_{asym}(kg/cm²)	σ^t_{max} (kg/cm²)	ε^t_{max}
0.012	56240	310	0.0227	140	98	0.0078

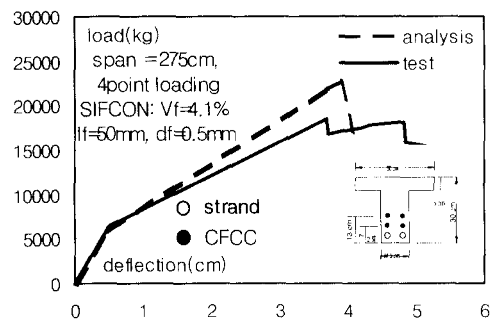

Fig.6(a) Beam TC6

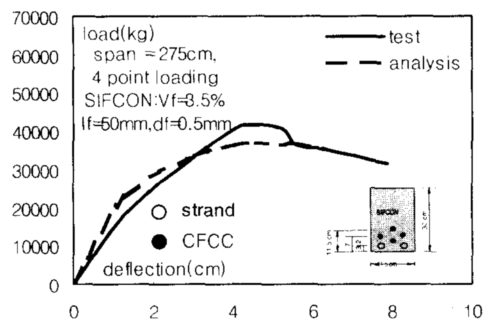

Fig.6(c) Beam RFCa-1

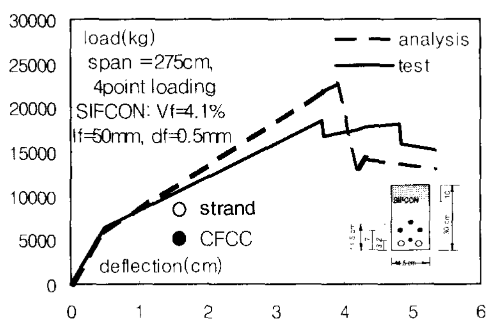

Fig.6(B) Beam RFC

Fig.6 Comparison between the Model and Test Results for Prestressed Beams with Bonded CFRPTendon(CFCC)[2]

5.3 Unbonded Prestressed Concrete Beam

Experimental results from Tao et.al[12,13] are used to compare with analytical model. Good agreements between model predictions and experimental results are shown in Fig.7

Table 3 Summary of reinforcing parameters[12,13]

Beam	Ref.	Tensile Steel			f'c (kg/cm²)	Tendon						
		Area(cm²)	ds(cm)	fy(kg/cm²)		Area (cm²)	fpu (kg/cm²)	Eps (kg/cm²)	ε_{pu}	fpy (kg/cm²)	dps(cm)	Pe/bar(kg)
A-1	12	1.57	25	4387	312	0.98	18251	210900	0.07	14936	22.0	9049
A-2	12	2.36	25	4387	312	1.59	18251	210900	0.07	14936	22.0	13114
B-1	13	1.57	25	4387	312	0.58	18251	210900	0.07	14936	22.0	5144
B-2	13	4.65	25	4077	312	1.57	18251	210900	0.07	14936	22.0	13613

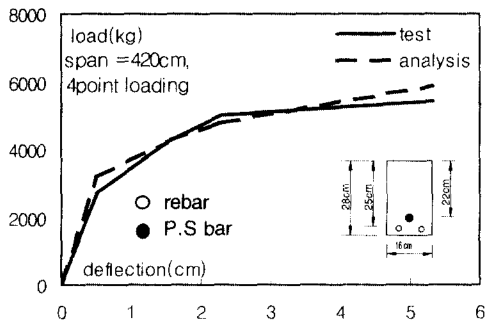

Fig.7(a) Beam A-1

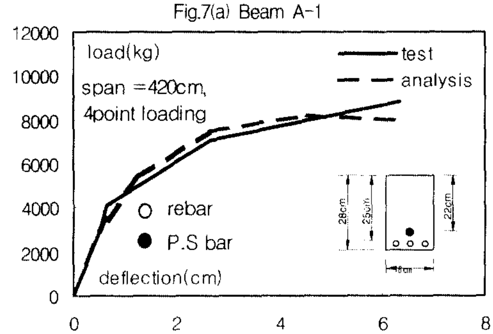

Fig.7(b) Beam A-2

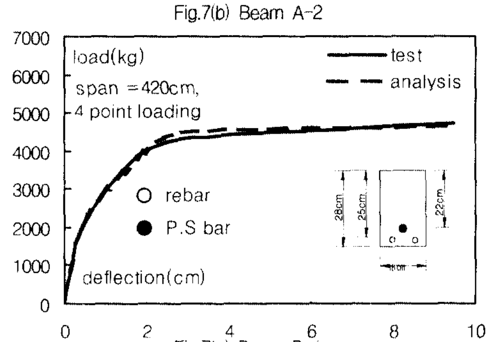

Fig.7(c) Beam B-1

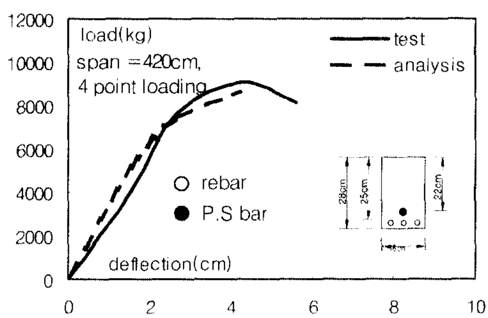

Fig.7(d) Beam B-2

Fig.7 Comparison between the Model and Test Results for Prestressed Beams with Unbonded Tendon[12,13]

6. CONCLUSION

The analytical model is developed in order to predict the flexural responses of beam which contains either conventional reinforcements, advanced composite materials, or their combinations. Main algorithm has been developed based on the strain compatibility for simply supported beams. The block concept is used, which can be regarded as an intermediate modeling method between the couple method with one block and the layered method with multiple sliced blocks in a section.

The main features of the model can be summarized as follows : 1) the model can predict the flexural behavior of composite bonded prestressed beam and unbonded prestressed concrete beam; 2) a beam having various combinations of reinforcing materials - i.e., linear elastic material like FRP rebar or tendon and strain hardening material like conventional rebar or steel tendons - can be modeled; and 3) the model can trace the residual flexural response of a beam after one (or more) of the main reinforcements is (are) fractured. This feature is useful when an overall flexural behavior of the beam reinforced with both brittle FRP bars (prestressed or nonprestressed) and conventional ductile reinforcements are to be simulated.

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요 약

최근에 섬유보강 콘크리트와 섬유보강 플라스틱과 같은 복합소재를 기존의 구조부재와 연루하여 적용하고자 하는 연구가 많이 진행되고 있다. 이러한 첨단 복합소재를 적절히 사용하기 위해서는 이들 소재를 구조물 또는 그 일부에 적용할 시에 저항 메카니즘과 파괴양상에 대한 이해가 요구된다.

본 연구에서는 특별히 단면이 시멘트 복합소재를 층으로 갖거나 FRP 텐던 등으로 보강된 Bonded 및 Unbonded 프리스트레스트 콘크리트보의 비선형 휨 거동을 예측할 수 있는 이론모델을 개발하고자 하였다. 본 모델은 한개의 단면으로 해석하는 Couple Method와 여러개의 적층으로 나눈 Layered Method의 중간적인 모델이라고 할 수 있는 블록개념(Block Concept)이 적용되었다. 주어진 하중에 대한 처짐을 구하기 위하여 N개의 축력에 대한 평형조건과 N개의 휨에 대한 평형조건을 이용하여 보 전체의 2N개의 변수를 구하였다. 본 모델은 여러형태로 배근된 Bonded 그리고 Unbonded 프리스트레스트 콘크리트보의 휨 거동을 성공적으로 예측하였다. 또한 취성적인 FRP 텐던이 파괴됨에 따른 보의 갑작스런 하중저하 이후의 점진적인 내력증가도 성공적으로 모사하였다. 이는 취성적인 FRP텐던으로 보강된 프리스트레스트 보의 전반적인 하중-처짐을 추적하는데 유용하다.

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