Failure Analysis Model for Tensioned FRP Dowels

인장을 받는 FRP 다우얼의 파괴 해석 모델

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요 약

본 연구에서는 콘크리트 속에서 인장과 전단을 받는 FRP 다우얼의 거동과 파괴를 예측할 수 있는 수리적인 파괴 해석 모델을 개발하였다. 다우얼 파괴해석 모델은 다우얼 작용과 파괴기준에 대한 두개의 하위 모델로 구성되어 있는데 이들을 수정, 결합하여 만들어졌다. 다우얼 작용에 대한 모델로는 BEF 모델을 기초로 하여 두가지의 지수를 새로이 정의, 사용하였는데 하나는 콘크리트 지지 강성을 변화시키기 위한 변위 정도 지수이고 다른 하나는 긴장된 케이블의 반력을 고려하기 위한 인장 지수이다. 인장과 전단이 작용하는 FRP 다우얼의 파괴 모델로는 Tsai-Hill 파괴기준이 사용되었고 이 기준을 적용하기 위하여 파괴 계수를 정의하였다. 개발된 파괴 해석 모델은 긴장된 FRP 다우얼의 극한 전단력과 극한 변위를 예측하는데 사용하였고, 해석결과는 여러 인장응력을 가진 FRP 다우얼의 시험결과와 비교하였다.

1. Introduction

Few mathematical models of dowel action exist in the technical literature. This is primarily because dowel action in the concrete matrix involves many parameters. Furthermore, until now the dowel model for tensioned dowels is not available. Modeling a

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• 본 논문에 대한 토의를 1998년 6월 30일까지 학회로 보내주시면 1998년 8월호에 토의회답을 게재하겠습니다.

dowel as a beam resting on an elastic and cohesionless foundation(BEF model) is still the most expedientway to describe the dowel mechanism, although it roughly predict the dowel behavior for steel bars. The BEF model may be used successfully in analyzing the dowel action as long as the embedding concrete is elastic and an appropriate concrete subgrade stiffness is selected.

According to an experimental study by Dei Poli et al.(1), the behavior of a dowel is mostly elastic both in the reinforcing bar and the concrete embedment for shear force not exceeding 40% of the ultimate capacity. In the dowel tests carried out by Park(5,6), fiber reinforced plastic(FRP) dowel tendons showed an almost elastic behavior and ruptured at 40% and 50% of the ultimate capacity of the steel tendons and reinforcing bar, respectively. Therefore, it is believed that the BEF model can be used successfully to model dowel behavior of FRP tendons, provided an appropriate subgrade stiffness of the concrete embedment is used and the cable effect of tensioned dowels is considered.

Since fiber reinforced plastic(FRP) material is elastic up to failure, a failure criterion is needed to predict the failure of FRP dowels. The Tsai-Hill failure criterion is widely quoted in textbooks on composite materials and often used in laminate analysis(3). The experi-mental data on laminates by Sinclair and Chamis(7) and Knappe and Schneider (4) showed that the Tsai-Hill criterion gives a more reliable prediction than the maximum stress criterion. Therefore, it was expected that the Tsai-Hill failure criterion would give better predictions for FRP dowels. The Tsai-Hill failure criterion takes account of the interaction of tensile and shear stress of dowels at failure.

2. BEF Model

BEF model is a linear elastic model in which the dowel is modeled as a beam element resting on a line of spring-like point supports describing the surrounding concrete. This model has the advantage of lumping together all characteristics of the dowel-to-concrete interface in a single parameter, the subgrade stiffness of the embedment, k. Fig. 1 shows the BEF model for a dowel in concrete matrix.

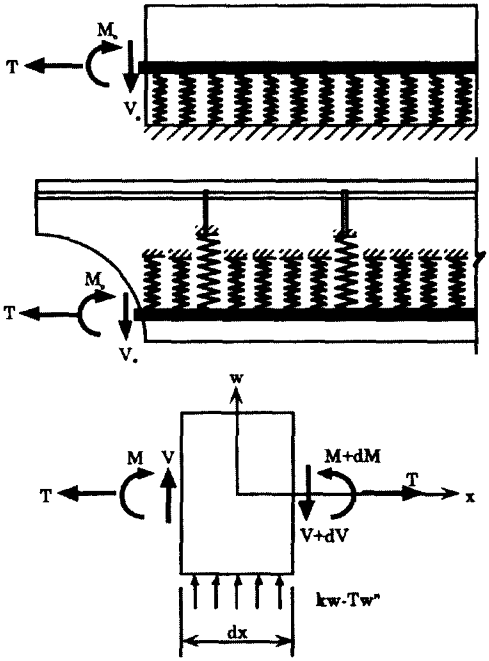

Fig. 1 BEF model for a dowel in concrete

The equations for predicting dowel deflection, shear force, and bending moment are derived next using the BEF model. This version of the BEF model, called Filonenko-Borodich Foundation(2), accounts for the effect of axial tension force in the tendon, which is not considered in the conventional BEF model (Winkler Foundation model(13)). The differential equations describing the deflection, shear force, and bending moment are given by:

$$EIw'''' - Tw'' + kw = 0 \qquad (1)$$

where EI is the stiffness of a beam

For the practical case $T \le \sqrt{4kEI}$, the general solution of Eq. 1 may be expressed as:

$$w(x) = e^{-\phi\beta x}(C_1 \cos\phi\alpha x + C_2 \sin\phi\alpha x) + e^{\phi\beta x}(C_3 \cos\phi\alpha x + C_4 \sin\phi\alpha x) \qquad (2)$$

where ϕ, α, β, and ψ are given by:

$$\phi = \sqrt[4]{\frac{k}{EI}}$$

$$\alpha = \sin\frac{\psi}{2}, \qquad \beta = \cos\frac{\psi}{2}$$

$$\psi = \arctan\left(\frac{\sqrt{4kEI - T^2}}{T}\right), \quad (0 < \psi < \frac{\pi}{2})$$

We consider the semi-infinite beam on an elastic foundation. The concentrated loads, P_0 and M_0, act at the end, $x=0$. The constants of integration C_1, C_2, C_3, and C_4, are determined from boundary conditions. Because deflection, w, must vanish at $x=0$, we must have $C_3=0$ and $C_4=0$. The flowing two conditions are needed to solve for C_1 and C_2.

$$M_0 = -EI\frac{d^2w}{dx^2}\bigg|_{x=0} \quad and \quad -p_0 = -EI\frac{d^3x}{dx^3}\bigg|_{x=0} \qquad (3)$$

Accordingly, from Eqs. 2 and 3, we find

$$C_1 = \frac{2\beta P_0 - \phi(3\beta^2 - \alpha^2)M_0}{\phi^3 EI},$$

$$C_2 = \frac{(\beta^2 - \alpha^2)P_0 + \phi\beta(3\alpha^2 - \beta^2)M_0}{\phi^3 \alpha EI} \qquad (4)$$

Therefore, the vertical displacement of the dowel at a distance x from the shear plane along its axis is

$$w(x) = e^{-\phi\beta x}(C_1 \cos\phi\alpha x + C_2 \sin\phi\alpha x) \qquad (5)$$

The bending moment of the dowel at a distance x is given by:

$$M(x) = -EI\phi^2 e^{-\phi\beta x}\Big[\big\{(\beta^2 - \alpha^2)C_1 - 2\alpha\beta C_2\big\}\cos\phi\alpha x + \big\{(\beta^2 - \alpha^2)C_2 + 2\alpha\beta C_1\big\}\sin\phi\alpha x\Big] \qquad (6)$$

The shear force of the dowel at a distance x is given by:

$$V(x) = -EI\phi^3 e^{-\phi\beta x}\Big[\big\{\beta(3\alpha^2 - \beta^2)C_1 + \alpha(3\beta^2 - \alpha^2)C_2\big\}\cos\phi\alpha x + \big\{\alpha(\alpha^2 - 3\beta^2)C_1 + \beta(3\alpha^2 - \beta^2)C_2\big\}\sin\phi\alpha x)\Big] \qquad (7)$$

2.1 Dowel Test Results

The dowel test results carried out by Park (5,6) were used to derive the following displacement level index and tension index. Also, the test results were compared with the dowel failure analysis results using the developed model. The dowel test was performed using block type specimen with 7.5 mm diameter carbon fiber reinforced plastic (CFRP) tendons. The CFRP tendon had a specified strength of 21.63 ton/cm^2. Table 1 shows the dowel test results of CFRP tendons.

Table 1 Tension ratios and results of dowel test with FRP tendons

Specimen ID	Jacking force ratio (%)	Effective prestressing force (ton)	Concrete strength (kg/cm^2)	Ultimate dowel shear force (ton)	Ultimate displace-ment(mm)
1	F_i=0%	0	400	1.220	2.41
2	F_i=40%	1.87(26.7%)	400	1.137	1.85
3	F_i=40%	1.87(26.7%)	400	1.184	1.65
4	F_i=60%	3.01(43.1%)	390	1.110	1.33
5	F_i=60%	3.01(43.1%)	390	1.100	1.20
6	F_i=80%	4.10(58.7%)	370	0.957	1.02
7	F_i=80%	4.10(58.7%)	370	1.020	1.07

where F_i=jacking force ratio

2.2 Displacement Level Index

A displacement level index was introduced and the concrete subgrade stiffness was varied according to the displacement level. The displacement level index was defined as:

$$\lambda = \frac{k}{k_{uo}} \tag{8}$$

where

λ : displacement level index

k : subgrade stiffness of concrete at a certain level of displacement

k_{uo} : ultimate subgrade stiffness of concrete when a non-tensioned FRP tendon reaches its maximum shear displacement

Therefore, the subgrade stiffness of the concrete at a certain level of shear displacement is

$$k = \lambda k_{uo} \tag{9}$$

The displacement level index for the concrete subgrade stiffness was derived from the measured load-displacement curve of the dowel tendon with no tensile force. The relationship was formulated as follows:

$$\lambda = 1.7 - 0.7\left(\frac{\delta}{\delta_{uo}}\right) \tag{10}$$

where

δ : shear displacement

δ_{uo} : ultimate shear displacement of non-tensioned tendon

Fig. 2 shows the displacement level indices observed at the ultimate state for the different tension ratios. A relationship between the displacement level index and the tension ratio was also derived from the dowel test results and was expressed as follows:

$$\lambda = 1.0 + 0.7\left(\frac{T}{T_u}\right) \tag{11}$$

where

T : applied tensile force of tendon

T_u : ultimate tensile strength of tendon

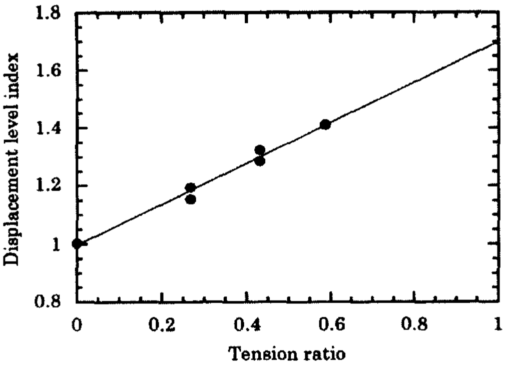

Fig. 2 Displacement level index and tension ratio

2.3 Tension Index

To analyze the dowel action of the tensioned tendon, it was necessary to consider the cable effect, which is a resisting reaction of a tensioned cable against a change in its curvature. Eqs. 5, 6, and 7 in the BEF model were originally derived for a semi-infinite beam on an elastic foundation. Thus, they do not include the effect of the upward resisting reaction force at the shear plane. The resisting force against the total curvature change in the deflected tendon is already included in the applied dowel force, that is

$$V_I = V_s + R_c \tag{12}$$

where

V_I : applied dowel shear force to infinite tendon

V_s : dowel shear force in case of semi-infinite tendon

R_c : upward resisting force due to cable effect

Theoretically, the resisting force, R_c, is the product of tensile force, T, and the slope of the tendon, θ, at the shear plane.

$$R_c = T\sin\theta \approx T\theta \quad (\theta \approx small),\ (\theta\ in\ radian) \tag{13}$$

In this dowel analysis the cable effect was accounted for by introducing a tension index, which was analytically derived from the results of the dowel test. The tension index was defined as:

$$\xi = \frac{\delta_I}{\delta_S}\ (when\ V_I = V_S)\ \text{or}\ \xi = \frac{V_S}{V_I}\ (when\ \delta_I = \delta_S) \quad (14)$$

where
ξ : tension index
δ_I : shear displacement of infinite dowel tendon
δ_S : shear displacement of semi-infinite tendon
V_S : dowel shear force in case of semi-infinite dowel
V_I : applied dowel shear force to infinite tendon

Fig. 3 shows a plot of the tension indices for different tension ratios obtained from the dowel tests. As shown by the line in Fig. 3, the relationship was described as follows:

$$\xi = 1.0 - 0.7\left(\frac{T}{T_u}\right) \quad (15)$$

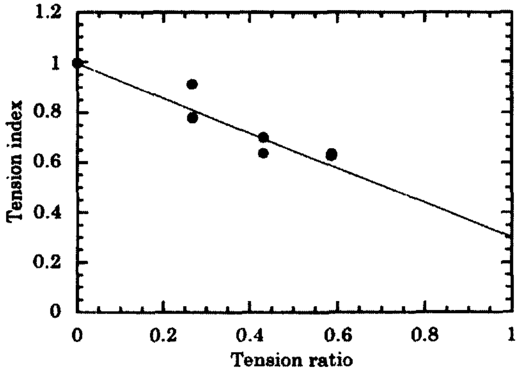

Fig. 3 Tension index and tension ratio

Therefore, the equations for the dowel displacement, bending moment, and shear force may be expressed as :

$$w_I(x) = \xi w(x) \quad (16)$$
$$M_I(x) = \xi M(x) \quad (17)$$
$$V_I(x) = V(x) \quad (18)$$

where
$w_I(x)$: dowel displacement for infinite tendon
$M_I(x)$: dowel bending moment for infinite tendon
$V_I(x)$: dowel shear force for infinite tendon
$w(x)$: dowel displacement in Eq. 5 for semi-infinite dowel
$M(x)$: dowel bending moment in Eq. 6 for semi-infinite dowel
$V(x)$: dowel shear force in Eq. 7 for semi-infinite dowel
x : distance from the shear plane

3. Failure Model

The failure of FRP tendons subjected to tensile and shear stresses can be simulated using one of two models: the maximum stress criterion and the Tsai-Hill criterion.

3.1 Failure Criteria

3.1.1 Maximum Stress Criterion

In the simple maximum stress criterion, it is assumed that failure occurs when a stress parallel or normal to the fiber axis reaches the critical value, that is, when one of the following is satisfied(3):

$$\sigma_1 \geq \sigma_{1u} \quad (19)$$
$$\sigma_2 \geq \sigma_{2u} \quad (20)$$
$$\tau_{12} \geq \tau_{12u} \quad (21)$$

where σ_1, σ_2 and τ_{12} are the imposed tensile and shear stresses, referring to the orthogonal directions in the plane, and the material properties σ_{1u}, σ_{2u} and τ_{12u} are the measured failure stresses, in tension and shear, when each is applied in isolation.

3.1.2 Tsai-Hill Criterion

The strength of a unidirectional tendon is determined by the interaction of tensile strength along and across the tendon, and in-plane shear strength. One of the best-known failure criteria which take account of such interaction is the maximum work theory, commonly referred to as the Tsai-Hill criterion. It is based on the von Mises yield criterion which was originally applied to homogeneous and isotropic materials. It was then expanded and modified by Hill to anisotropic bodies and applied to composite materials by Tsai (3,10,11). The criterion is expressed as:

$$\left(\frac{\sigma_1}{\sigma_{1u}}\right)^2 - \left(\frac{\sigma_1\sigma_2}{\sigma_{1u}\sigma_{2u}}\right) + \left(\frac{\sigma_2}{\sigma_{2u}}\right)^2 + \left(\frac{\tau_{12}}{\tau_{12u}}\right)^2 \leq 1 \qquad (22)$$

In most conditions of FRP tendons under tension and shear, the tensile or compressive stress across a tendon, σ_2, is small (i.e., the confinement effect of the surrounding concrete is negligible). Therefore, the tendon failure criterion can be written in a reduced form as follows:

$$\left(\frac{\sigma_1}{\sigma_{1u}}\right)^2 + \left(\frac{\tau_{12}}{\tau_{12u}}\right)^2 \leq 1 \qquad (23)$$

3.2 Failure Factor

For the failure analysis for tensioned FRP dowel tendons, a failure factor was defined as:

$$F_f = \sqrt{\left(\frac{\sigma}{\sigma_u}\right)^2 + \left(\frac{\tau}{\tau_u}\right)^2} \qquad (24)$$

where

F_f : failure factor

σ' : applied tensile stress of tendon

σ_u : ultimate tensile stress of tendon

τ : applied shear stress of tendon

τ_u : ultimate shear stress of tendon

According to the Tsai-Hill criterion, the FRP tendons subjected to tensile and shear stresses fail when the failure factor reaches 1.0. The failure factor is the distance from the origin to the failure envelope in the figure of tensile stress and shear stress ratios.. In dowel failure analysis the failure factor was used to locate the critical section, at which the tensioned tendon would fail, by calculating the failure factor along the tendon.

4. Dowel Failure Analysis

Failure analysis of the dowel test results was performed using the dowel failure analysis model, which was developed by modifying the BEF Model and the Tsai-Hill failure criterion, and combining them.

4.1 Assumptions

For the failure model of the tensioned dowel the following assumptions were made. The tensile stress due to effective prestressing force is given by:

$$\sigma_t = \frac{T}{A_t} \qquad (25)$$

where At is the area of FRP tendon

The bending stress due to dowel bending moment is calculated as follows:

$$\sigma_b = \frac{M}{J} \qquad (26)$$

where M is the dowel bending moment due to dowel force and J is the section modulus of the tendon.

Thus, the total tensile stress is

$$\sigma = \sigma_t + \sigma_b \tag{27}$$

The shear stress due to dowel shear force is determined as follows: According to the theory of elasticity(9), the shear stress along the vertical diameter of the circular cross section is

$$\tau = \frac{(3+2\nu)V_w}{8(1+\nu)I_w}(r^2 - y^2) \tag{28}$$

where

V_w : applied shear force to each wire (=V/7)

V : applied dowel shear force to each tendon

I_w : moment of inertia of wire

ν : Poisson's ratio

r : radius of wire

y : distance from the middle of wire in the vertical direction

From the above equation, it can be seen that the magnitude of the shear stress depends on the magnitude of Poisson's ratio. The Poisson's ratio of FRP composites can be predicted by a simple "rule of mixtures" for axial stressing of the composite(3).

$$\nu = f\nu_f + (1-f)\nu_m \tag{29}$$

where

f : volume fraction of fibers

ν_f : Poisson's ratio of fiber

ν_m : Poisson's ratio of matrix.

Therefore, the CFRP tendons used in this study, trade-name CFCC, which are made of carbon fibers and epoxy resin, have a Poisson's ratio of 0.268.

4.2 Analysis Results

Fig. 4 shows the failure factors along the tendons with the different tension ratios. The dowel failure analysis shows that all the tendons with the tension ratio ranging from 0 to 0.8 are critical in the middle of the top wire at the shear plane. However, when the tension ratio increases more than about 0.8, the critical point moves from the shear plane to a section distant about 8.9 mm from the shear plane, at which the dowel bending moment has its maximum value (at which the dowel shear force is zero). The failure factor at that section reaches its maximum value at the top fiber of the upper wire of the tendon. This transition of the critical points from the shear plane to the maximum bending moment section means that the tendons with high tension ratio break due only to the tensile stress rather than to the interaction of tensile and shear stresses.

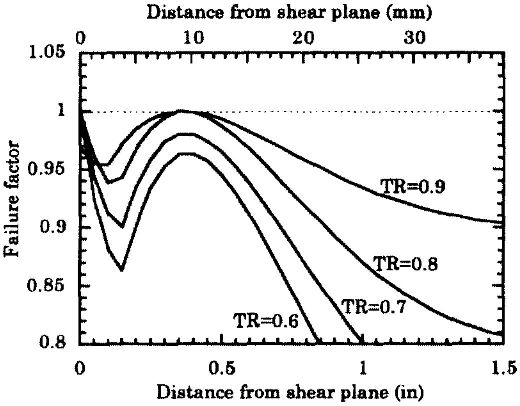

Fig. 4 Failure factors along the FRP dowels

Figs. 5 and 6 show the dowel test results with the failure envelopes, which were derived by the failure analysis model. The solid line in the figures is the failure envelope for the failure factor to be 1.0 at a certain point along the tendon, while the dotted line is the failure envelope simply at the shear plane. As can be seen in the figures, the failure envelopes are both in very good agreement with the

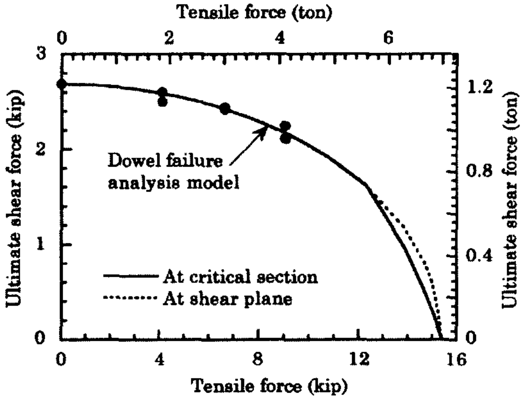

Fig. 5 Failure envelope by failure analysis model and test results

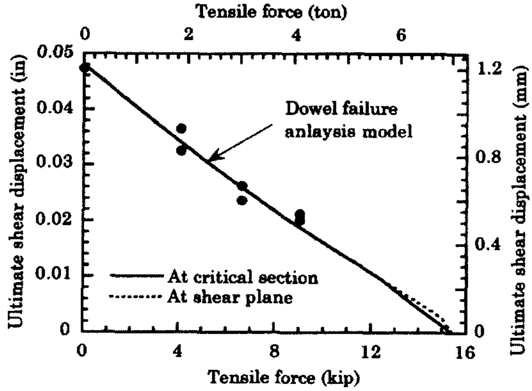

Fig. 6 Ultimate shear displacement curve by failure analysis model and test results

experimental test results.

The failure envelopes and test results in Fig. 5 shows the relationship of the tensile and ultimate dowel shear forces. The ultimate dowel force of the tendon decreases elliptically as the tensile force increases. The failure envelope in Fig. 6 shows the rela-tionship of the tensile force and ultimate dowel displacement. The ultimate dowel shear displacement decreases linearly with an increase of the tension force. This relationship in the figure can be simply expressed as:

$$\frac{\delta_u}{\delta_{uo}} = 1 - \frac{T}{T_u} \qquad (30)$$

Eq. 29 can be also derived by combining Eqs. 10 and 11.

5. Conclusions

Based on the analytical study on the mathematical model for tensioned FRP dowel, the following conclusions were made.

1) The developed FRP dowel failure analysis model, which is based on the BEF model and the Tsai-Hill failure criterion, can be used successfully to predict the dowel behavior and FRP dowl failure of tensioned FRP tendons. In this model the BEF model was modified as suggested in this study by the displacement level index and tension index. To use the Tsai-Hill failure criterion for predicting FRP dowl failure, a failure factor was defined to locate the critical section in tensioned FRP dowels.

2) The displacement level index, which was defined as the ratio of the subgrade stiffness of the concrete to a base value, gave a more accurate prediction of dowel displacement. The analytically derived index was observed to decrease linearly with the displacement ratio and increase linearly with the tension ratio.

3) In analyzing the tensioned dowel using the BEF model for a semi-infinite beam it is necessary to consider the cable effect. The analytically derived tension index, which was introduced to consider the cable effect, was very satisfactory for the behavior of tensioned dowel tendons. It was observed to decrease linearly with the tension ratio.

4) The test results showed that the dowel failure analysis model gives a highly reliable prediction of FRP tendon failure under combined tensile and shear forces for practical purposes. The criterion states that

the ultimate dowel shear stress decreases elliptically as the tensile stress increases.

5) The ultimate dowel displacements predicted by the failure analysis model developed in this study agree well with the experimental dowel test results of this study. They decrease linearly as the tensile force increases.

6) The critical section, at which the tensioned tendons fail, is the shear plane or the maximum dowel bending moment section. When the tension ratio is less than about 0.8, the dowel tendon is likely to break at the shear plane, while when it is more than about 0.8, the critical section is the maximum bending moment section.

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ABSTRACT

A mathematical failure analysis model was developed to predict the behavior and failure of FRP dowels subjected to tensile and shear forces. The model consisted of two sub-models: one for the dowel action and the other for the failure criterion of the FRP dowels. These two sub-models were modified and combined to develop a failure analysis model for tensioned FRP dowels. To model dowel action, the BEF (Beam on an Elastic Foundation) model was adopted and modified. In the modification, two indices were introduced: one is a displacement level index to vary the concrete subgrade stiffness, and the other is a tension index to consider the cable effect. To model the failure of FRP dowels, the Tsai-Hill failure criterion was used as the failure criterion for the dowels subjected to tensile and shear forces. To apply the Tsai-Hill failure criterion for the tensioned FRP dowels, a failure factor was defined. Finally, the failure analysis model was used to predict the ultimate dowel shear force and corresponding displacement. The analysis results were compared with the test results of the FRP dowels with different tensile stresses.

Keywords : FRP, dowel, dowel failure analysis model, BEF model, Tsai-Hill failure criterion

(접수일자 : 1997. 10. 13)