기동시 회전축계의 지진응답 거동

Seismic Behavior of Rotating Shaft System at Start-up

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국문요약

회전축계가 기동시 지진하중을 받을 때의 응답 거동을 조사하였다. 지진과 기동으로 인하여 회전축계가 불안정하다면, 과다한 진동이 발생할 뿐만 아니라 회전기계의 회전자가 고정자와 부딪혀 기계 성능을 발휘하지 못하게 할 것이다. 그래서 면진장치를 갖춘 기초에 지진동이 작용할 때에, 회전축계의 응답을 모사하였다. 기동시 회전축계의 과도 응답을 얻기 위하여 우선 회전축계의 운동방정식을 유도하였다. 유도한 운동방정식은 비선형이어서 Runge-Kutta 수치해석법을 이용하여 응답을 계산하였으며, 기동 운전모드에 따른 거동뿐만 아니라 면진스프링의 강성을 매개변수로 취하여 회전축계의 응답거동을 고찰하였다.

주요어 : 회전축계, 운전모드, 면진, 지진응답거동

ABSTRACT

A rotating shaft system subjected to seismic motions has been investigated for the various operating modes at start-up. During an earthquake excitation, the rotor may hit the stator of machines due to the excessive deformation of shaft, and thus the response of rotating shaft system of which foundation is supported by the vibration isolation devices has been simulated. In order to examine the transient response of the rotating shaft system at the start-up to both the various operating conditions and the seismic excitation simultaneously, nonlinear equations of motion are derived and solved numerically using Runge-Kutta method. The response of the rotating shaft system is calculated according to the operating modes as recommended by the machine and the system parameters such as the spring stiffness of isolation devices.

Key words *: rotating shaft system, operating mode, seismic isolation, seismic response*

1. Introduction

The rotating shaft systems are widely used in a variety of machines ranging from the small precession motor to the turbine-generator in power plants. For the small rotating machines the critical speed is greater than the operating speed, and thus the stability at start-up is not important. However the critical speed of the large rotating machine are less than the operating speed, and the stability problem is occurred because the machine speed should pass the critical speed at start-up. In addition to that, if the machine foundation is installed on the vibration/seismic isolation device, the behavior of the rotating shaft system at start-up could be investigated corresponding to the operating mode and the seismic excitation. Also it is necessary that the isolation device should be designed by considering the characteristics of earthquake and of machines and that the response of supporting structure including isolation device be accurately predicted by a proper

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method.

In order to predict the transient response of the rotating shaft system, a set of five nonlinear differential equations of motion have been derived from the kinetic and the potential energies of system. There are two coordinate systems as shown in Fig. 1. One is x_g and y_g coordinate of which origin is on the ground, the other is x and y originated at bearing or foundation. Actually the coordinate $x_g - y_g$ stands for the seismic ground motion. As a result the rotating shaft system is considered to be five degree of freedom system and the governing equation of motions have five variables. Two represents the plane motion of rotating disk, two represent the plane motion of foundation, and one represents the rotation angle of rotor.

The transient motion of the rotating shaft system at start-up can be investigated according to the operating modes. An operating mode can be determined by the torque input. The reason is that the angular acceleration of rotor is proportional to torque. In the study, two-step operation mode is adopted to investigate the dynamic response of the rotating shaft system under both the imbalance, which is defined by the eccentricity(e) in Fig. 1, and the seismic ground motion. The unbalance is given by the eccentricity from the center of rotation. The first step is from zero to speed between 0.8 and 0.9 of the critical speed(w_{cr}), which is called the intermediate speed, and the second step is from the end of the first step to the normal operating speed of machine (Fig. 2). More torque may be required to pass the critical speed as fast as possible. If the shaft rotates near the critical speed, the amplitude of shaft vibration increases very fast and the system finally may be in danger.

Thus, the torque rate at the second step is greater than that of the first step. If the torque input at the second step were not enough, the rotating speed might not reach the operating speed and the shaft rotates near the critical speed, which is called the phenomena of limited power supply. Since the machine speed is near the critical speed, the amplitude of rotor is so large that the rotor hits the stator.

For the low-tuned foundation, the vertical fundamental frequency is much lower than the running speed of machine and the ratio of the machine speed to the fundamental frequency of foundation is greater than 3. But the fundamental frequencies of the low-tuned foundations generally coincide with the predominant frequency range of earthquake, and thus the fundamental frequency of foundation with seismic isolation device should be less than 1 Hz. Therefore in this study the spring stiffness of isolation has been treated as a parameter.

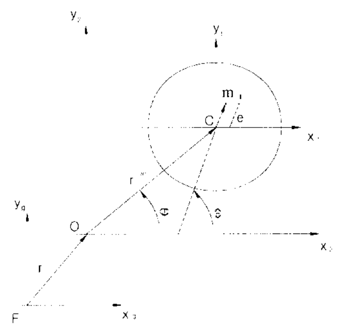

Fig. 1 Coordinate system of rotating shaft :
$\varphi=$ precession angle, $\theta=$ rotating angle
$C=$ center of rotation, $e=$ eccentricity

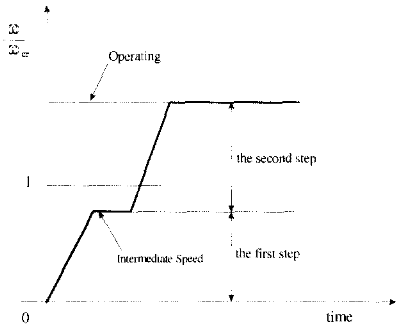

Fig. 2 Two-steps operating mode

2. Equations of Motion

The position vector to the center of rotor as shown in Fig. 1 can be expressed as

$$\mathbf{r}^{m_1/F} = \mathbf{r}^{O/F} + \mathbf{r}^{C/O} + \mathbf{r}^{m_1/O} \tag{1}$$

where $\mathbf{r}^{O/F}$ is the position vector to bearing housing relative the fixed reference frame and can be expressed in terms of the seismic ground motion as

$$\mathbf{r}^{O/F} = x_g\ \mathbf{n}_x + y_g\ \mathbf{n}_y \tag{2}$$

where $\mathbf{n}_x$, $\mathbf{n}_y$ are the unit vectors in the directions of x_g and y_g, respectively. $\mathbf{r}^{C/O}$ is the position vector of rotating center relative the bearing housing or foundation,

$$\mathbf{r}^{C/O} = x_1\ \mathbf{n}_x + y_1\ \mathbf{n}_y \tag{3}$$

and the position vector from the elastic center to the mass center is given

$$\mathbf{r}^{m_1/O} = e\cos\theta\ \mathbf{n}_x + e\sin\theta\ \mathbf{n}_y \tag{4}$$

in which the rotation angle θ is measured from the x-axis and e is the eccentricity of mass center to the rotation center.

Substitution of Eqs. (2), (3) and (4) into Eq.(1) may be expressed by

$$\begin{aligned}\mathbf{r}^{m_1/F} &= (x_g + x_1 + e\cos\theta)\ \mathbf{n}_x \\ &\quad + (y_g + y_1 + e\sin\theta)\ \mathbf{n}_y\end{aligned} \tag{5}$$

Similarly the position vector of the foundation of machine can be expressed by

$$\mathbf{r}^{m_2/F} = (x_g + x_2)\ \mathbf{n}_x + (y_g + y_2)\ \mathbf{n}_y \tag{6}$$

The velocities of masses m_1 and m_2 can be obtained by differentiating Eq. (5) and (6) with respect to time and can be written as

$$\begin{aligned}\mathbf{v}^{m_1/F} &= (\dot{x}_g + \dot{x}_1 - e\dot{\theta}\sin\theta)\ \mathbf{n}_x \\ &\quad + (\dot{y}_g + \dot{y}_1 + e\dot{\theta}\cos\theta)\ \mathbf{n}_y\end{aligned} \tag{7}$$

$$\mathbf{v}^{m_2/F} = (\dot{x}_g + \dot{x}_2)\ \mathbf{n}_x + (\dot{y}_g + \dot{y}_2)\ \mathbf{n}_y \tag{8}$$

The total kinetic energy of system is given

$$\begin{aligned}T &= \frac{1}{2} m_1 (\mathbf{v}^{m_1/F} \cdot \mathbf{v}^{m_1/F}) \\ &\quad + \frac{1}{2} m_2 (\mathbf{v}^{m_2/F} \cdot \mathbf{v}^{m_2/F}) + \frac{1}{2} I\omega \cdot \omega\end{aligned} \tag{9}$$

where m_1 and m_2 are the masses of rotor and of foundation including machine, respectively.

The potential energy of system is composed of the strain energy due to the deformation of shaft, the vertical position of rotor center and the deformation of isolation device such as spring.

$$\begin{aligned}V &= \frac{1}{2}[k_{x1}(x_1 - x_2)^2 + k_{y1}(y_1 - y_2)^2] \\ &\quad + \frac{1}{2}[k_{x2}x_2^2 + k_{y2}y_2^2] + m_1 g(y_1 + e\sin\theta)\end{aligned} \tag{10}$$

where k_{x1} and k_{y1} can be obtained from the flexibility of shaft. The stiffness of shaft is determined by assuming that the shaft is simply supported at bearings and the rotor is at the mid-point of shaft. If the cross-section of shaft is circle, the horizontal stiffness k_{x1} should be the same as the vertical stiffness k_{y1}, $k_{x1} = k_{y1}$. And the horizontal and vertical stiffnesses of isolation devices beneath foundation bed are denoted by k_{x1} and k_{y2}, respectively. The gravitational acceleration in the last term is denoted by the symbol g.

The total dissipation energy can be written as

$$D = D_1 + D_2 \ .$$

The dissipation energy caused by the isolation devices is given by

$$D_1 = \frac{1}{2}[c_{x2}\dot{x}_2 + c_{y2}\dot{y}_2] \tag{11}$$

where c_{x2} and c_{y2} are the damping coefficients of isolation device in the direction of x and y, respectively. The dissipation energy caused by the internal rotor friction can be obtained from the force relationship and is given by[11]

$$D_2 = c_1\left[\frac{(\dot{x}_1 - \dot{x}_2)^2 + (\dot{y}_1 - \dot{y}_2)^2}{2} + \dot{\theta}\rho^2\frac{d\varphi}{dt}\right]$$

where c_1 is the damping coefficient associated with the interface between shaft and bearing, and φ is the rotor attitude angle defined as

$$\varphi = \tan^{-1}\frac{x_1 - x_2}{y_1 - y_2}$$

$\dot{\varphi}$ is the rotor precession rate, and

$$\rho = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Then the dissipation due to rotor friction can be written as

$$D_2 = c_1\Big[\frac{(\dot{x}_1 - \dot{x}_2)^2 + (\dot{y}_1 - \dot{y}_2)^2}{2} + \dot{\theta}[(y_1 - y_2)(\dot{x}_1 - \dot{x}_2) - (x_1 - x_2)(\dot{y}_1 - \dot{y}_2)]\Big] \tag{12}$$

Thus when the rotor precession rate is zero, the internal friction dissipation energy is assumed to be the conventional viscous damping. It is very important to note that only in the case when the dissipation function has the special characteristic of being dependent upon the rotor precession rate, the self-excited whirl instability can be developed.

The governing equations of motion of the system can be obtained from Lagrange's equation, which states

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_r}\right) - \frac{\partial L}{\partial q_r} + \frac{\partial D}{\partial \dot{q}_r} = F_{qr} \tag{13}$$

where $L = T - V$, q_r is the generalized coor- dinate, and F_{qr} is the generalized force.

Application of the above equation (13) for the five generalized coordinates, x_1, y_1, x_2, y_2 and θ, yields the following equations;

$$\begin{aligned}
&m_1(\ddot{x}_g + \ddot{x}_1 - e\sin\theta\ddot{\theta} - e\cos\theta\ddot{\theta}^2) \\
&+ c_1[(\dot{x}_1 - \dot{x}_2) + \dot{\theta}(y_1 - y_2)] + k_1(x_1 - x_2) = F_{x1} \\
&m_1(\ddot{y}_g + \ddot{y}_1 + e\cos\theta\ddot{\theta} - e\sin\theta\ddot{\theta}^2) \\
&+ c_1[(\dot{y}_1 - \dot{y}_2) + \dot{\theta}(x_1 - x_2)] + k_1(y_1 - y_2) = F_{y1} \\
&m_2(\ddot{x}_g + \ddot{x}_2) + c_{x2}\dot{x}_2 + c_1[(\dot{x}_2 - \dot{x}_1) \\
&+ \dot{\theta}(y_2 - y_1)] + (k_1 + k_{x2})x_2 - k_1x_1 = F_{x2} \\
&m_2(\ddot{x}_g + \ddot{y}_2) + c_{y2}\dot{y}_2 + c_1[(\dot{y}_2 - \dot{y}_1) \\
&+ \dot{\theta}(x_2 - x_1)] + (k_1 + k_{x2})y_2 - k_1y_1 = F_{y2}
\end{aligned} \tag{14}$$

$$
\begin{aligned}
(I+m_1 e^2)\ddot{\theta} - m_1 e(\ddot{x}_1 + \ddot{x}_g)\sin\theta + m_1 e(\ddot{y}_1 + \ddot{y}_g)\cos\theta \\
- m_1 e(\dot{x}_1 + \dot{x}_g)\cos\theta - m_1 e(\dot{y}_1 + \dot{y}_g)\sin\theta \\
+ c_1 [(x_1 - x_2)(\dot{y}_1 - \dot{y}_2) - (y_1 - y_2)(\dot{x}_1 - \dot{x}_2)] \\
+ m_1 eg\cos\theta = F_\theta
\end{aligned}
$$

Now the equations of motion are solved numerically by using Runge-Kutta method.

3. Numerical Example

In order to verify the validity of Eq. (14) and to investigate the effectiveness of isolation spring for the rotating shaft system under seismic motion, the specifications of an example system are listed in Table 1. As mentioned before the spring constant of isolation device and the torque in Eq. (14) are treated as the system parameters.

At first the rotating speeds are calculated according to the constant torque rates. For convenience the operating mode at start-up is divided into two steps with respect to the ratio of machine speed(ω)to the critical speed($\omega_{cr} = \sqrt{k_1/m_1}$). The speed range of the first step is from 0 to the 'intermediate speed' which is between 0.8 and 0.9, and the second step starts from the intermediate speed to the operation speed. The intermediate speed in this study is selected to be 0.85(2). The torque rate of the first step in the simulation is usually less than that of the second step because the rotating speed should pass the critical speed as fast as possible.

Fig. 3 shows that the rotation speed and the stability are affected by the torque rate of the second step. If the torque rate is not sufficient[3], the machine speed reaches its operating speed after the several cycles of oscillations near the critical speed as shown in Fig. 3. The number of oscillation, which can be used as the measure of the system instability, decreases with increasing the torque rate on the second step. Although the speed reaches the operating speed, the displacement of rotor during oscillation increases so great that the rotor hit the stator and thus the rotating machine are not going to perform its function properly.

Table 1 Specifications of example system

symbol	description	numerical value
m_1	mass of rotor disk	$2,000 kg$
I	mass moment of inertia	$8,000\ kg-cm^2$
k_1	stiffness of shaft	$5.0\times10^7\ kg/cm$
m_2	mass of foundation	$10\,m_1$
$\frac{c_{x2}}{m_2} = \frac{c_{y2}}{m_2}$	damping coefficient	0.075
e	eccentricity	$0.01\ cm$
$\triangle t$	time increment	1.0×10^{-6} sec
ω_{cr}	critical speed	$\sqrt{k_1/m_1}$
ω_{oper}	operation speed	$3\omega_{cr}$

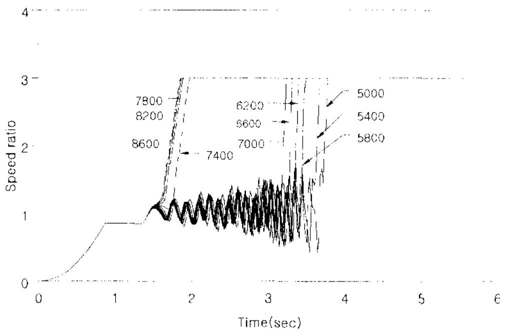

Fig. 3 Rotating speed at start-up corresponding to torque rate of second step

Also the speed of rotation has been calculated according to the fundamental horizontal frequencies, $f_n = \frac{1}{2\pi}\sqrt{k_{x2}/(m_1+m_2)}$ of isolation spring, and Fig. 4 shows that the number of oscillations decreases with increasing the stiffness of isolation spring. In simulation the vertical stiffness of isolation spring is considered to be 1.5 times of the horizontal stiffness, $k_{y2} = 1.5k_{x2}$. Because the vertical static deflection in the gravitational field is 24.8 cm for the vertical natural frequency of 1 Hz, the vertical stiffness is taken greater than the horizontal stiffness.

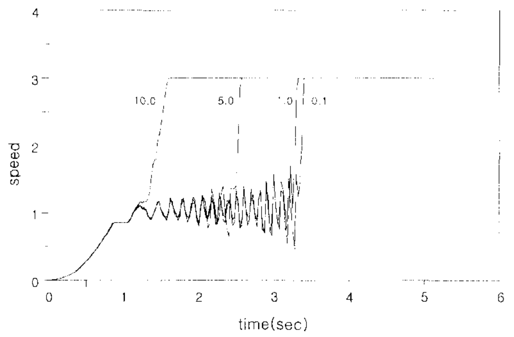

Fig. 4 Rotating speed at start-up corresponding to stiffness of isolated spring

The isolated rotating system under Taft earthquake(Fig. 5) has been simulated with respect to the stiffness of isolation spring. The relations between the fundamental frequencies and the maximum horizontal displacements under the horizontal ground motion are shown in Fig. 6. As expected, the maximum displacements of rotor and foundation decrease with increasing the stiffness of isolation spring, but in Fig. 7 the relative displacement of rotor to foundation is not affected by the spring stiffness. The reason is that the ratio of the machine speed to the fundamental frequency is much large as the low-tuned spring foundation.

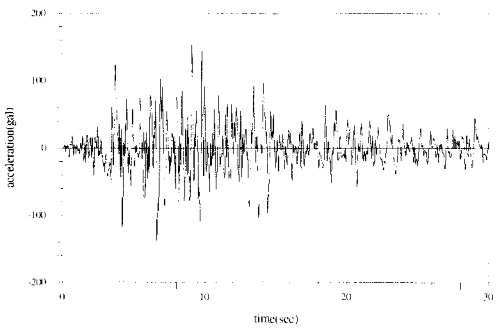

Fig. 5 Time history of taft earthquake

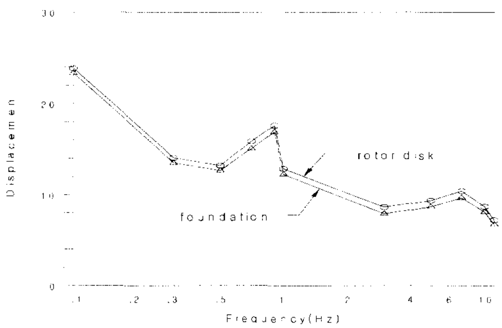

Fig. 6 Maximum displacement of system corresponding to stiffness of isolation spring

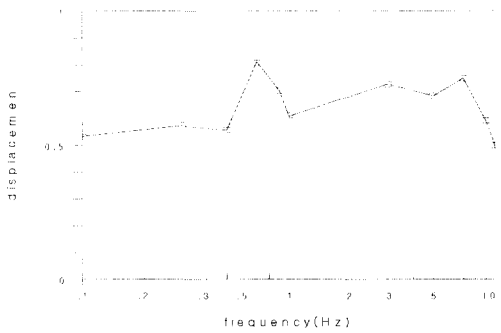

Fig. 7 Relative displacement of rotor to foundation with respect to isolating spring

Finally, the seismic response of rotating shaft system under the horizontal ground motion has been simulated according to the time lag between the arrival of the seismic motion and the start-up, and the maximum displacement of rotor disk with respect to the time lag is shown in Fig. 8. It can be determined that the time lag does not affect the seismic behavior of rotor system.

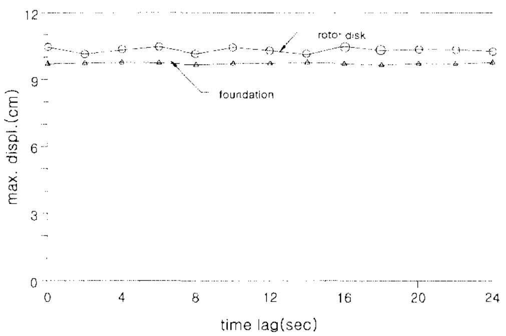

Fig. 8 Maximum horizontal displacement corresponding to time lag

4. Conclusion

The transient response of rotating shaft system having isolation device under an earthquake at start-up has been numerically simulated after deriving the equation of motion. Considering the simulated results, the seismic motion may not affect on the behavior of rotating system if the operating speed is much greater than the fundamental frequency of isolation device. However, the equation of motion derived can be appropriately used to simulate the transient response and to design the foundation of rotating machine.

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