地域硏究, 第14卷 第2號
1998年 12月 pp. 77~83

Energy Intensity by Means of the General Relation between Two Different Total Requirements Matrices*

Ho Un Gim
Department of Economics
Keimyung University

1. Introduction

In energy input-output analysis, the energy intensity or the total energy requirement can be defined as the measure of the direct and indirect energy required to produce a unit of each sector's output. The total energy requirement measured in physical units is the sum of the direct energy requirement and the indirect energy requirement. The direct energy is the energy needed for an industry's production process. The indirect energy is the energy embodied in that industry's inputs.

Nevertheless, until now the Leontief inverse matrix of the traditional input-output model has been successfully used to compute the energy intensity of each output. As it is well known, each element of the Leontief inverse, c_{ij}, represents the direct and indirect output requirements to support a unit of final demand. At this point, we can find a significant error in computing the energy intensity. We cannot put the identical meaning between the notion of direct and indirect input requirements to support a unit of final demand and that to produce a unit of gross output in the open input-output model. Therefore, we cannot consider two different notions as identical because there is a difference between the notion for a unit of final demand and that for a unit of gross output. Identically, we can find the difference between the total requirement matrix for a unit of final demand and that for a unit of gross output.

On the basis of the issues mentioned above, the study objectives of this paper are as follows. (1) We introduce the general relation, verified by the authors, between two different notions of direct

---

* This is the revised version of the paper presented at the twelfth international conference on input-output techniques, New York University, U. S. A., 18-22 May 1998. The present research has been conducted by the Bisa Research Grant of Keimyung University in 1998.

and indirect input requirements, or between two different total requirements matrices. It greatly helps us improve the accuracy of energy intensity and distinguish clearly the exact meaning between two different inverse matrices. (2) We can obtain a more meaningful energy intensity the same as the definition described above through the general relation. There are two different input-output accounts:

commodity-by-commodity and commodity-by-industry. We derive the energy intensity by both energy input-output models expressed in hybrid units: energy flows in British thermal units and nonenergy flows in dollars.

This article has five main sections. Section 1 is an introduction to this paper. Section 2 explains briefly the general relation. Section 3 describes "energy intensity" in energy input-output models by using the general relation. Section 4 presents numerical illustrations for the accuracy and usefulness of energy intensity. Section 5 contains a brief summary of this paper and concluding remarks.

2. The General Relation[1)]

Let us consider the open static input-output model:

$$(I_n - A)x = d, \tag{1}$$

where I_n is an nth order identity matrix, A the technical coefficient matrix, x the gross output vector, and d the final demand vector. If $|I_n - A| \neq 0$, then $(I_n-A)^{-1}$ exists and the unique solution is given by

$$x = C\,d, \tag{2}$$

where $C = (I_n-A)^{-1}$. Hence, $C\,(I_n-A) = I_n$.

In this basic input-output model, we proposed the general relation between the notion of direct and indirect input requirements of commodity i to support a unit of final demand of commodity j, γ^f_{ij}, and that to produce a unit of gross output of commodity j, γ^g_{ij}. We generalized γ^f_{ij} and γ^g_{ij} by dividing the elements into two parts.

For the diagonal elements, we verified that the two different notions are related by

$$\gamma^f_{ii} = c_{ii}\,\gamma^g_{ii} \tag{3}$$

$$\text{or} \quad \gamma^g_{ii} = 1 - \frac{1}{c_{ii}}, \tag{4}$$

and as for the nondiagonal elements by

$$\gamma^f_{ij} = c_{ii}\,\gamma^g_{ij} \tag{5}$$

$$\text{or} \quad \gamma^g_{ij} = \frac{c_{ij}}{c_{ii}}, \quad i \neq j, \tag{6}$$

where c_{ii} is the diagonal element of the Leontief inverse. This means the direct and indirect output requirements of that commodity itself to support a unit of final demand of commodity i.

Let us use the example given in Miller and Blair(1985) to illustrate the validity of the general relation obtained in equations (3) and (5). In it, the matrices are given as

$$A = \begin{pmatrix} .15 & .25 & .05 \\ .20 & .05 & .40 \\ .30 & .25 & .05 \end{pmatrix}, \qquad (7)$$

$$C = \begin{pmatrix} 1.365 & 0.425 & 0.251 \\ 0.527 & 1.348 & 0.595 \\ 0.570 & 0.489 & 1.289 \end{pmatrix}. \qquad (8)$$

Then, we have

$$\Gamma^{f} = \begin{pmatrix} .365 & .425 & .251 \\ .527 & .348 & .595 \\ .570 & .489 & .289 \end{pmatrix}, \qquad (9)$$

$$\Gamma^{g} = \begin{pmatrix} .267 & .311 & .184 \\ .391 & .258 & .441 \\ .442 & .379 & .224 \end{pmatrix}. \qquad (10)$$

The values of the diagonal elements are related by equations (3) and (4), and for the nondiagonal elements by equations (5) and (6).

Until now, we inevitably have used the Leontief inverse matrix, as in equation (9), to compute the energy intensity of each output. However, as indicated above, these values do not exactly coincide with the definition of energy intensity. From now on, I suggest that we can calculate more accurate energy intensities by using inverse matrix modified by the notion of each sector's output as in equation (10). Further, we can find a large difference in each element between two different inverse matrices.

3. Energy input-output models

1) Commodity-by-commodity input-output accounts

The open static input-output identities (1) and (2) can be rewritten in hybrid units, energy flows in British thermal units and non-energy flows in dollars, as

$$(I_n - A^{\circ})\, x^{\circ} = d^{\circ}, \qquad (1)'$$

$$x^{\circ} = C^{\circ} d^{\circ}, \qquad (2)'$$

where $C^{\circ} = (I_n - A^{\circ})^{-1}$. It is also known that the notion of direct and indirect input requirements of that commodity itself i to support a unit of final demand of commodity i is given as

$$\gamma_{ii}^{f(\circ)} = c_{ii}^{\circ} - 1,$$
$$i = 1, 2, \ldots\ldots, n. \qquad (11)$$

And moreover we know that

$$\gamma_{ij}^{f(*)} = c_{ij}^{*},$$

$$i, j = 1,\ 2,\ \ldots,\ n,\quad i \neq j. \qquad (12)$$

By substituting (11) and (12) for C^{*}, we will get $\Gamma^{f(*)}$, which is total energy requirements matrix expressed by a unit of final demand. Therefore, we can draw $\Gamma^{g(*)}$, energy intensities matrix expressed by a unit of output, by using (4) and (6). This is the same meaning as the definition of energy intensity. The energy intensity of each sector can be computed by column sum of energy intensities matrix for only energy sectors.

2) Commodity-by-industry input-output accounts

In a standardized system of national accounts, national economic accounts are compiled by the two notions of "commodity" versus "industry" accounts. The following Table 1 is an outline of commodity.

Table 1. Outline of commodity and industry accounts

	Commo-dities	Indust-ries	Final demand	Total output
Commo-dities	A	U	d	x
Industries	V			Q
Value added		W		
Total input	x′	Q′		

Let us take the following notion for m commodities and n industries in the accounts:

A : technical coefficient matrix,
U : use matrix,
d : final demand vector,
x : commodity gross output vector,
V : make matrix,
Q : industry total output vector,
W : industry value-added input vector.

From this accounts we can calculate total requirements matrices based on the assumptions: industry-based technology, commodity-based technology, and mixed technology. Moreover, each assumption has 4 different types of total requirements matrices: commodity-by-commodity, commodity-by- industry, industry-by-commodity, and industry-by-industry total requirements matrices.

In this commodity-by-industry model, let us introduce one example based on an industry-based technology assumption, where we can get a commodity-by-commodity total requirements matrix, Eq. (13):[2)]

$$x = (I - BE)^{-1} d, \qquad (13)$$

where B is the technical coefficient matrix in commodity-by-industry terms, and E the commodity output proportion matrix. In this case, the matrix $(I - BE)^{-1}$ is known as the commodity-by-commodity total require ments matrix. The ijth element of this matrix means the production of commodity i required to support a unit of final demand of commodity j. If we need to calculate energy intensities from (13), x and d are measured in hybrid units, as before. And then, we get

$$x^* = (I-B^*E^*)^{-1}d^*, \qquad (13)'$$

where * represents the values converted to hybrid units. Of course, to get an energy intensities matrix with the same meaning as the definition, as we did in commodity-by-commodity input-output accounts, the matrix $(I-B^*E^*)^{-1}$ must be modified by using (4) and (6).

4. Illustration of Energy Intensities

Let us take the example given in Miller and Blair[3)] to illustrate the accuracy of energy intensities obtained in section 3.

According to the example, consider a four-sector economy, in which I, II, and III sectors are energy sectors, namely crude oil, refined petroleum, and electric power respectively. The fourth sector, autos, is the only nonenergy sector. The monetary transactions for the economy are given in millions of dollars; the energy flows in the economy are measured in 10^{15} Btus.

The Leontief inverse for the hybrid-units, $C^*=(I-A^*)^{-1}$, is given as

$$C^* = \begin{pmatrix} 1.109 & 1.391 & 1.183 & 0.217 \\ 0.033 & 1.217 & 0.035 & 0.065 \\ 0.076 & 0.174 & 1.148 & 0.152 \\ 0.000 & 0.000 & 0.000 & 1.000 \end{pmatrix}.$$

Using (11) and (12), then, we have

$$\Gamma^{f(*)} = \begin{pmatrix} 0.109 & 1.391 & 1.183 & 0.217 \\ 0.033 & 0.217 & 0.035 & 0.065 \\ 0.076 & 0.174 & 0.148 & 0.152 \\ 0.000 & 0.000 & 0.000 & 0.000 \end{pmatrix}.$$

Also, using (4) and (6), we obtain

$$\Gamma^{g(*)} = \begin{pmatrix} 0.098 & 1.254 & 1.067 & 0.196 \\ 0.027 & 0.178 & 0.029 & 0.053 \\ 0.066 & 0.152 & 0.129 & 0.132 \\ 0.000 & 0.000 & 0.000 & 0.000 \end{pmatrix}.$$

The term $\gamma_{14}^{g(*)} = 0.196$ is the total primary energy intensity of producing automobiles in the economy. That is, it takes 0.196×10^9 Btus of crude oil to produce (including both direct and indirect energy requirements) one dollar's worth of output in the automobile sector. Similarly, $\gamma_{24}^{g(*)} = 0.053$ is the secondary energy intensity of automobile production. Thus, the total energy intensity of the automobile sector is the sum of primary and secondary energy intensities for only energy sectors, that is 0.196+0.053+0.132 = 0.381; it takes 0.381×109 Btus of energy to produce one dollar's worth of automobile output.

5. Conclusions

The energy intensity is the measure of the direct and indirect energy, measured in physical units, required to produce a unit of each sector's output. However, the Leontief inverse matrix of the traditional input-output model still has been used to compute the energy intensity of output, because we cannot find another inverse matrix which fits in the definition so far. Each element of the Leontief inverse represents the direct and indirect output requirements to support a unit of final demand. Therefore, we must convert the Leontief inverse into the other inverse, which means the direct and indirect input requirements to produce a unit of output. We proposed the general relation between two different notions of direct and indirect input requirements already.

In conclusion, we intend to calculate the energy intensity in commodity-by-commodity and commodity-by-industry energy input-output models expressed in hybrid units by using the modified Leontief inverse through (4) and (6). Trying to do that, we can get the accurate values of energy intensities, which is precisely the definition.

Notes

1) "The general relation" was derived by the authors. For more details, refer to Gim and Kim(1998).
2) See Miller and Blair(1985), pp. 166-167.
3) *Ibid*, pp.204-206.

References

Gim, Ho Un, 1994, "A Study on Energy Intensities by Means of Input-Output Analysis for the Korean Manufacturing Industry : With Attention to the 1975-1985-1990 Link Input-Output tables," *The Journal of Korean Planners Association, 29(2), 43-62.*

_____, 1995, "The Transition Analysis of the Changes in Energy Intensities by Means of Link Input-Output Tables, 1975-1990: The Case of the Republic of Korea," *St. Andrew's University Bulletin of Research Institute,* 20(3), 19-45.

_____, 1997, "The Logical Relation between Three Different Notions of Fixed Input Coefficients, Fixed Input Proportions and Proportionality: With Reference to the Basic Assumptions of the Input-Output Model," *The Keimyung Journal of Behavioral Sciences*, 9(1), 59-68.

_____ and K. Kim, 1998, "The General Relation between Two Different Notions of Direct and Indirect Input Requirements," *Journal of Macroeconomics*, 20(1), 199-208.

Miller, R. E. and Blair, P. D., 1985, *Input-Output Analysis: Foundations and Extensions*, Prentice-Hall, Englewood, 104.

Percebois, J., 1979, "Is the Concept of Energy Intensity Meaningful?," *Energy Economics*, 1. 148-155.

Proops, J. L. E., 1977, "Input-Output Analysis and Energy Intensities," *Applied Mathematical Modeling*, 1(4).

_____, 1984, "Modelling the Energy-Output Ratio," *Energy Economics*, 6(1), 1984.

ABSTRACT

The energy intensity or the total energy requirement can be defined as the measure of the direct and indirect energy required to produce a unit of each sector's output.

Nevertheless, until now the Leontief inverse has been used to compute the energy intensity of each output. At this point, we can find a significant error in computing the energy intensity. We cannot put the identical meaning between the notion of direct and indirect input requirements to support a unit of final demand and to produce a unit of gross output.

The study objectives of this paper are as follows. (1) We introduce "the general relation," verified by the authors, between two different total requirements matrices. (2) We can obtain a more meaningful energy intensity the same as the definition through "the general relation."

In conclusion, we intend to calculate the energy intensity in commodity-by-commodity and commodity-by-industry energy input-output models expressed in hybrid units by using "the general relation."

Key words : *energy intensity, the general relation, total energy requirement, direct energy requirement, indirect energy requirement*