중복근을 갖는 비비례 감쇠시스템의 고유치 해석

Solution of Eigenvalue Problems for Nonclassically Damped Systems with Multiple Frequencies

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요 지

본 논문에서는 중복근을 갖는 비비례 감쇠 시스템의 고유치 해석 방법을 제안하였다. 2차 고유치 문제의 행렬 조합을 통한 선형 방정식에 수정된 Newton-Raphson 기법과 고유벡터의 직교성을 적용하여 제안방법의 알고리즘을 유도하였다. 벡터 반복법 또는 부분공간 반복법과 같은 기존의 반복법에서는 수렴성을 향상시키기 위해 변위법을 적용하였으며, 이 값이 시스템의 고유치에 근사하게 되면 행렬분해 과정에서 특이성이 발생한다. 그러나 제안방법은 구하고자 하는 고유치가 중복근이 아닐 경우에, 변위값이 시스템의 고유치 일지라도 항상 정칙성을 유지하며, 이것을 해석적으로 증명하였다. 제안방법은 수정된 Newton-Raphson 기법을 이용하기 때문에 초기값을 필요로 한다. 제안방법의 초기값으로는 반복법의 중간결과나 근사법의 결과를 사용할 수 있다. 이들 방법중 Lanczos 방법이 가장 효율적으로 좋은 초기값을 제공하기 때문에 Lanczos 방법의 결과를 제안방법의 초기값으로 사용하였다. 제안방법의 효율성을 증명하기 위하여 두가지 예제 구조물에 대해 해석시간 및 수렴성을 가장 많이 사용하고 있는 부분공간 반복법과 Lanczos 방법의 결과와 비교하였다.

Abstract

A solution method is presented to solve the eigenvalue problem arising in the dynamic analysis of nonclassically damped structural systems with multiple eigenvalues. The proposed method is obtained by applying the modified Newton-Raphson technique and the orthonormal condition of the eigenvectors to the linear eigenproblem through matrix augmentation of the quadratic eigenvalue problem. In the iteration methods such as the inverse iteration method and the subspace iteration method, singularity may be occurred during the factorizing process when the shift value is close to an eigenvalue of the system. However, even though the shift value is an eigenvalue of the system, the proposed method provides nonsingularity, and that is analytically proved. Since the modified Newton-Raphson technique is adopted to the proposed method, initial values are need. Because the Lanczos method effectively produces better initial values than other methods, the results of the Lanczos method are taken as the initial values of the proposed method. Two numerical examples are presented to demonstrate the effectiveness of the proposed method and the results are compared with those of the well-known subspace iteration method and the Lanczos method.

Keywords : *eigenanalysis, nonclassically damped system, modified Newton-raphson technique*

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1. Introduction

In the analysis of dynamic response of structural systems, the equation of motion of a damped system can be expressed as

$$M\ddot{u}(t)+C\dot{u}(t)+Ku(t)=f(t) \tag{1}$$

where M, K and C are the n by n mass, stiffness and nonproportional damping matri ces[1], respectively, and $\ddot{u}(t)$, $\dot{u}(t)$ and $u(t)$ are the n by 1 acceleration, velocity and displacement vectors, respectively. To find the free vibration solution of the system, we first solve Eq. (1) for the homogeneous solution, which is of the form

$$u(t)=\phi e^{\lambda t} \tag{2}$$

Substituting Eq. (2) into Eq. (1), it yields the quadratic eigenproblem such as

$$\lambda^2 M\phi+\lambda C\phi+K\phi=0 \tag{3}$$

in which λ and ϕ are the eigenvalue and eigenvector of the system. There are $2n$ eigenvalues for the system with n degrees of freedom and these occur either in real pairs or in complex conjugate pairs, depending upon whether they correspond to overdamped or underdamped modes.

The common practice is to reformulate the quadratic system of equation to a linear one by doubling the order of the system such as

$$\begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}\begin{Bmatrix} \phi \\ \lambda\phi \end{Bmatrix}=\lambda\begin{bmatrix} C & M \\ M & 0 \end{bmatrix}\begin{Bmatrix} \phi \\ \lambda\phi \end{Bmatrix} \tag{4}$$

which may be rewritten as

$$A\Psi=\lambda B\Psi \tag{5}$$

with

$$A=\begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix},\ B=\begin{bmatrix} C & M \\ M & 0 \end{bmatrix} \text{ and}$$

$$\Psi=\begin{Bmatrix} \phi \\ \lambda\phi \end{Bmatrix} \tag{6}$$

Transformation methods such as QR[2], LZ[3] or Jacobi[4] determine all the eigenvalues and the associated eigenvectors in an arbitrary sequence. This is not very efficient in situations where only the lowest frequencies are of interest and there is a large number of degrees of freedom. Also transformation methods by their nature modify the initial matrices during the solution process and can not take full advantage of the sparseness of these matrices.

Perturbation method[5)-9)] is used for the eigenvalue problem of lightly damped systems. Since weak damping implies that the eigensolution of the damped system will differ only slightly from that of the corresponding undamped system, it is to set the eigensolution of the undamped system as the zero order approximation of that of the damped system and let the higher order terms account for the slightly damping effect.The classical inverse iteration method[10)-12)] is commonly used to solve for only a small number of desired modes. However, the method requires a great deal of complex arithmetic operations for each eigenvalue sought. The subspace iteration method[13),14)] is a more efficient alternative than the inverse iteration method. It yields all modes requested simultaneously and does not have the drawback that the higher modes are less accurate

than the lower modes because it avoids the round-off errors of the inverse iteration method due to the deflation process. However, as in the inverse iteration method, a large number of complex arithmetic manipulations are required in the iteration process for general structural systems. Furthermore, when the shift value becomes close to an eigenvalue of the system, singularity may be encountered during triangularization process.

In recent years there has been considerable interest in the Lanczos algorithm and its applications. The Lanczos algorithm for the computation of eigenvalues and eigenvectors of a real symmetric matrix was presented in reference 15 and improved in references 16 ~20. The Lanczos algorithm to solve the eigenvalue problem of nonclassically damped system is dealt with in references 21~26. Two sided-Lanczos algorithm[21)-24)] requires the generation of two sets of Lanczos vectors, left and right, and the symmetric Lanczos algorithm[25),26)] uses a set of Lanczos vectors to reduce a large eigenvalue problem in a much smaller one. Although only real arithmetic is solved during the Lanczos recursive process, in contrast to the case of real symmetric eigenproblems, there will be a possibility of serious breakdown and the accuracy of the solutions obtained is low[27)].

Although the authors presented the solution method for an eigenvalue problem with distinct eigenvalues[28)], the method has demerit that singularity is occurred if the eigenvalue desired is multiple. Therefore, in the present paper we develop the method to solve an eigenproblem with guaranteed nonsingularity for a damped structural system with multiple eigenvalues as well as distinct ones.

In the second section, the basic concept of the proposed method, which applies the modified Newton-Raphson technique to a linear eigenproblem, and the analytical proof of its nonsingularity are presented. In the third section, two numerical examples are presented to identify the efficiency of the proposed method and the results of the proposed method are compared with those of the well-known subspace iteration method[14)] and the Lanczos method[25)].

2. Method of analysis

2.1 Problem Definition

In this paper, we consider an eigenproblem of which the eigenvalue λ_i has multiplicity. m. For simplicity let us assume that the first eigenvalues are equal

$$\lambda \equiv \lambda_1 = \lambda_2 = \cdots = \lambda_m \tag{7}$$

Then Eq. (5) can be presented in matrix form for the m multiple eigenvalues as follows

$$A\Psi = B\Psi\Lambda \tag{8}$$

where $\Lambda = diag(\lambda_1, \cdots, \lambda_m) = \lambda I_m$ and $\Psi = [\Psi_1, \cdots, \Psi_m]$is, a n by m matrix satisfying the orthonormal condition with respect to matrix B such as

$$\Psi^T B \Psi = I_m \tag{9}$$

where I_m is an unitary matrix of order m.

The objective is to develop an efficient solution method with guaranteed nonsingularity for an eigenproblem described by Eqs (8) and (9).

2.2 Proposed Method

Let us assume that initial approximate solutions of Eq. (8) $\Lambda^{(0)}$ and $\Psi^{(0)}$ are known.

Denoting the approximate eigenvalues and the associated eigenvectors after k iterations by $\Lambda^{(k)}$ and $\Psi^{(k)}$, the residual vector becomes as follows;

$$R^{(k)} = A\Psi^{(k)} - B\Psi^{(k)}\Lambda^{(k)} \tag{10}$$

and

$$(\Psi^{(k)})^T B\Psi^{(k)} = I_m \tag{11}$$

where the residual matrix $R^{(k)} = [r_1{}^{(k)} \cdots r_m{}^{(k)}]$ denotes the error for each eigenpair, and is not generally zero because of substitution of approximate values into Eq. (8).

In order to get the solutions converged to the multiple eigenvalues and the associated eigenvectors of the system, the residual vectors should be removed. For the purpose of that, the Newton-Raphson technique is applied such as

$$R^{(k+1)} = A\Psi^{(k+1)} - B\Psi^{(k+1)}\Lambda^{(k+1)} = 0 \tag{12}$$

and

$$(\Psi^{(k+1)})^T B\Psi^{(k+1)} = I_m \tag{13}$$

where

$$\Lambda^{(k+1)} = \Lambda^{(k)} + \Delta\Lambda^{(k)} \text{ and} \tag{14}$$

$$\Psi^{(k+1)} = \Psi^{(k)} + \Delta\Psi^{(k)} \tag{15}$$

Substituting Eqs (14) and (15) into Eqs (12) and (13) and neglecting the nonlinear terms $B\Delta\Psi^{(k)}\Delta\Lambda(k)$ and $(\Delta\Psi^{(k)})^T B\Delta\Psi^{(k)}$, it yields the linear simultaneous equations for unknown incremental values $\Delta\Lambda^{(k)}$ and $\Delta\Psi^{(k)}$ as follows;

$$A\Delta\Psi^{(k)} - B\Delta\Psi^{(k)}\Lambda^{(k)} - B\Psi^{(k)}\Delta\Lambda^{(k)} = -R^{(k)} \tag{16}$$

and

$$(\Psi^{(k)})^T B\Delta\Psi^{(k)} = 0 \tag{17}$$

Since the eigenvalue is multiple, the offdiagonal elements of $\Lambda^{(k)}$ are zero or very small compared with its diagonal at kth iteration step, and the diagonal element very close. Thus, the second term in right side of Eq. (16) may be approximated by $\lambda_1^{(k)}B\Delta\Psi^{(k)}$, which yields

$$A\Delta\Psi^{(k)} - \lambda_1^{(k)}B\Delta\Psi^{(k)} - B\Psi^{(k)}\Delta\Lambda^{(k)} = -R^{(k)} \tag{18}$$

The Matrix form of Eqs (18) and (17) can be written such as

$$\begin{bmatrix} (A-\lambda_1^{(k)}B) & -B\Psi^{(k)} \\ (-B\Psi^{(k)})^T & 0 \end{bmatrix} \begin{bmatrix} \Delta\Psi^{(k)} \\ \Delta\Lambda^{(k)} \end{bmatrix} = -\begin{bmatrix} R^{(k)} \\ 0 \end{bmatrix} \tag{19}$$

Because the new coefficient matrix should be reformed and refactorized in each iteration step, the proposed method, despite of its rapid convergence, is not efficient.

These blemishes may be overcome by applying the modified Newton-Raphson technique to Eq. (19) such as

$$\begin{bmatrix} (A-\lambda_1^{(0)}B) & -B\Psi^{(k)} \\ (-B\Psi^{(k)})^T & 0 \end{bmatrix} \begin{bmatrix} \Delta\Psi^{(k)} \\ \Delta\Lambda^{(k)} \end{bmatrix} = -\begin{bmatrix} R^{(k)} \\ 0 \end{bmatrix} \tag{20}$$

The symmetric coefficient matrix of Eq. (20) is of order $(2n+m)$. While singularity occurs in factorization process of the iteration methods such as the inverse iteration method[10)-12)]

and the subspace iteration method[13),14)] when the shift is close to an eigenvalue of the system, nonsingularity of the proposed method is always guaranteed by means of including a side condition $(\Psi^{(k)})^T B \Delta\Psi^{(k)} = 0$ as shown in Eq. (20). This is the main difference compared with the iteration method with shift. The complete procedure of the proposed method for calculating the eigenpairs is summarized in table 1.

2.3 Nonsingularity of the Proposed Method

In the iteration methods such as the inverse and the subspace iteration methods, the shifting algorithm is adopted to improve the convergence. However, singularity may be occurred during the factorizing process when the shift value is close to an eigenvalue of the system. One of the characteristics of the proposed method is that its nonsingularity is also guaranteed in this situation. If the proposed method is nonsingular when the shift values is an eigenvalue itself, the coefficient matrix encountered in the iteration process must necessarily be nonsingular. Therefore, the nonsingularity of the proposed method is proved by introducing the new eigenproblem of the resulting matrix such as

Table 1 The algorithm of the proposed method

1. Calculate initial values $\Lambda^{(0)} = diag(\lambda_1^{(0)} \cdots \lambda_m^{(0)})$ and $\Psi = (\Psi_1^{(0)} \cdots \Psi_m^{(0)})$.
   (a) For $k = 0$
   (b) Define $\begin{bmatrix} A-\lambda_j^{(0)}B & -B\Psi_j^{(k)} \\ -(B\Psi_j^{(k)})^T & 0 \end{bmatrix}$.
   (c) Compute $-\begin{bmatrix} R^{(k)} \\ 0 \end{bmatrix}$,
   where $R^{(k)} = [r_1^{(k)} \cdots r_m^{(k)}]$.
   $= A\Psi^{(k)} - B\Psi^{(k)}\Lambda^{(k)}$
   (d) Compute $\begin{bmatrix} (A-\lambda_i^{(0)}B) & -B\Psi^{(K)} \\ (-B\Psi^{(k)})^T & 0 \end{bmatrix}\begin{bmatrix} \Delta\Psi^{(k)} \\ \Delta\Lambda^{(k)} \end{bmatrix}$
   $= -\begin{bmatrix} R^{(k)} \\ 0 \end{bmatrix}$ for $\begin{bmatrix} \Delta\Psi^{(k)} \\ \Delta\Lambda^{(k)} \end{bmatrix}$
   (e) Compute $\Lambda^{(k+1)} = \Lambda^{(k)} + \Delta\Lambda^{(k)}$ and
   $\Psi^{(k+1)} = \Psi^{(k)} + \Delta\Psi^{(k)}$
   (f) If the norm of the residual vector does not satisfy the predetermined error limit, then go to (b) with $k = k+1$, otherwise stop.

$$Eu_i = \gamma_i F u_i \qquad i=1, \cdots, 2n+m \tag{21}$$

where γ_i and u_i are the ith eigenvalue and the associated eigenvector of the new eigenproblem, respectively, and

$$E = \begin{bmatrix} A-\lambda_i B & -B\Psi \\ (-B\Psi)^T & 0 \end{bmatrix} \tag{22}$$

$$F = \begin{bmatrix} C & M & 0 \\ M & 0 & 0 \\ 0 & 0 & I_m \end{bmatrix} \tag{23}$$

or collectively

$$EU = FU\Gamma \tag{24}$$

where $\Gamma = diga(\gamma_1 \cdots \gamma_{2n+m})$ and $U = (u_1 \cdots u_{2n+m})$ The eigenpairs of the Eq. (24) are as follows;

- Eigenvalue γ_i

$$\begin{cases} -1 & : ms \\ 1 & : ms \\ (\lambda_i - \lambda_k)k = m+1, \cdots, 2n & : (n-m)s \end{cases} \tag{25}$$

- Eigenvector u_i

$$\left\{\begin{matrix} \Psi_j \\ e_j \end{matrix}\right\}, \left\{\begin{matrix} \Psi_j \\ -e_j \end{matrix}\right\}, \left\{\begin{matrix} \Psi_k \\ 0 \end{matrix}\right\} \quad \begin{matrix} j=1, \cdots, m \\ k=m+1, \cdots, 2n \end{matrix} \tag{26}$$

where e_j^T is an unit vector of order m such as

$$e_j^T = <0 \cdots 0\ 1\ 0 \cdots 0> \rightarrow j\text{th loaction} \quad (27)$$

Considering the determinant of Eq. (24),

$$\det[E]\det[U] = \det[F]\det[U]\det[\Gamma]$$

or

$$\det[E] = \det[F]\det[\Gamma] = (-1)^m \det[F] \prod_{k=m+1}^{2m} (\lambda_1 - \lambda_k) \neq 0 \quad (28)$$

because of

$$\det[F] = \begin{bmatrix} C & M & 0 \\ M & 0 & 0 \\ 0 & 0 & I_m \end{bmatrix} = \begin{bmatrix} C & M \\ M & 0 \end{bmatrix} = (-1)^m \det[M]\det[M] \neq 0 \quad (29)$$

The determinant of E is not equal to zero because $\det[M] \neq 0$ by definition. The nonsingularity of the proposed method is proved analytically.

2.4 Starting Values of the Proposed Method

Initial values of the proposed method can be obtained as the intermediate results of the iteration methods[11)-14)] or results of approximate methods[21)-26)]. In this paper, the starting values are taken as the results of the symmetric Lanczos method[25)] with selectively reorthogonalization process because the method does not need complex arithmetic in the Lanczos recursive process, and because the multiplicity of the desired eigenvalues can be checked by the results of the $4p$ Lanczos vectors (p : the number of desired eigenvalues). In the Lanczos method, the initial Lanczos vectors is set equal to $A^{-1}\langle 1 \cdots 1\rangle^T$ and is normalized with respect to matrix B.

3. Numerical Examples

In this section two test problems with multiple eigenvalues are used to assess the performance of the proposed method for generalized eigenproblems. The CPU time spent for the first twelve eigenvalues and the associated eigenvectors and the variation of the error norm to each iteration step of the proposed method are compared with those of the subspace iteration method[14)]. The least subspace dimension to effectively calculate required eigenpairs is $2p\ (=24)$. Each method is stopped when the error norms are reduced by the factor of 10^{-6}, which yields a stable eigensolution and sufficient accuracy in the calculated eigenvalues and eigenvectors for practical analysis[30)]. The error norm[30)] is defined as

$$\varepsilon_i^{(k)} = \frac{\|r_i^{(k)}\|_2}{\|A\Psi_i^{(k)}\|_2} \quad (30)$$

where

$$R^{(k)} = [\, r_1^{(k)} \cdots r_m^{(k)} \,]$$
$$A\Psi^{(k)} - B\Psi^{(k)}\Lambda^{(k)} \quad (31)$$

All executions are done on the CONVEX C3420 with 100 MIPS and 200 MFLOPS.

3.1 Plane Frame Structure with Lumped Dampers

The finite element model of a plane frame

is used as the first example. The Dimensionless values of the geometric configuration and material properties are shown in Fig. 1. The model is discretized in 200 beam elements resulting in the system of dynamic equations with a total of 590 degrees of freedom. Thus, the order of the associated eigenproblem is 1,180. The consistent mass matrix is used for M. Its damping matrix is derived from the proportional damping expression given by $C = \alpha M + \beta K$ and concentrated dampers.

The eigenvalues of the model are shown in Table 2. All the eigenvalues of the model are multiple. The variations of the error norms to increasing the iteration step are shown in Figs 2 to 4. The error norms of the initial values obtained by using the $4p\,(=48)$ Lanczos vectors are about 0.7 to 10^{-7}, which are possible to check the multiplicity of the desired eigenvalues. The number of iterations for the proposed method applied to the initial values that do not satisfy the error norm 10^{-6} is only one. The results in Figs 2 to 3 indicate that the convergence of the proposed method is much better than that of the subspace iteration method. The CPU time for the proposed method is compared with that

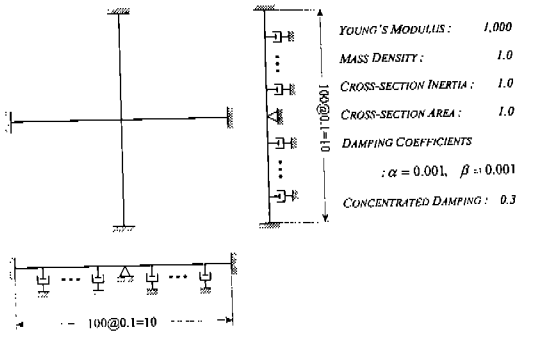

Fig. 1 Plane frame structure with lumped dampers

of the subspace iteration method in Table 3. If we let the solution time for the proposed method be 1, it takes 3.55 times for the subspace iteration method. In Table 4, the CPU time for the Lanczos method is summarized. Because the method does not need complex operations, the less solution time is required. However, the some results of the Lanczos

Table 2 Eigenvalues of the plane frame structure with multi-lumped dampers

Mode Number	Eigenvalues
1	−0.09590 + j 8.66792
2	−0.09590 + j 8.66792
3	−0.09590 − j 8.66792
4	−0.09590 − j 8.66792
5	−0.60556 + j 15.5371
6	−0.60556 + j 15.5371
7	−0.60556 − j 15.5371
8	−0.60556 − j 15.5371
9	−0.57725 + j 20.7299
10	−0.57725 + j 20.7299
11	−0.57725 − j 20.7299
12	−0.57725 − j 20.7299

Table 3 CPU time spent for the first twelve eigenvalues of the plane frame structure with multi-lumped dampers

Methods	CPU time in seconds (Ratio)
Proposed method	872.69 (1.00)
Subspace iteration method	3,096.62 (3.55)

Table 4 CPU time for the Lanczos method vs. the number of generated Lanczos vectors

The number of generated Lanczos vectors	CPU time in seconds
24	116.20
36	185.54
48	260.37
60	332.90
72	408.63
84	492.83
96	664.27

method as shown in Fig. 4 are not improved in spite of the increase of the number of the Lanczos vectors.

3. 2 Three-Dimensional Frame Structure with Concentrated Dampers

In this example a three-dimensional building structure with concentrated dampers is

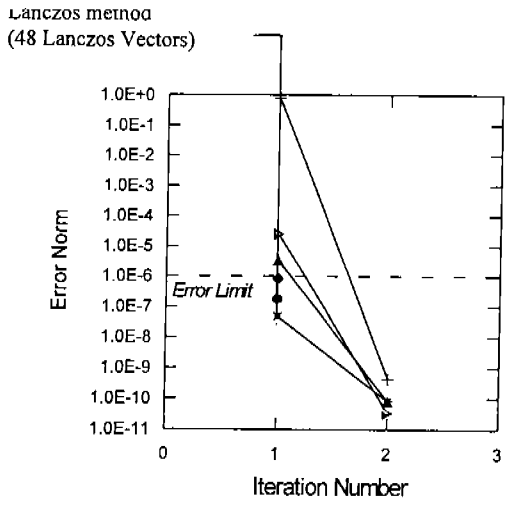

Fig. 2 Variation of the error norm of the frame model by the proposed method

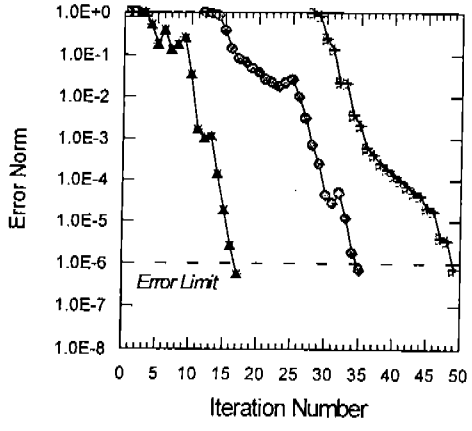

Fig. 3 Variation of the error norm of the frame model by the subspace iteration method

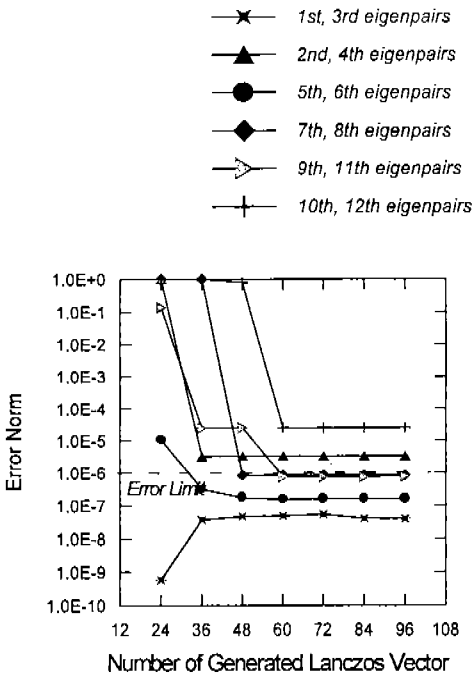

Fig. 4 Variation of the error norm of the frame model by the Lanczos method

presented. The geometric configuration and material properties are shown in Fig. 5. The model is divided into 436 beam elements and has 1,128 degrees of freedom. The order of the associated eigenproblem is 2,256. The consistent mass matrix is used to define M.

The damping matrix consists of the Rayleigh damping and concentrated dampers.

The results of the proposed method are summarized in Table 5. The first and second eigenvalue are coincident, and also the ninth and tenth eigenvalues and their conjugate eigenvalues coincident. The variations of the error norms to increasing the iteration step are shown in Figs 6 to 8. The first step of the proposed method denotes the results of the Lanczos algorithm. The error norms of the initial values obtained by using the 48 Lanczos vectors are about 10^{-4} to 10^{-7}. The

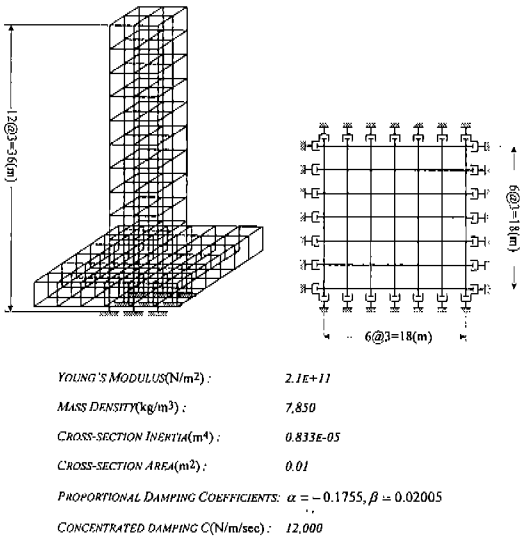

Fig. 5 (a) Three-dimensional building structure (b) Damping from two-layer foundation

Table 5 Eigenvalues of the 3-dimensional building structure with concentrated dampers

Mode Number	Eigenvalues
1	−0.13811 + j 3.09308
2	−0.13811 + j 3.09308
3	−0.13811 − j 3.09308
4	−0.13811 − j 3.09308
5	−3.53017 + j 2.20867
6	−3.53017 − j 2.20867
7	−0.24297 + j 4.16980
8	−0.24297 − j 4.16980
9	−1.65509 + j 7.04244
10	−1.65509 + j 7.04244
11	−1.65509 − j 7.04244
12	−1.65509 − j 7.04244

number of iterations for the proposed method applied to initial values that do not satisfy the error norm 10^{-6} is one or two. The results in Figs 6 to 8 indicate that the convergence of the proposed method is much better than that of the subspace iteration method. The CPU time for the proposed method is compared with the subspace iteration method in Table 6. If we let the solution time for the proposed method be 1, it takes 1.09 times for the subspace iteration method. In Table 7, the CPU time for the Lanczos method summarized is summarized. Because the method does not need the complex operations, the less solution time is required. However, the some results of the Lanczos method as shown Fig. 8 are not improved in spite of the increase of the number of the Lanczos vectors.

Table 6 CPU time spent for the first twelve eigenvalues of the 3-dimensional building structure with concentrated dampers

Methods	CPU time in seconds (Ratio)
Proposed method	7,641.94 (1.00)
Subspace iteration method	8,337.60 (1.09)

Table 7 CPU time for the Lanczos method vs. the number of generated Lanczos vectors

The number of generated Lanczos vectors	CPU time in seconds
24	613.33
36	933.51
48	1,246.60
60	1,572.73
72	2,000.39
84	2,227.23
96	2,582.77

4. Conclusions

An efficient method for solving damped structural dynamic eigenproblems with multiple eigenvalues as well as distinct ones is presented. Characteristics of the proposed method identified by the numerical results

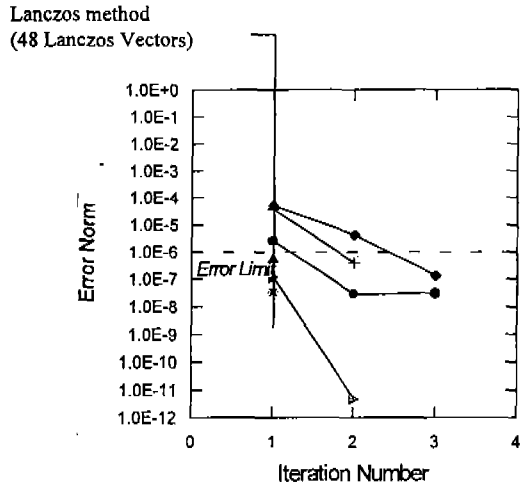

Fig. 6 Variation of the error norm of the 3-dimensional building by the proposed method

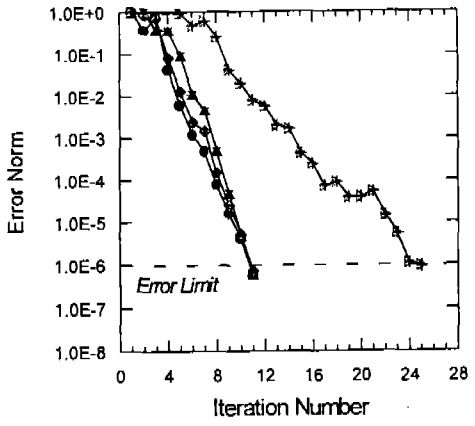

Fig. 7 Variation of the error norm of the 3-dimensiona building by the subspace iteration method

from test problems are identified as follows;

① Since the convergence rate of the proposed method is high, the proposed method is very effective for solving damped dynamic systems with a large number of degrees of freedom.

② Nonsingularity of the proposed method

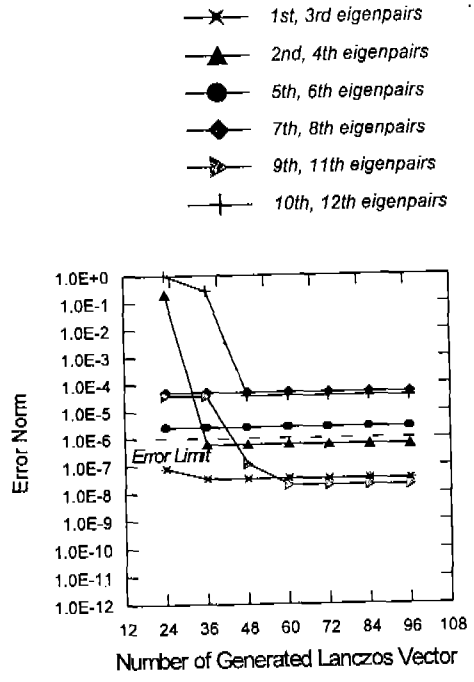

Fig. 8 Variation of the error norm of the 3-dimensional building by the Lanczos method

is always guaranteed, which is proved analytically.

③ The algorithm of the proposed method is simple.

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