Fluid Flow Analysis of the Threshold based Leaky Bucket Scheme

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Abstract

We investigate a Leaky Bucket(LB) scheme with a threshold in the data buffer, where leaky rate changes depending on the contents of data buffer. We use the fluid flow model for the analysis of the LB scheme with a threshold. We model the bursty input source as Markov modulated fluid flow(MMFF). As performance measures we obtain loss probability and mean delay. We present some numerical results to show the effects of the level of a threshold, the rate of token generation, the size of token pool, and the size of data buffer on the performances of the LB scheme with a threshold.

I. Introduction

The Broadband Integrated Services Digital Network (B-ISDN) will be high speed network, which can support various services such as voice, data, audio, and video. By using the Asynchronous Transfer Mode(ATM), information flow in B-ISDN is carried by packets of a fixed size, called cell. The use of the ATM technique emphasizes the importance of traffic control. The ATM technique aims to allow the network to support cost effectively a great variety of services with different traffic characteristics. Under statistical multiplexing, the cell flow of a connection may significantly alter the cell flow of the other connections. To guarantee the network performances required by the different services, a set of traffic control procedures based on the appropriate parameters must be defined. The future of ATM networks depends on the development of an effective traffic and congestion control framework.

Rathgeb[1] showed that the Leaky Bucket(LB) scheme is more effective than window-based mechanisms such as the Jumping window, the Moving window, and its variants. Butto *et al.*[2] presented the exact analysis of this LB scheme by Markov chain(MC) and fluid flow model. To reduce the cell loss probability, a data buffer is provided in the LB scheme. The motivation of having data buffer is to better control the trade-off between cell delays and cell loss probabilities. Park *et al.*[3] proposed the LB scheme with a threshold in the data buffer and described the LB scheme which can control the cell loss probability and the buffer size to the reasonable level and manage the bandwidth utilization efficiently under the given input conditions by choosing the leaky rates appropriately.

Sidi *et al.*[4] studied a continuous time analysis of the LB scheme whose arrival process is a Poisson process, and Kim *et al.*[5] studied a continuous time analysis of the LB scheme whose arrival process is a Markov modulated Poisson process(MMPP). Choi *et al.*[6] proposed the LB scheme with a dynamic leaky rate control in which the leaky rate changes according to the contents of data buffer. Sohraby *et al.*[7] and Wu *et al.*[8] studied a discrete time analysis of the LB scheme with a data buffer.

In ATM environment, a bursty source may stay in a relatively long duration in both active (ON) and inactive (OFF) periods. It is known [7] that the LB without a threshold is not appropriate for traffic with a long ON period, which gives a relatively high cell loss probability due to the shortage of data buffer. Our model has better performance of cell loss probability for traffic with a relatively long ON period than the LB without a threshold while keeping the multiplexing gain constant.

The more parameters that can be negotiated, the more exact the representation of the behavior of the service will be and the more accurate the policing function can react. Different sorts of policing functions exist based on the monitoring of the peak cell rate, the mean cell rate and the monitoring of other parameters such as burstiness and burst period. The policing function by the static token generation rate has a relatively poor performance in the cell loss probability when the input source has a long ON period. Our model overcomes this defect of the LB scheme without a

Manuscript received August 25, 1997; accepted February 23, 1998.
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threshold.

In ATM environment, the small and uniform cell size and the constant cell interarrival time give us fundamental reasons why fluid models are useful to analyze the system[9]. Elwalid *et al.*[9] studied the features of the LB scheme by modeling a bursty source as a Markov modulated fluid source.

In this paper, we analyze the LB schemes with a dynamic token rate control by the fluid flow model. In the section Ⅱ, we describe the system model of the LB scheme with a threshold in the data buffer and study the performance analysis of this system using the fluid flow model in which the input source is modeled as a Markov modulated fluid flow(MMFF). We give numerical results in the section Ⅲ. We finally have a conclusion in the section Ⅳ.

Ⅱ. Fluid Flow Model for the LB Scheme

1. System model

The system model under consideration is shown in Fig.1. The size of token pool is M and the size of data buffer is B. The data buffer has one threshold of level T_1 $(\leq B)$. A new token is stored in the pool when the token pool contains less than M tokens, otherwise, the newly generated token is discarded.

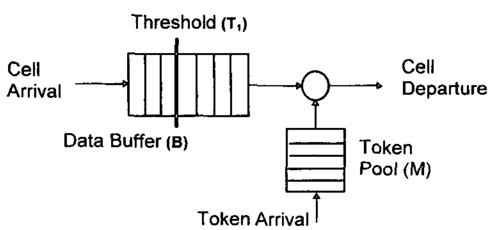

Fig. 1. Queueing model for LB scheme with a threshold.

An arriving cell is first queued in a data buffer and transmitted into the network immediately only when a token is available and one token is removed from the token pool. If there are no tokens in the token pool, cells in the buffer must wait until the next instant of token generation. Cells arriving when the data buffer is full are blocked and lost, and cells queued in the buffer are served on the FIFO basis.

The next token generation rate is r_1 when the occupancy of data buffer just after the token generation instants is less than or equal to the threshold T_1 and it is r_2 when the occupancy is greater than T_1.

2. Analysis

In this section, the bursty sources to the LB scheme with a threshold are modeled as Markov modulated fluid flow(MMFF) sources in which the state of an irreducible continuous time Markov chain(CTMC) determines the rates of fluid. The fundamental reasons why fluid models are appropriate in the ATM environment are the small and uniform size of cells, the constant interarrival time, and mathematical tractability[9].

Let $Q=(q_{ij})$ be the $m\times m$ infinitesimal generator of the underlying CTMC. There are only m feasible values, $\lambda_1,\cdots,\lambda_m$, where λ_i is the rate of cell generation when the CTMC is in state i. Let $\lambda=(\lambda_1,\cdots,\lambda_m)$. Then an MMFF source can be characterized by the parameters (Q,λ). The data buffer has one threshold of value T_1. The token generation rate is r_1 when the level of data buffer is less than equal to T_1, otherwise, it is r_2.

Let $N_b(t)$ $(0\leq N_b(t)\leq B)$ and $N_p(t)$ $(0\leq N_p(t)\leq M)$ which have continuous states, denote the content of the data buffer and token pool at time t respectively. Since $N_b(t)N_p(t)=0$, we can define $X(t)=M+N_b(t)-N_p(t)$ as the virtual fluid content of the data buffer and token pool at time epoch t and let $J(t)$ be the state of the CTMC at time epoch t. Let $F_i(t,x)$, $1\leq i\leq m$, $0\leq t,x$, be the probability that the underlying CTMC is in state i and the virtual content $X(t)$ does not exceed x, at time t *i.e.*,

$$F_i(t,x)=P\{X(t)\leq x, J(t)=i\}.$$

By a simple inspection of our model, we have

$$\begin{aligned} F_i(t+\triangle t,x) &= \sum_{j\neq i} q_{ji}\triangle t F_j(t,x) \\ &\quad +(1-q_i\triangle t)F_i(t,x-(\lambda_i-c)\triangle t) \\ &\quad +o(\triangle t), \end{aligned}$$

where $q_i=-q_{ii}$, $i=1,\cdots,m$, and c is a service rate.

Then we have

$$\begin{aligned} F_i(t+\triangle t,x)&-F_i(t,x-(\lambda_i-c)\triangle t) \\ &=\sum_{j\neq i} q_{ji}\triangle t F_j(t,x)-q_i\triangle t\times \\ &\quad F_i(t,x-(\lambda_i-c)\triangle t)+o(\triangle t) \end{aligned}$$

Dividing two sides by $\triangle t$ and letting $\triangle t\rightarrow 0$, we have the following differential equations, for $0\leq x< L_1$, (for simplicity, hereafter we use $L_1=M+T_1$ and $L=M+B$)

$$\begin{aligned} \frac{\partial}{\partial t}F_i(t,x)+(\lambda_i-r_1)\frac{\partial}{\partial x}F_i(t,x) \\ =\sum_{j\neq i} q_{ji}F_j(t,x)-q_iF_i(t,x). \end{aligned}$$

Let $F_i(x)=\lim_{t\rightarrow\infty}F_i(t,x), i=1,\cdots,m$, then we obtain, for $0\leq x< L_1$,

$$\frac{d}{dx}F_i(x)(\lambda_i-r_1)=\sum_j F_j(x)q_{ji}.$$

In the matrix notation, for $0< x< L_1$, we have

$$\frac{d}{dx}\overline{F}(x)\,\underline{D}=\overline{F}(x)Q, \tag{1}$$

where $\underline{D}=\text{diag}\,(\lambda_1-r_1,\cdots,\lambda_m-r_1)$ and $\overline{F}(x)=(F_1(x),\cdots,F_m(x))$ Similarly, for $L_1<x<L$, we have

$$\frac{d}{dx}\overline{F}(x)\,\overline{D}=\overline{F}(x)Q, \tag{2}$$

where $\overline{D}=\text{diag}\,(\lambda_1-r_2,\cdots,\lambda_m-r_2)$.

Now consider the drift $\frac{d}{dt}X(t)$ of the virtual fluid content at time t, which is given by

$$\frac{d}{dt}X(t)=\begin{cases}\lambda_{J(t)}-r_1, & 0<X(t)<L_1,\\ \lambda_{J(t)}-r_2, & L_1<X(t)<L.\end{cases} \tag{3}$$

If $(\lambda_{J(t)}-r_1)(\lambda_{J(t)}-r_2)>0$, then the equation (3) implies that $\frac{d}{dt}X(t)$ is nonzero and has the same sign for both $X(t)<L_1$ and $X(t)>L_1$. On the other hand, if $\lambda_{J(t)}-r_1>0$ and $\lambda_{J(t)}-r_2<0$, then we have the special equation as follows

$$\frac{d}{dt}X(t)>0,\ \text{if}\ X(t)<L_1,$$
$$\frac{d}{dt}X(t)<0,\ \text{if}\ X(t)>L_1.$$

Hence $\frac{d}{dt}X(t)=0$ if $X(t)=L_1$ so that there is a confluence of drift[10] at $X(t)=L_1$. Assume that $\underline{D}_{ii}\neq 0$ and $\overline{D}_{ii}\neq 0$, $i=1,\cdots,m$. We obtain

$$\overline{F}(x)=\begin{cases}\sum_{i=1}^{m}\underline{c}_i\,\underline{\phi}_i\exp(\underline{z}_i x), & 0<x<L_1,\\ \sum_{i=1}^{m}\overline{c}_i\,\overline{\phi}_i\exp(\overline{z}_i x), & L_1<x<L,\end{cases} \tag{4}$$

where $\{\underline{z}_i,\underline{\phi}_i\}$ and $\{\overline{z}_i,\overline{\phi}_i\}$ are two eigenvalue and eigenvector pairs which satisfy the following equations

$$\underline{z}_i\,\underline{\phi}_i\,\underline{D}=\underline{\phi}_i Q,\ i=1,\cdots,m,$$
$$\overline{z}_i\,\overline{\phi}_i\,\overline{D}=\overline{\phi}_i Q,\ i=1,\cdots,m.$$

In (4), the coefficients $\underline{c}_i$ and $\overline{c}_i$ are obtained from the boundary conditions. We define $\underline{S}_U(\underline{S}_D)$ and $\overline{S}_U(\overline{S}_D)$ be the set of source states giving upward (downward) drifts to the virtual fluid contents according to the buffer occupancy levels, *i.e.*,

$$\underline{S}_D=\{i\in J\,|\,\lambda_i-r_1<0\},$$
$$\underline{S}_U=\{i\in J\,|\,\lambda_i-r_1>0\},$$
$$\overline{S}_D=\{i\in J\,|\,\lambda_i-r_2<0\},$$
$$\overline{S}_U=\{i\in J\,|\,\lambda_i-r_2>0\}.$$

A confluence of drift implies that there is a probability mass at L_1 which causes $F_i(x)$ to have discontinuity at $x=L_1$ for $i\in\underline{S}_U\cap\overline{S}_D$. But in the case of $J(t)\in\underline{S}_D$ and $J(t)\in\overline{S}_U$, there is no confluence of drifts. Consequently, $F_i(x)$ is continuous at $x=L_1$ for $i\in\underline{S}_D\cup\overline{S}_U$. We can obtain the complete set of boundary conditions as follows

$$\begin{cases}F_i(0+)=0 & \text{if } i\in\underline{S}_U,\\ F_i(L_1-)=F_i(L_1+) & \text{if } i\in\underline{S}_D\cup\overline{S}_U,\\ F_i(L-)=x_i & \text{if } i\in\overline{S}_D,\end{cases}$$

where $x=(x_1,\cdots,x_m)$ is the steady-state distribution of the underlying CTMC.

The probability mass at $x=L_1$ and some performance measures are given by

$$P\{X=L_1,J=i\}=F_i(L_1+)-F_i(L_1-),$$
$$P\{X=0,J=i\}=F_i(0+),$$
$$P\{X=L,J=i\}=x_i-F_i(L-).$$

The rate of cells to network is given by

$$\gamma=\sum_{i=1}^{m}\lambda_i x_i-\sum_{i=1}^{m}(\lambda_i-r_2)[x_i-F_i(L-)].$$

Thus the cell loss probability P_B is given by

$$P_B=1-\gamma\Big(\sum_{i=1}^{m}\lambda_i x_i\Big)^{-1}$$

The mean cell delay W is obtained from

$$\begin{aligned}\gamma W=&\sum_{i=1}^{m}\Big\{\int_M^{L_1}(t-M)dF_i(t)+\int_{L_1}^{L}(t-M)dF_i(t)\Big\}\\ &+(L_1-M)\sum_{i=1}^{m}[F_i(L_1+)-F_i(L_1-)]\\ &+(L-M)\sum_{i=1}^{m}[x_i-F_i(L)].\end{aligned}$$

That is, we have

$$\begin{aligned}\gamma W=\sum_{i=1}^{m}\Big\{&\int_{[M,L)}(t-M)\,dF_i(t)\\ &+(L-M)(x_i-F_i(L-))\Big\}.\end{aligned} \tag{5}$$

Finally as shown in Appendix, we have

$$\gamma W=\int_M^L\Big(1-\sum_{i=1}^{m}F_i(t)\Big)dt. \tag{6}$$

III. Numerical Results

In this section we present some numerical results to investigate the performances of the LB scheme with a threshold using the fluid flow model. We use a simple two-state Markov process Q as the underlying process, whose the infinitesimal generator is given by

$$Q=\begin{pmatrix}-r_{12} & r_{12}\\ r_{21} & -r_{21}\end{pmatrix}.$$

We use (Q,λ) to characterize an MMFF source where λ has only two feasible values λ_1 and λ_2 depending on the state of the above underlying CTMC.

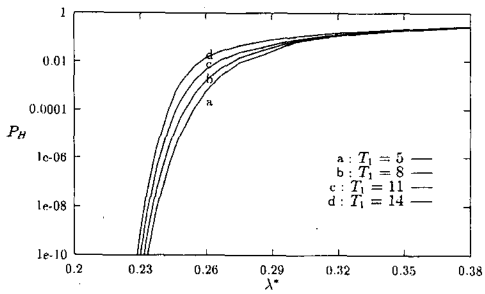

Fig. 2. Loss Probability vs. λ^* for the LB with a threshold, ($M=3, B=17, K_1=4.0, K_2=3.5, r_{21}=0.05, \lambda_1=0.5$ and $\lambda_2=0.2$)

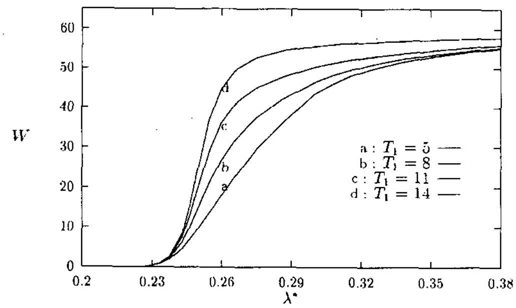

Fig. 3. Mean delay vs. λ^* for the LB with a threshold, ($M=3, B=17, K_1=4.0, K_2=3.5, r_{21}=0.05, \lambda_1=0.5$ and $\lambda_2=0.2$)

In Fig. 2 and 3, we show the cell loss probability and the mean cell delay of the LB scheme with a threshold for 4 cases of the threshold value, as the effective arrival rate varies, when the other parameters $M, B, K_1, K_2, \lambda_1, \lambda_2$, and r_{21} are fixed. From these figures, we see that the cell loss probability for the LB scheme with a threshold decreases and the mean cell delay increases as the threshold value(T_1) increases.

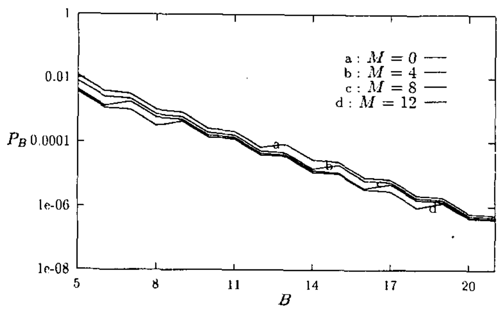

Fig. 4. Loss Probability vs. buffer size B for the LB with a threshold ($K_1=4.0, K_2=3.0, r_{12}=0.25, r_{21}=0.05, T_1=[(B+1)/2], \lambda_1=0.5$ and $\lambda_2=0.2$)

In Fig. 4 and 5, we show the cell loss probability and the mean cell delay of the LB scheme with a threshold for 4 cases of the token pool size, as the buffer size and the threshold value vary, when the other parameters $K_1, K_2, \lambda_1, \lambda_2, r_{12}$, and r_{21} are fixed. In these figures, the notation $[x]$ means the largest integer which does not exceed x. From these figures, we see that the cell loss probability for the LB scheme with a threshold decreases and the mean cell delay increases as the buffer size(B) increases.

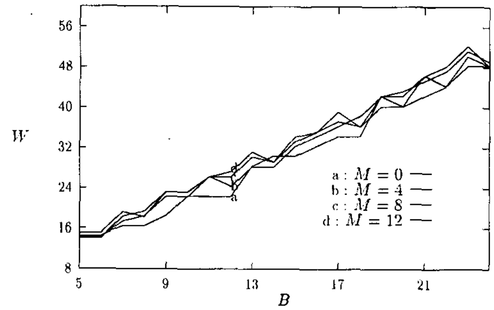

Fig. 5. Mean delay vs. buffer size B for the LB with a threshold ($K_1=4.0, K_2=3.0, r_{12}=0.25, r_{21}=0.05, T_1=[(B+1)/2], \lambda_1=0.5$ and $\lambda_2=0.2$)

The LB scheme with a dynamic leaky rate control is more effective and flexible to control the traffic into the network than the LB scheme without a threshold, because it is more convenient to change the level of threshold and leaky rate rather than the sizes of token pool and data buffer.

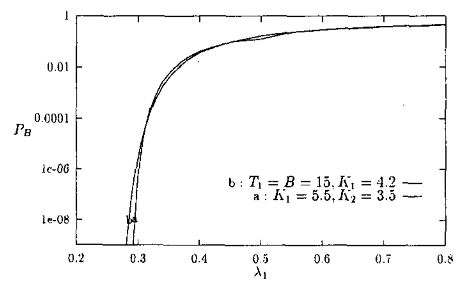

Fig. 6. Loss Probability vs. λ_1 for the LB with and without a threshold, ($M=5, B=15, T_1=8, r_{12}=0.04, r_{21}=0.05$ and $\lambda_2=0.0$)

In Fig. 6 and 7, we use a bursty source of ON-OFF type as input process. In Fig. 6, we show the cell loss probability for the LB scheme with a threshold(a) and the LB scheme without a threshold(b) as the arrival rate λ_1 varies. In this figure, to achieve the cell loss probability 10^{-6} when the token rate in LB without a threshold is 0.24, λ_1 should be about 0.3. Increase of λ_1 means that user sends more cells than negotiated one. Curve represents detection capability and two curves are almost the same. Thus our model has almost the same detection function as the LB scheme without a threshold.

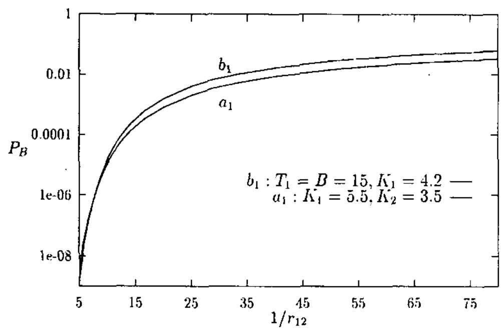

Fig. 7. Loss Probability vs. λ_1 for the LB with and without a threshold, ($M=5, B=15$, $T_1=8$, $r_{12}=0.04$, $r_{21}=0.05$ and $\lambda_2=0.0$)

In Fig. 7, since $r_{21}=r_{12}$, input traffic has the same mean length of ON and OFF period and thus effective arrival rate is independent of $r_{21}=r_{12}$. As $r_{21}=r_{12}$ increases, arrival rate does not change, but burst length of input traffic increases. Fig. 7 shows the cell loss probability for our model is lower than that of LB scheme without a threshold, and thus our model gives better performance for conforming traffic.

IV. Conclusion

In this paper we analyze the performances of the LB schemes with a threshold. We use an MMFF to model the burst input traffic. We obtain the closed-form solutions of the cell loss probability and the mean cell delay. We give some numerical examples to show the effects of the size of token pool, the size of data buffer, and the level of threshold on the cell loss probability and the mean cell delay.

We also find that in the LB scheme with a dynamic leaky rate control, the leaky rate in the ON periods becomes larger than that in the OFF periods while maintaining the same average leaky rate. Thus we can reduce the size of data buffer, and the size of token pool in order to decrease the mean cell delay and the cell loss probability of the input traffic while satisfying the required QoS.

V. Appendix

1. Derivation of equation (6)

To show that equation (6) holds, let $F(t)=\sum_{i=1}^{m} F_i(t)$. Then from equation (5), we have

$$\begin{aligned}\gamma W = & \int_M^{L_1}(t-M)dF(t)+\\ & \int_{L_1}^{L}[(t-L_1)+(L_1-M)]dF(t)\\ & +(L_1-M)[F(L_1+)-F(L_1-)]\\ & +(L-M)[1-F(L)]\end{aligned}$$

$$\begin{aligned}= & \int_M^{L_1}\int_M^t dxdF(t)+\int_{L_1}^{L}\int_{L_1}^{t} dxdF(t)\\ & +(L_1-M)[F(L)-F(L_1-)]\\ & +(L-M)[1-F(L)].\end{aligned}$$

By changing the order of integration, we have

$$\begin{aligned}\gamma W = & \int_M^{L_1}\int_x^{L_1} dF(t)dx+\int_{L_1}^{L}\int_x^{L} dF(t)dx\\ & +(L_1-M)[F(L)-F(L_1-)]\\ & +(L-M)[1-F(L)]\end{aligned}$$

$$\begin{aligned}= & \int_M^{L_1}[F(L_1)-F(x)]dx+\\ & \int_{L_1}^{L}[F(L)-F(x)]dx\\ & +(L_1-M)[F(L)-F(L_1-)]\\ & +(L-M)[1-F(L)]\end{aligned}$$

$$\begin{aligned}& = \int_M^{L_1}-F(x)dx-\int_{L_1}^{L}F(x)dx+(L-M)\\ & = \int_M^{L_1}[1-F(x)]dx+\int_{L_1}^{L}[1-F(x)]dx\\ & = \int_M^{L}[1-F(x)]dx.\end{aligned}$$

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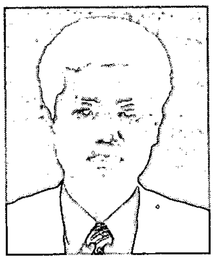

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