Decentralized Output-feedback Stabilization of Linear Time-invariant Interconnected Systems with Delays

Duk-Sun Shim

Abstract

We study the decentralized stabilization problem of linear time-invariant large-scale interconnected systems with delays without any system structure. We obtain sufficient stability conditions for interconnected systems which are equivalent to disturbance attenuation of some scaled system. A decentralized output-feedback controller is obtained using standard H_∞ control theory. The obtained controller is delay-independent. We also obtain an observer for the interconnected system.

I. Introduction

Decentralized stabilization of large scale interconnected systems has been an important topic in the past two decades. For large scale systems it is more efficient to treat those systems as interconnected systems rather than to treat those as a whole central system. Large-scale interconnected systems such as power systems and traffic systems are often formed from geographically separated systems and thus such systems are due to transport and computation times. Therefore we had better consider time delays for those interconnected systems. However, there are only a few papers which consider time delay for large scale interconnected systems.

Hu[2] and Lee and Radovic[3] studided decentralized stabilization problems of linear interconnected systems with delays and obtained decentralized state-feedback controllers. Lee and Radovic[3] imposed restriction on the interconnected system structure and bound on time delays between subsystems. Hu[2] used the same system as in Lee and Radovic[3], without any assumption of the system structure and bound on time delays. However, Trinh and Aldeen[6] pointed out that the result of Hu[2] can be applied only to a very restrictive class of systems for which the number of inputs and outputs is equal to that of states. There are some more work on the time delay problem for large-scale interconnected systems (refer to [8] and references therein).

In this paper we studied the decentralized stabilization problem for the same system model as in [3], but do not impose restriction on the interconnected system structure and bound on time delays between subsystems. We obtain sufficient decentralized stabilization conditions for the unforced interconnected system, which is given as an H_∞ disturbance attenuation condition of some scaled systems and then use the conditions to solve the decentralized state-feedback and output-feedback stabilization problem using standard H_∞ control theory in [1]. We also apply the above result to obtain the observers for interconnected systems.

II. Stability Analysis of Interconnected LTI Systems with Delays

Consider a large scale linear time-invariant (LTI) interconnected system composed of N subsystems which are interacting with each other through interconnections with delays.

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j=1, j\neq i}^{N} A_{ij} x_j(t-\tau_{ij}) \quad (1)$$

$$y_i(t) = C_i x_i(t) \quad (2)$$

where $i=1,2,\ldots,N$ and $x_i(t) \in R^{n_i}$ is the state, $u_i(t) \in R^{m_i}$ is the control input, $y_i(t) \in R^{p_i}$ is the output, and the matrices

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The author is with the Department of Control and Instrumentation Eng., Chung-Ang University, Seoul, Korea.

A_i, B_i, C_i and A_{ij} are real constant matrices with compatible dimensions and τ_{ij} denotes the time delay from j-th subsystem to i-th subsystem.

The decentralized stabilization problem for system of (1) and (2) can be stated as follows:

Design a decentralized linear feedback controller $G_C(s)=diag\{G_{C_1}, G_{C_2},,,G_{C_N}\}$ *for system (1) and (2) such that with* $u_i=G_{C_i}(s)y_i,\ i=1,2,,,N$ *the resulting interconnected system is asymtotically stable.*

Suppose that in system (1) and (2), (A_i, B_i) is controllable and (C_i, A_i) is observable. We recall the notion of H_∞ disturbance attenuation defined in [4].

Definition : Consider the linear system $\Sigma : \dot{x}(t)=Ax(t)+Bw(t),\ z(t)=Cx(t)$ where matrices A, B, C are real constant matrices with compatible dimensions. Given a scalar $\gamma>0$, the system Σ is said to have H_∞ disturbance attenuation γ if the matrix A is stable and $\|C(sI-A)^{-1}B\|_\infty<\gamma$ where $\|\cdot\|_\infty$ denotes the infinity norm of a stable transfer matrix.

The following lemma gives equivalent conditions to H_∞ disturbance attenuation γ as Algebraic Riccati Inequality (ARI) and Algebraic Riccati Equation(ARE).

Lemma 1 [1,9] Consider the asymptotically stable linear time-invariant system: $\dot{x}=Ax+Bw,\ z=Cx$. Let $T(s)=C(sI-A)^{-1}B$. Then the followings are equivalent.

(i) $\|T\|_\infty < \gamma$

(ii) There exists a symmetric matrix $P>0$ such that satisfies

$$A^TP+PA+\frac{1}{\gamma^2}PBB^TP+C^TC<0.$$

(iii) There exists a symmetric matrix $P>0$ such that satisfies $A^TP+PA+\frac{1}{\gamma^2}PBB^TP+C^TC=0$ and $A+\frac{1}{\gamma^2}BB^TP$ is Hurwitz.

First, we study the stability analysis of the unforced system (1) (by setting $u_i(t)\equiv 0$, for $i=1,2,\dots,N$).

Theorem 2 Consider the unforced interconnected system (1). Then the interconnected system is asymptotically stable if the matrices $A_i,\ i=1,2,,,N$ are stable and there exist real symmetric matrices $P_i>0$ such that for $i=1,2,,,N$

$$A_i^TP_i+P_iA_i+P_i\{\sum_{j=1,j\neq i}^{N}A_{ij}A_{ij}^T\}P_i+(N-1)I<0 \tag{3}$$

where N is the number of subsystems.

Proof. Let a Lyapunov function V(x,t) be as follows:

$$V(x,t)=\sum_{i=1}^{N}x_i^T(t)P_ix_i(t)+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t-\tau_{ij}}^{t}x_j^T(\sigma)x_j(\sigma)\,d\sigma$$

Then we obtain the derivative of V(x,t) with respect to time using ARI (3)

$$\begin{aligned}\frac{d}{dt}V(x,t) &= \sum_{i=1}^{N}x_i^T(t)(A_i^TP_i+P_iA_i)x_i(t)\\ &+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\{x_j^T(t-\tau_{ij})A_{ij}^TP_ix_i(t)\\ &+x_i^T(t)P_iA_{ij}x_j(t-\tau_{ij})\}\\ &+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(x_j^T(t)x_j(t)-x_j^T(t-\tau_{ij})x_j(t-\tau_{ij}))\\ &< -\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(x_j(t-\tau_{ij})-A_{ij}^TP_ix_i(t))^T\\ &\quad(x_j(t-\tau_{ij})-A_{ij}^TP_ix_i(t))\end{aligned}$$

Thus $\frac{d}{dt}V(x,t)<0$ holds always for all $x_i(t)\neq 0$ and thus the interconnected system is asymptotically stable by the well-known Lyapunov stability theorem. ▽▽▽

Remarks: 1) Theorem 2 gives a sufficient condition for the stabilization of unforced interconnented system (1), which is ARI (3). It is interesting that ARI (3) does not contain time delay term.

2) ARI (3) is the equivalent condition of the unitary H_∞ disturbance attenuation according to Lemma 1 for the following system.

$$\begin{aligned}\dot{x}_i(t) &= A_ix_i(t)+\overline{B}_iw_i(t)\\ z_i(t) &= \overline{C}_ix_i(t)\end{aligned}$$

where $\overline{B}_i\overline{B}_i^T=\sum_{j=1,j\neq i}^{N}A_{ij}A_{ij}^T,\quad \overline{C}_i^T\overline{C}_i=(N-1)I.$

III. Decentralized State-Feedback Stabilization

Now we consider the design of a decentralized controller that solves the decentralized stabilization problem for system (1) and (2) assuming that the state is available.

Before we state Theorem 3 consider the following N scaled system which will be used for the decentralized state-feedback controller design.

$$\dot{\eta}_i(t) = A_i\eta_i(t)+\overline{B}_iw_i(t)+B_iu_i(t) \tag{4}$$

$$z_i(t) = \overline{C}_i\eta_i(t) \tag{5}$$

$$y_i(t) = \eta_i(t) \tag{6}$$

where $\overline{B}_i\overline{B}_i^T=\sum_{j=1,j\neq i}^{N}A_{ij}A_{ij}^T,\quad \overline{C}_i^T\overline{C}_i=(N-1)I.$

The following theorem gives a solution to the decentralized state-feedback stabilization of system (1) and (2).

Theorem 3 Consider the interconnected system (1) and (2) with $C_i=I$. Then the system is stabilizable via decentralized state-feedback controller $u_i(t)=-B_i^TP_ix_i(t)$ if there exist real symmetric matrices $P_i>0$ such that for $i=1,2,,,N$

$$A_i^TP_i+P_iA_i+P_i(\sum_{j=1,j\neq i}^{N}A_{ij}A_{ij}^T-B_iB_i^T)P_i+(N-1)I<0 \tag{7}$$

Proof We obtain the following closed loop system for $i=1,2,\dots,N$ using the state-feedback control $u_i(t)=F_ix_i(t)$

$$\dot{x}_i(t) = (A_i+B_iF_i)x_i(t)+\sum_{j=1,j\neq i}^{T} A_{ij}x_j(t-\tau_{ij})$$
$$= \overline{A}_ix_i(t)+\sum_{j=1,j\neq i}^{T} A_{ij}x_j(t-\tau_{ij})$$

where $\overline{A}_i=A_i+B_iF_i$.
Then according to the result of Theorem 2, the sufficient condition that the interconnected system is stable is given as follows.

$$(A_i+B_iF_i)^TP_i+P_i(A_i+B_iF_i)+P_i\{\sum_{j=1,j\neq i}^{N} A_{ij}A_{ij}^T\}P_i+(N-1)I<0 \quad (8)$$

The closed loop system (4)-(6) can be described by the following system.

$$\dot{\eta}_i(t) = (A_i+B_iF_i)\eta_i(t)+\overline{B}_iw_i(t) \quad (9)$$
$$z_i(t) = \overline{C}_i\eta_i(t) \quad (10)$$

Note that the condition of unitary H_∞ disturbance attenuation of the system (9) and (10) is given exactly in (8). Thus the state-feedback controller for system (4)-(6) to have $\|T_{z_iw_i}\|_\infty<1$ where $T_{z_iw_i}$ denotes the transfer function from w_i to z_i is $u_i(t)=-B_i^TP_ix_i(t)$ with ARI (7). ▽▽▽

Theorem 3 shows that the decentralized control problem of the interconnected system (1) and (2) can be solved by decomposing the interconnected system to N decoupled systems as in (4)-(6) and apply the standard state-feedback H_∞ control theory when the state is available.

Remark: Lee and Ladovic[3] imposed the condition $A_{ij}=B_iL_{ij}C_j$ for the structure of the interconnected system which may not be applicable in some situations. Hu[2] did not imposed the above condition for the same problem. However, Trihn and Aldeen[6] pointed out that the number of inputs and outputs should be equal to that of states to use the result of Hu[2], which is very restrictive condition. Lee and Ladovic[3] and Hu[2] suggested state-feedback controller, thus if the states of the interconnected system are not available, they can not obtain the decentralized controller. In this paper we do not imposed the structure of the interconnected system as in Lee and Ladovic[3] and obtain an output feedback controller(see chapter V).

IV. An Observer for Interconnected LTI Systems with Delays

This section shows that we can apply the state-feedback control theory obtained in section II to the decentralized observers for large scale interconnected system with delays. We can use the following observer to determine the state of each local system.

$$\dot{\hat{x}}_i(t)=A_i\hat{x}_i+B_iu_i(t)+\sum_{j=1,j\neq i}^{N} A_{ij}\hat{x}_j(t-\tau_{ij})$$
$$+H_i[C_i\hat{x}_i(t)-y_i(t)] \quad (11)$$

for $i=1,2,\dots,N$.s

Define the observation error as $e_i(t)=x_i(t)-\hat{x}_i(t)$, then we obtain the following dynamic error equation

$$\dot{e}_i(t)=(A_i+H_iC_i)e_i(t)+\sum_{j=1,j\neq i}^{N} A_{ij}e_j(t-\tau_{ij})$$

for $i=1,2,\dots,N$.

The problem of the decentralized observer is to find the observer gains H_i's. The following theorem gives a solution to the observer problem.

Theorem 4 Consider the system (11). Then observer gains $H_i=-Q_iC_i^T$, $i=1,2,\dots,N$ provide asymptotically stable observer if there exist real symmetric matrices $Q_i>0$ such that

$$A_iQ_i+Q_iA_i^T+Q_i((N-1)I-C_i^TC_i)Q_i+\sum_{j=1,j\neq i}^{N} A_{ij}A_{ij}^T<0$$

$i=1,2,\dots,N$.

Proof Note that the observer problem is the dual of the state-feedback control problem and thus the proof uses the same procedure for $(A_i, B_i, C_i, F_i)\Leftrightarrow(A_i^T, C_i^T, B_i^T, H_i^T)$ as in the proof of Theorem 3. ▽▽▽

V. Decentralized Output Feedback Stabilization

This chapter shows the main result of this paper. We consider a partially interconnected system in which only some parts of states in a subsystem affects other subsystem. For example, when we use decentralized output-feedback control, the original states of a subsystem affect other subsystems, but the controller states do not. We call the states which affect other subsystems active states and the states which do not affect other subsystems passive states. For convenience, we decompose the state $x_i(t)\in R^{n_i}$ of i-th subsystem into active state $x_{i_1}(t)\in R^{n_{i_1}}$ and passive state $x_{i_2}(t)\in R^{n_{i_2}}$ with $x_i(t)=[x_{i_1}^T(t)\ x_{i_2}^T(t)]^T$ and $n_{i_1}+n_{i_2}=n_i$.
Consider the following LTI interconnected unforced system with delays.

$$\dot{\overline{x}}_i(t) = \overline{A}_i\overline{x}_i(t)+\sum_{j=1,j\neq i}^{N}\overline{A}_{ij}\overline{x}_{j_1}(t-\tau_{ij}) \quad (12)$$

Note that in (12),

$$\sum_{j=1,j\neq i}^{N}\overline{A}_{ij}\overline{x}_{j_1}(t-\tau_{ij})=\sum_{j=1,j\neq i}^{N}[\overline{A}_{ij}\ \ 0]\overline{x}_j(t-\tau_{ij}).$$

where $\overline{A}_{ij} \in R^{n_i \times n_{j_1}}$, $0 \in R^{n_i \times n_{j_1}}$.

The following theorem shows that when only parts of states affect the other subsystems, the decentralized stabilization condition becomes a little different from theorem 2.

Theorem 5 Consider the unforced interconnected system (12). Then the interconnected system is asymptotically stable if the matrices $\overline{A}_i$, $i=1,2,,,N$ are stable and there exist real symmetric matrices $\overline{P}_i > 0$ such that for $i=1,2,,,N$

$$\overline{A}_i^T \overline{P}_i + \overline{P}_i \overline{A}_i + \sum_{j=1, j\neq i}^{N} \overline{P}_i \overline{A}_{ij} \overline{A}_{ij}^T \overline{P}_i + (N-1)\begin{bmatrix} I_{n_{i_1}} & 0 \\ 0 & 0_{n_{i_2}} \end{bmatrix} < 0 \quad \textbf{(13)}$$

where $I_{n_{i_1}} \in R^{n_{i_1} \times n_{i_1}}$ is an identity matrix and $0_{n_{i_2}} \in R^{n_{i_2} \times n_{i_2}}$ is an zero square matrix.

Proof. Let a Lyapunov function $V(\overline{x}, t)$ be as follows:

$$V(\overline{x}, t) = \sum_{i=1}^{N} \overline{x}_i^T(t) \overline{P}_i \overline{x}_i(t) + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \int_{t-\tau_{ij}}^{t} \overline{x}_{j_1}^T(\sigma) \overline{x}_{j_1}(\sigma) d\sigma$$

Then we obtain the derivative of $V(\overline{x}, t)$ with respect to time using ARI (13)

$$\frac{d}{dt} V(\overline{x}, t) < -\sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \{ \overline{x}_{j_1}(t-\tau_{ij}) - \overline{A}_{ij}^T \overline{P}_i \overline{x}_i(t) \}^T \{ \overline{x}_{j_1}(t-\tau_{ij}) - \overline{A}_{ij}^T \overline{P}_i \overline{x}_i(t) \}$$

Thus $\frac{d}{dt} V(\overline{x}, t) < 0$ always for all $\overline{x}_i(t) \neq 0$ and the interconnected system is asymptotically stable by the well-known Lyapunov stability theorem. ▽▽▽

Next, we consider the decentralized output feedback stabilization problem of the system (1). Consider the following N scaled LTI systems.

$$\dot{x}_i(t) = A_i x_i(t) + \overline{B}_i w_i(t) + B_i u_i(t) \quad \textbf{(14)}$$

$$z_i(t) = \overline{C}_i x_i(t) \quad \textbf{(15)}$$

$$y_i(t) = C_i x_i(t) \quad \textbf{(16)}$$

where $\overline{B}_i \overline{B}_i^T = \sum_{j=1, j\neq i}^{N} A_{ij} A_{ij}^T$, $\overline{C}_i^T \overline{C}_i = (N-1)I$.

Theorem 6 Consider the interconnected system (1) and (2). Then the system is stabilizable via decentralized output feedback controller $u_i = G_{C_i}(s) y_i$ if the following conditions hold for $i=1,2,,,N$.

(i) there exist a real symmetric matrices $P_i > 0$ such that

$$A_i^T P_i + P_i A_i + P_i \{ (\sum_{j=1, j\neq i}^{N} A_{ij} A_{ij}^T) - B_i B_i^T \} P_i + (N-1) I < 0$$

(ii) there exist a real symmetric matrices $Q_i > 0$ such that

$$A_i Q_i + Q_i A_i^T + Q_i ((N-1)I - C_i^T C_i) Q_i + \sum_{j=1, j\neq i}^{T} A_{ij} A_{ij}^T < 0$$

(iii) $\rho(P_i Q_i) < 1$.

where $\rho()$ denotes maximum spectral radius.

If conditions (i) - (iii) hold, one controller $G_{C_i}(s)$ for i-th subsystem is given as follows:

$$G_{C_i} = \left[\frac{A_C \mid B_C}{C_C \mid 0} \right]$$

where

$$\begin{aligned} A_C &= A_i + (\sum_{j=1, j\neq i}^{N} A_{ij} A_{ij}^T - B_i B_i^T) P_i - (I - Q_i P_i)^{-1} Q_i C_i \\ B_C &= -(I - Q_i P_i)^{-1} Q_i C_i^T \\ C_C &= -B_i^T P_i \end{aligned} \quad \textbf{(17)}$$

Proof. Suppose that the controller $G_{C_i}(s)$ has the following realization.

$$\dot{x}_{C_i}(t) = A_{C_i} x_{C_i}(t) + B_{C_i} y_i(t), \quad x_{C_i}(0) = 0 \quad \textbf{(18)}$$

$$u_i(t) = C_{C_i} x_{C_i}(t) \quad \textbf{(19)}$$

where $x_{C_i} \in R^{n_{C_i}}$ and the dimensions of A_{C_i}, B_{C_i} and C_{C_i} are compatible.

The closed loop system of (1), (2), (18) and (19) is given by

$$\dot{\overline{x}}_i(t) = \overline{A}_i \overline{x}_i(t) + \sum_{j=1, j\neq i}^{N} \overline{A}_{ij} x_j(t - \tau_{ij})$$

where $\overline{A}_i = \begin{bmatrix} A_i & B_i C_{C_i} \\ B_{C_i} & A_{C_i} \end{bmatrix}$, $\overline{A}_{ij} = \begin{bmatrix} A_{ij} \\ 0 \end{bmatrix}$, $\overline{x}_i = \begin{bmatrix} x_i \\ x_{C_i} \end{bmatrix}$

The above closed loop system is same as (12) and thus the sufficient stability condition of the interconnected system is given as follows:

$$\overline{A}_i^T \overline{P}_i + \overline{P}_i \overline{A}_i + \sum_{j=1, j\neq i}^{N} \overline{P}_i \overline{A}_{ij} \overline{A}_{ij}^T \overline{P}_i + (N-1)\begin{bmatrix} I_{n_i} & 0 \\ 0 & 0_{n_{C_i}} \end{bmatrix} < 0 \quad \textbf{(20)}$$

The closed loop system (14)-(16) and the controller $G_{C_i}(s)$ can be described by the following system.

$$\dot{\overline{x}}_i(t) = \overline{A}_i \overline{x}_i(t) + \overline{B}_i w_i(t) \quad \textbf{(21)}$$

$$z_i(t) = \sqrt{N-1}\, [I_{n_i} \ 0] \overline{x}_i(t) \quad \textbf{(22)}$$

where $\overline{B}_i \overline{B}_i^T = \sum_{j=1, j\neq i}^{N} \overline{A}_{ij} \overline{A}_{ij}^T$.

Note that the condition of unitary H_∞ disturbance attenuation of system (21) and (22) is given exactly in (20). Thus the output-feedback controller for system (14)-(16) to have $\|T_{z_i w_i}\|_\infty < 1$ where $T_{z_i w_i}$ denotes the transfer function from w_i to z_i is $G_{C_i}(s)$ in (17). ▽▽▽

Note that for the system (1) and (2), the decentralized output-feedback stabilization problem becomes the standard H_∞ problem of N scaled systems.

V. Conclusions

We considered a large scale interconnected linear time-invariant system with delays. The system has no restriction on the interconnected system structure and no bound on time delays.

For unforced systems, the decentralized stabilization of the interconnected system can be given as the condition of H_∞ disturbance attenuation of scaled systems. We obtained decentralized state-feedback and output-feedback controller using standard H_∞ control theory and used it to obtain a decentralized observer for the interconnecred system.

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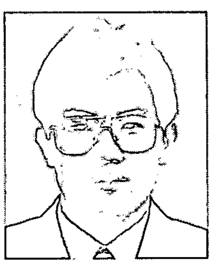

Duk-Sun Shim was born on Oct. 18, 1961. He received the B.S. and M.S. degrees in control and instrumentation engineering from Seoul National University in 1984 nad 1986, respectively. He received the Ph. D. degree in aerospace engineering from the University of Michigan in 1993. He was with the Department of Electrical Engineering and Computer Science in the University of Michigan as a visiting scholar from Jan. 1994 to Jan. 1995. He has been with the Department of Control and Instrumentation Engineering in Chung-Ang University since Mar. 1995 and currently an associate professor. His research fields include robust control and inertial navigation system.