횡변위 구속조건을 받는 고층철골구조물의 이산형 최적설계

Discrete Optimal Design of Tall Steel Structures subject to Lateral Drift Constraints

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요 지

본 연구는 횡변위 구속조건을 받는 고층철골구조물의 이산형 최적설계를 위해 효율적인 쌍대알고리즘을 제시하고자 한다. 양함수형태의 횡변위 구속조건을 설정하기 위해 가상일의 원리가 적용되며 고층철골조의 설계변수의 수를 줄여주기 위해 쌍대알고리즘내에 단면특성관계식이 추가된다. 이 알고리즘의 검증을 위하여 횡하중을 받는 네 가지 형태의 고층철골조 예제가 제시되며, 반복과정에서 수렴된 최종물량을 기존의 최적설계방법과 비교해 봄으로써 제시된 알고리즘의 효율성이 검토된다.

핵심용어 : 쌍대알고리즘, 이산형 최적화, 횡변위 구속조건, 고층철골구조물, 최적정기준법

Abstract

This study presents an effective dual algorithm for the discrete optimal design of tall steel structures subject to lateral drift constraints. Specifically, the principle of virtual work is introduced to obtain the drift constraint equations with the forms of explicit functions and the linear regression equation between the cross-section properties is added within dual algorithm to reduce the number of design variables. Four practical examples of tall steel frameworks subject to lateral loads are presented. The results such as the final weights converged within iteration history are compared with those of the conventional optimal design method so as to evaluate the efficiency of the proposed dual algorithm.

Keywords : *dual algorithm, discrete optimization, lateral drift constraints, tall steel structures, optimality criteria method*

1. INTRODUCTION

The design criterion of lateral drifts such as top-storey and interstorey drift is an important requirement for both serviceability and safety in the optimal design of tall steel buildings.

While many design software packages exist to satisfy member strength requirements ac-

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cording to design codes, very little work has been done to formalize the difficult and dominating problem of satisfying lateral drift criteria. Very often, the discrete optimal design of tall steel structures is carried out by the designer based on intuition, guess and educated trial and error.

Therefore, this study presents the dual algorithm formulation that is often referred to in the literature[1),3)] as a generalized optimality criteria technique, among a number of different algorithms that have been presented to solve the above optimization problem. The dual algorithm approach is well known and quite respected in the mathematical programming community, and it subsequently led to a reconciliation of optimality criteria techniques and mathematical programming methods[2),4)].

Unlike the conventional optimality criteria (OC) method, the dual algorithm directly accounts for discrete sizing variables and the corresponding constraints instead of using a pseudo-discrete algorithm based on the optimality criterion. This method also employs mixed design variables (either direct or reciprocal) in order to get first-order, conservative approximations to the objective function and to the drift constraints. The primary optimization problem is therefore replaced with a sequence of explicit approximate subproblems having a simple algebraic structure. Since each explicit subproblem is convex and separable, it can be solved efficiently by using a dual algorithm formulation.

However, practical schemes are additionally required to perform the dual algorithm efficiently and to reduce the computational efforts for many constraints and design variables given in the optimal design of tall buildings. To this end, this study introduces the principle of virtual work to obtain the drift constraint equations with the forms of explicit functions and adds the linear regression equation between the cross-section properties within dual algorithm so as to reduce the number of design variables.

Several practical examples for tall steel buildings are presented to illustrate the features of the design method. Wind loads are considered as equivalent static loads applied horizontally on the structure according to the provisions of most current building codes. Top lateral and interstorey drift limits are given as drift constraints. The results of this study are compared with the ones of the conventional OC method.

2. DESIGN PROBLEM FORMULATION

2.1 Lateral Drift Constraints

The minimum weight design of a tall steel framework under lateral drift constraints can be generally stated as the following discrete optimization problem.

$$\text{Minimize} : W(a_i) = \sum_{i=1}^{n} w_i a_i \tag{1a}$$

$$\text{Subject to} : (\delta_j - \delta_{j-1}) \le d_j^u h_j \quad (j=1, 2, \cdots, m) \tag{1b}$$

$$\delta_t \le d_t^u h_t \tag{1c}$$

$$a_i \in A_i \quad (i=1, 2, \cdots, n) \tag{1d}$$

Eq. (1a) defines the weight of the structure, where a_i is the cross-section area for member i and w_i is the corresponding weight coefficient related to material density and length for member i. Eqs. (1b) define the interstorey drift constraints for the structure, where δ_j and δ_{j-1} are the lateral drifts at two adjacent storey levels, h_j is the corresponding storey height and d_j^U is the allowable interstorey drift. Eq.(1c) defines the top drift constraint, where δ_t is the top lateral drift, h_t is the height of the structure and d_t^U is the allowable top lateral drift. Eqs.(1d) show each cross-sectional area a_i to belong to the set of areas A_i prevailing for the standard steel section profile specified for member i.

Since Eqs.(1b) and (1c) are implicit functions for the variables a_i, it is necessary to reformulate these constraints as explicit functions of the a_i so as to facilitate computer analysis. To this end, the displacement at any point of a two- dimensional steel framework can be expressed as, by the principle of virtual work,

$$\delta_j = \sum_{i=1}^{n}\int_0^{L_i}\left(\frac{F_X f_{Xj}}{Ea} + \frac{M_Z m_{Zj}}{EI_Z}\right)_i dx \tag{2}$$

where E=elastic modulus ; a=axial area ; I_Z =flexural moment of inertia ; F_X and M_Z= member force and moment due to the actual loading condition ; f_{Xj} and m_{Zj}=member force and moment due to a unit virtual load applied to the framework at the location of displacement δ_j.

Now, for commercial standard steel sections, the cross-section properties I_Z may be expressed in terms of the axial area 'a' using a linear regression equation. Here, among the several conventional linear regression equations, Eq.(3) is adopted, which has been applied in the optimality criteria method by Grierson and Chan[5),7)].

$$\frac{1}{I_Z} = \frac{C_{IZ}}{a} + C'_{IZ} \tag{3}$$

where the coefficients C_{IZ} and C'_{IZ} are determined by linear regression analysis and have different values depending on the type and size of the section. (e.g. see Table 1 for W14 and W24 sections from the AISC code[10)])

From Eq.(2), displacement δ_j can be expressed in terms of the cross-section areas a_i.

$$\delta_j = \sum_{i=1}^{n}\left(\frac{C_{ij}}{a_i} + C'_{ij}\right) \tag{4}$$

where the coefficients C_{ij} and C'_{ij} are given by

$$C_{ij} = \int_0^{L_i}\left(\frac{F_X f_{Xj} + M_Z m_{Zj} C_{IZ}}{E}\right)_i dx$$

$$C'_{ij} = \int_0^{L_i}\left(\frac{M_Z m_{Zj} C'_{IZ}}{E}\right)_i dx \tag{5}$$

From Eq.(4), the drift constraints (1b) and (1c) can be formulated as explicit functions of the variables a_i.

Table 1 C_{IZ} and C'_{IZ} relating a and I_Z for W14 and W24

Group	Sections	No. of sections	C_{IZ}	C'_{IZ}
1	W14×22-26	2	0.03924	-0.00102
2	W14×30-38	3	0.03533	-0.00057
3	W14×43-53	3	0.03197	-0.00020
4	W14×61-82	4	0.02988	-0.00011
5	W14×90-132	5	0.02903	-0.00010
6	W14×145-176	3	0.02852	-0.00008
7	W14×193-257	4	0.02808	-0.00008
8	W14×283-426	6	0.02714	-0.00007
9	W14×455-730	6	0.02444	-0.00004
1	W24×55-62	2	0.01409	-0.00013
2	W24×68-84	3	0.01343	-0.00012
3	W24×94-103	2	0.01196	-0.00006
4	W24×104-131	3	0.01101	-0.00004
5	W24×146-192	4	0.01067	-0.00003
6	W24×207-306	5	0.00994	-0.00002
7	W24×335-492	5	0.00978	-0.00002

$$(\delta_j-\delta_{j-1})=\sum_{i=1}^{n}\left(\frac{e_{ij}}{a_i}+e'_{ij}\right)(j=1,\ 2,\ \ldots,\ m)$$

$$\delta_t=\sum_{i=1}^{n}\left(\frac{C_{it}}{a_i}+C'_{it}\right) \tag{6}$$

where the coefficients e_{ij} and e'_{ij} are given as follows

$$e_{ij}=C_{ij}-C_{ij-1},\ e'_{ij}=C'_{ij}-C'_{ij-1} \tag{7}$$

2. 2 Optimization Algorithm Formulation

From Eqs. (1) and (6), by adopting the reciprocal variables $x_i=1/a_i$ $(i=1,\ 2,\ \cdots,\ n)$, the discrete optimization problem becomes

$$\text{Minimize : } W(x_i)=\sum_{i=1}^{n} w_i/x_i \tag{8a}$$

$$\text{Subject to : } \sum_{i=1}^{n}(e_{ij}x_i+e'_{ij})\le d_j^u h_j \tag{8b}$$

$$(j=1,\ 2,\ \cdots,\ m)$$

$$\sum_{i=1}^{n}(C_{it}x_i+C'_{it})\le d_t^u h_t \tag{8c}$$

$$x_i\in X_i \qquad (i=1,\ 2,\ \cdots,\ n) \tag{8d}$$

Since Eqs.(8a), (8b) and (8c) are all separable functions of the variables x_i, the dual algorithm directly applies for the solution of the above problem.

Now, we formulate the Lagrangian function so as to solve Eqs.(8).

$$L(x_i,\ \lambda_j)=\sum_{i=1}^{n}\frac{w_i}{x_i}$$
$$+\sum_{j=1}^{m}\lambda_j\left[\sum_{i=1}^{n}(e_{ij}x_i+e'_{ij})-d_j^u h_j\right]$$
$$+\lambda_{m+1}\left[\sum_{i=1}^{n}(C_{it}x_i+C'_{it})-d_t^u h_t\right] \tag{9}$$

$$\lambda_j\ge 0\ \ (j=1,\ 2,\ \cdots,\ m,\ m+1) \tag{10}$$

If a solution point $(x',\ \lambda')$ is a saddle point of Eq.(9), and if x' satisfies Eqs.(8d) while λ' satisfies Eqs.(10), then x' is a solution of the primal optimization problem posed by Eqs.(8). The saddle point is given by the solution of the min-max problem.

$$(x',\ \lambda')=\max_{\lambda_j}[\min_{x_i}L(x_i,\ \lambda_j)] \tag{11}$$

From Eq.(11), the dual function which can be expressed solely in terms of λ_j is defined as

$$L_m(\lambda_j)=\min_{x_i}L(x_i,\ \lambda_j)$$
$$=\min_{x_i}\sum_{i=1}^{n}L_i(x_i,\ \lambda_j)-L_o \tag{12}$$
$$(j=1,\ 2,\ \cdots,\ m,\ m+1)$$

where L_O and $L_i(x_i,\ \lambda_j)$ are given by

$$L_o=\sum_{j=1}^{m}\lambda_j\left[\sum_{i=1}^{n}e'_{ij}-d_j^u h_j\right]$$
$$+\lambda_{m+1}\left[\sum_{i=1}^{n}C'_{it}-d_t^u h_t\right] \tag{13a}$$

$$L_i(x_i,\ \lambda_j)=\frac{w_i}{x_i}+x_i\left[\sum_{j=1}^{m}\lambda_j e_{ij}+\lambda_{m+1}C_{it}\right]$$
$$(i=1,\ 2,\ \cdots,\ n) \tag{13b}$$

Since Eq.(9) can be expressed as the summation of separable functions of the variables x_i, Eq.(12) is readily formulated by finding and summing together the individual minimum of Eq.(13). The min-max problem Eq.(11) becomes, from Eqs.(9) and (10).

$$\text{Maximize : } L_m(\lambda)=\sum_{i=1}^{n}\frac{w_i}{x_i^k}+\sum_{j=1}^{m}\lambda_j\left[\sum_{i=1}^{n}(e_{ij}x_i^k+e'_{ij})\right.$$
$$\left.-d_j^u h_j\right]+\lambda_{m+1}\left[\sum_{i=1}^{n}(C_{it}x_i^k+C'_{it})-d_t^u h_t\right] \tag{14a}$$

Subject to : $\lambda_j \geq 0$ $(j=1, 2, \cdots, m, m+1)$ (14b)

where the x_i^k are the discrete values of the variables x_i corresponding to the individual minimum Eqs.(13). A dual gradient projection algorithm is applied to iteratively modify the dual variables in solving the dual optimization problem of Eqs.(14).

$$\lambda_j^{\nu+1} = \lambda_j^{\nu} + \alpha S_j \quad (j=1, 2, \cdots, m) \qquad (15)$$

where S_j is the search direction, α is the step length and λ_j^{ν} is the variable value found at the end of the previous search step. As the continuous variable λ_j varies, the value of the sizing variable x_i for which the corresponding function $L_i(x_i, \lambda_j)$ is minimum shifts from one discrete value x_i^k to the next discrete value x_i^{k+1} in the set X_i. When this happens, continuity of the dual function is maintained by the following identity,

$$L_i(x_i^k, \lambda_j) = L_i(x_i^{k+1}, \lambda_j) \qquad (16)$$

which reduces to, by differentiating Eq.(9) with respect to x_i.

$$\sum_{j=1}^{m} \lambda_j e_{ij} + \lambda_{m+1} C_{il} = \frac{w_i}{x_i^k x_i^{k+1}} \qquad (17)$$

$$(i=1, 2, \cdots, n)$$

From Eq.(17), the specific discrete value of each sizing variable is given by

$$x_i^{\nu} = x_i^k \qquad (i=1, 2, \cdots, n) \qquad (18a)$$

if

$$\frac{w_i}{x_i^{k-1} x_i^k} < \sum_{j=1}^{m} \lambda_j^{\nu} e_{ij} + \lambda_{m+1}^{\nu} C_{il} < \frac{w_i}{x_i^k x_i^{k+1}}$$

$$(i=1, 2, \cdots, n) \qquad (18b)$$

The steps of the dual algorithm to solve the optimization problem are as follows ;

1) Adopt an initial set of reciprocal sizing variables x_i^k. (e.g., select the smallest values allowed by the discrete sizing constraints Eqs.(8d), which corresponds to selecting the largest available cross-section areas for the steel section shapes specified for the members.)
2) For the current x_i^k, analyze the structure and establish the coefficients e_{ij}.
3) Adopt an initial set of the dual variables λ_j, which correspond to the currently most active constraints from among Eqs.(8b).
4) For the current λ_j^{ν}, determine discrete x_i^k values for the variables x_i from Eqs.(18).
5) Substitute the current x_i^k values into the dual objective function Eq.(14a), and use the dual gradient projection method to find the new set of dual variables $\lambda_j^{\nu+1}$ through Eqs.(15).
6) Check convergence of the dual gradient projection method. That is, if the directional derivative of the dual function Eq.(14a) is zero, go to step 7 ; otherwise set $\nu=\nu+1$ and return to step 4.
7) Check convergence of the overall design process. That is, if the structure weight from Eq.(8a) is the same for two successive discrete optimizations, terminate with the minimum weight structure ; otherwise, return to step 2.

The above design process is programmed by using MATLAB[9] that facilitates the umerical analysis and matrix calculation. Also, some subroutines for the discrete optimization problem are introduced from SODA (Structural Optimization Design and Analysis) program[8].

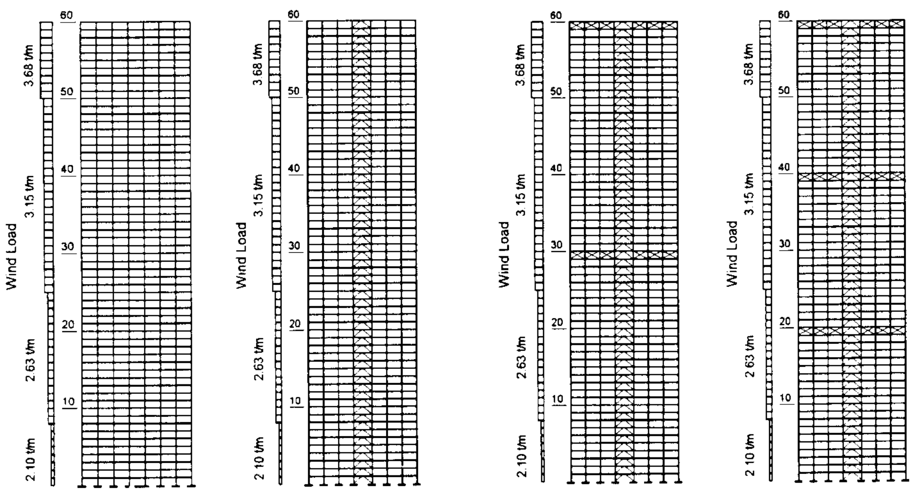
3. EXAMPLES

To illustrate the proposed dual algorithm, compared with the conventional OC method, consider four planar steel frameworks shown in Fig. 1. Example 1 is a rigid frame, for which lateral stiffness is supplied solely by the beams and columns. Example 2 is a rigid frame with K-braced trusses, where additional flexural capacity is supplied by the diagonal members. Example 3 and 4 are rigid frames with both K-braced and outrigger trusses, where the outrigger trusses provide additional lateral stiffness and two types of the locations of outriggers are considered.

No gravity loads are considered for these example. The lateral wind loading is applied to the framework as horizontal point loads at each floor level. The bay width is 6m and the storey height is 3.6m. The material elastic modulus for all examples is 2.1×10^3 t/cm^2. The frameworks are to be designed such that interstorey drift is within a typical limit of 1/400. American AISC standard wide-flange (W) steel sections are used to size the members. The columns, diagonals and outrigger trusses are to be selected from the range of sections W14×22 to W14×730, while the beams are to be chosen from the range of section W24×55 to W24×492. To satisfy practical construction requirements, beams are grouped together as having a common section for each storey, as are diagonals and outriggers, and the exterior columns are grouped together over two adjacent storeys, as are symmetrical pairs of interior columns. The optimization process starts with the maximum sizes for all members as the initial trial design.

3. 1 Example 1 and 2

Example 1 is a rigid frame which consists of columns and beams joined by moment- resistant connections, as shown in Fig. 1(a). In Example 2, all beams and columns for the



(a) Example 1 (b) Example 2 (c) Example 3 (d) Example 4

Fig. 1 Multistorey building frameworks

frame are rigidly connected, while the diagonals of the K-braced truss are simply connected, as shown in Fig. 1(b). Two examples are the same models used in the optimality criteria method by Grierson and Chan[6]. This optimality criteria method constitutes a recursive algorithm by the minimization of the Lagrangian function so that optimality critetia based on stationary conditions are satisfied through an iterative pseudo-discrete optimization procedure in order to size members directly using discrete commercial standard steel sections. Specifically, the results by OC method in Example 1 and 2 have been directly obtained from Grierson and Chan[6] for the comparison with the proposed dual method.

The results by the proposed dual algorithm are shown in Table 2 and 3, Fig. 2. Table 2 and 3 show a comparison of the initial and final weights for the proposed dual method and the conventional optimality criteria method, according to the design cycle. Fig. 2 shows the iteration history due to the results from Table 2 and 3. The proposed dual method finds directly the discrete variables for section sizes, while the conventional OC method requires the process that transfers the continuous design solution progressively to discrete section sizes using the pseudo- discrete OC method. From Table 2 and 3, the percentage differences in the total weight of the overall structure between the proposed dual method and the conventional OC method are within 4% and 7% for Example 1 and 2, respectively. This means that the proposed dual method results in little difference in the total weight of the overall structure and provides the effects of saving the weights somewhat, compared with the conventional OC method.

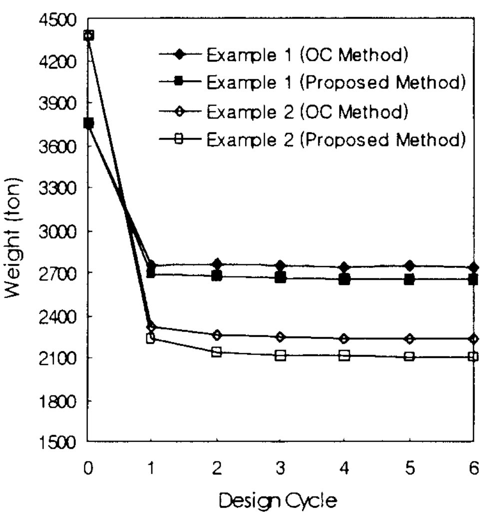


Fig. 2 Iteration history of Example 1 & 2

Table 2 Comparison of weights (Example 1)

Design Cycle	OC Method (1) (ton)	Proposed Method (2) (ton)	Diff. ((1) & (2)) (%)
0	3754.3	3754.3	-
1	2748.2	2684.6	3.6
2	2763.3	2671.7	3.4
3	2743.5	2665.6	2.9
4	2735.1	2648.6	3.3
5	2744.5	2648.6	3.6
6	2734.2	2648.6	3.2

Table 3 Comparison of weights (Example 2)

Design Cycle	OC Method (1) (ton)	Proposed Method (2) (ton)	Diff. ((1) & (2)) (%)
0	4372.3	4372.3	-
1	2317.5	2238.0	3.6
2	2257.4	2140.1	5.5
3	2244.9	2115.4	6.1
4	2238.5	2110.5	6.1
5	2238.3	2105.2	6.3
6	2235.7	2105.2	6.2

The proposed dual method shows rapid and steady convergence, as shown in Fig. 2. The final design is obtained after the 4th and 5th design cycles for Example 1 and 2, respectively, in the proposed dual method, while obtained after 6th design cycles for two examples in the conventional OC method. Specifically, in the proposed dual method, the optimization process can be terminated after the 3rd design cycle, which design weighs only 0.5% heavier than that after the 5th cycle.

3. 2 Example 3 and 4

Two additional examples are presented to illustrate the efficiency of the proposed dual algorithm. Example 3 and 4 which outriggers are added within Example 2 of Fig. 1(b) are shown in Fig. 1(c) and (d). Example 3 has two outriggers in 30th and 60th storeys and Example 4 has three outriggers in 20th, 40th and 60th storeys.

Fig. 3 shows the iteration history of Example 3 and 4. Final design is obtained after the 6th and 7th design cycles in Example 3 and 4, respectively. Specifically, the final weights of four examples are shown in Table 4. From the results, we can find that Example 3 and 4 with outriggers give the large reduction of weights, compared with Example 1 and 2, and also Example 4 with three outriggers provides the most economic alternative in reducing weights and controlling the drifts. This shows that the outriggers can provide the effective means to obtain the lateral stiffness in resisting the lateral drifts of tall steel frameworks, as is well known in the comparison of structural systems for tall buildings.

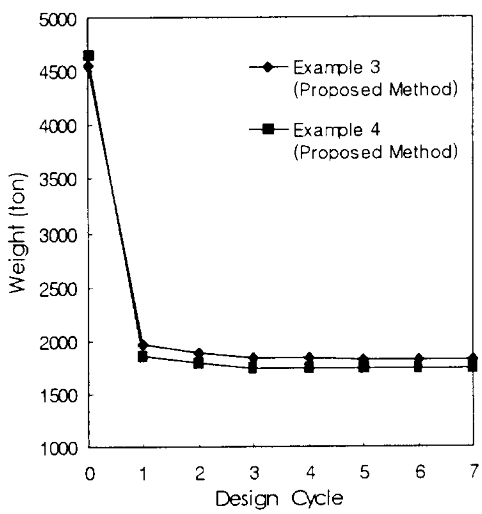


Fig. 3 Iteration history of Example 3 & 4

4. CONCLUSIONS

This study presents an effective dual algorithm for the discrete optimal design of tall steel structures subject to lateral drift constraints. Specifically, the concept of the principle of virtual work is provided to replace the drift constraint equations with the forms of explicit functions and the linear regression equation to express the relationships between the cross-section properties is additionally considered within dual algorithm so as to reduce the number of design variables.

The dual algorithm solves the drawbacks inherent to the conventional OC method. The selection of the set of active constraints does

Table 4 Final weights of four examples

Example	Final Weight (ton)	Diff. between Example 1 & others (%)
Example 1	2648.6	-
Example 2	2105.2	25.8
Example 3	1823.2	45.3
Example 4	1736.8	52.5

not introduce any difficulty by constructing explicit dual functions which are maximized subject to non-negativity constraints on the dual variables. The subdivision of the design variables in an active and passive group is intrinsically contained in the dual formulation. Furthermore, this method is effective because the explicit form of the design optimization problem is such that both the objective and constraints are separable functions of the member sizing variables. The proposed dual method directly accounts for discrete sizing variables instead of using a pseudo-discrete algorithm based on the optimality criterion, compared with the conventional OC method, and gives rapid and steady convergence to minimum weight structure since the number of iterations is independent of complexity of the structure. This can give the engineer the effects of saving the computational efforts due to trial and error methods in controlling the lateral drifts. Also, the proposed dual method shows that the total weights are somewhat reduced, compared with OC method. Therefore, the proposed dual method provides an effective strategy for discrete optimal design of tall steel structures subject to lateral drift constraints.

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