Optimum Design of Thinned Microphone Arrays Using a Modified Perturbation Approach

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※This work was supported by KOSEF(contract number : 95-0100-03-05-3).

Abstract

A modified perturbation method is proposed to optimize the beam pattern of thinned microphone arrays. Both microphone spacing and array weight are iteratively adjusted via successive perturbation to achieve an optimum beam pattern in a Dolph Chebyshev sense. To improve the sidelobe performance, an alternative perturbation with respect to microphone spacing and array weight is implemented. Also, a linear space-tapering is employed in the perturbation process. It is demonstrated that the proposed approaches successfully yield sidelobe performances comparable to that of a normal array. Computer simulation results are presented.

I. Introduction

An array of microphones may be used to receive a speech signal corrupted by directional or background noises with a high sensitivity. If the array elements are spaced uniformly every half wavelength, the array is called a filled array. A thinned array consists of fewer number of elements than the filled array with the same array length. If the array is large or the cost of a microphone is expensive, a thinned microphone array is very efficient in the sense that the array design cost is lower compared with that of the filled array due to the reduced number of elements while it performs well with broadband signal. The origin of the concept for the thinned arrays dates back to the work of Unz [1] in the 1950's. Ever since then, the thinned arrays have been used successfully in such areas as radar [2] and astronomy [3], etc. The optimum pattern for the thinned array may not be obtained as easily as for the filled array because the reduced number of elements results in the reduced number of degrees of freedom to control the beam pattern. Thus, the main concern in the design of thinned microphone array is to find an optimum set of microphone spacings or array weights which yields an array performance comparable to that of the corresponding filled array. In a teleconference environment, it is desirable to form a beam pattern with uniform sidelobe to deal with uniformly distributed interferences. A variety of optimization methods have been investigated to this end, such as perturbation method [4], least square [5], dynamic programming [6], minimax [7], etc.

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Manuscript Received : September 16, 1998.

In this paper, it is proposed to employ a modified perturbation method which is a modified version of the perturbation method [4] to optimize the beam pattern of the thinned microphone array. In this method, the beam pattern is optimized by perturbing either the element spacing or the array weight in such a way that the element spacing is perturbed with uniform array weights or the array weight is perturbed with uniform spacing. It is to be noted that the proposed method is very efficient in controlling the sidelobes of the thinned array due to the fact that the locations of the sidelobes are found numerically during the perturbation process rather than algebraically as in the conventional method. It is shown that the proposed method provides an equalized sidelobe successfully. To improve the sidelobe performance, an alternative perturbation of element spacing and array weight is implemented and its performance is compared with that of perturbing either one. Also, a spacetapered thinned array is employed in perturbing the array weights. It is shown that the sidelobe performance is improved with the space tapering compared with that of a uniform spacing.

II. Perturbation Method for Thinned Arrays

Consider a thinned linear array of omni directional microphones which is symmetric with respect to array weight and microphone spacing as shown in Fig. 1. Assuming that the number of microphones is odd and incoming signals are plane waves, the array factor is given by

$$H(\omega) = a_0 + 2 \sum_{i=1}^{N} a_i \cos(d_i \omega) \tag{1}$$

where $\omega = \pi \sin\theta$, θ is an angle from the array normal,

a_i is array weight, d_i is microphone spacing normalized by a half wavelength, and the number of microphones is $2N+1$, and i is microphone index.

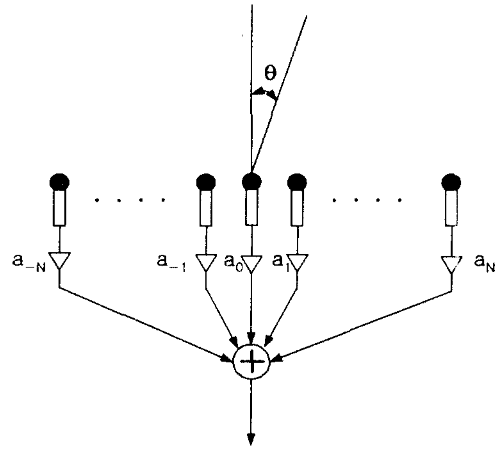

Figure 1. Thinned linear microphone array.

The basic idea of the perturbation method[4] is that for a specified threshold, the level of higher sidelobes is lowered by sacrificing lower sidelobes via perturbation of microphone spacing or array weight so that the sidelobes are equalized.

Suppose that L sidelobes are located at ω_j, $1 \le j \le L$ and the corresponding sidelobe levels are $H(\omega_j) = \varepsilon_j$. Using the fact that the derivative of the array factor with respect to ω at each ω_j is zero, we have the following equations.

$$\varepsilon_j = a_0 + 2\sum_{i=1}^{N} a_i \cos(d_i \omega_j), \quad 1 \le j \le L$$

$$\sum_{i=1}^{N} a_i d_i \sin(d_i \omega_j) = 0, \quad 1 \le j \le L \tag{2}$$

In the perturbation method, (1) and (2) are solved for ω_j and, a_i or d_i by perturbing the element spacing d_i or the array weight a_i iteratively such that L sidelobes get closer to a specified threshold level. In this method, the locations of the sidelobes are found algebraically based on the initially specified sidelobes. If the number of sidelobes is more than the number of microphone spacings or array weights as in the thinned arrays, some of the sidelobes can not be controlled. To overcome this shortcoming of the perturbation method, we propose a modified perturbation method.

III. Modified Perturbation Method

3.1 Perturbation of Microphone Spacing

For the perturbation of the microphone spacing with uniform weights, if the spacing at the kth iteration is perturbed by Δd_i^k, the $(k+1)$th spacing is given by

$$d_i^{k+1} = d_i^k + \Delta d_i^k \tag{3}$$

As a result, the position and the level of the L sidelobes are changed accordingly as follows,

$$\omega_j^{k+1} = \omega_j^k + \Delta\omega_j^k \tag{4}$$

$$\varepsilon_j^{k+1} = \varepsilon_j^k + \Delta\varepsilon_j^k \tag{5}$$

where $\Delta\varepsilon_j^k$ is a small fraction of the difference between the actual sidelobe level of the jth sidelobe and a specified sidelobe level. If (3), (4), and (5) are substituted into (1) and (2) and the resulting equation is linearized with the following approximations assuming a small perturbation,

$$\begin{aligned} &\sin(d_i^k\Delta\omega_j^k + \omega_j^k\Delta d_i^k + \Delta d_i^k\Delta\omega_j^k) \approx d_i^k\Delta\omega_j^k + \omega_j^k\Delta d_i^k \\ &\cos(d_i^k\Delta\omega_j^k + \omega_j^k\Delta d_i^k + \Delta d_i^k\Delta\omega_j^k) \approx 1 \end{aligned} \tag{6}$$

we have

$$\Delta\varepsilon_j^k = -2\omega_j^k \sum_{i=1}^{N} a_i \Delta d_i^k \sin(d_i^k \omega_j^k) \tag{7}$$

$$\begin{aligned} \Delta\omega_j^k \sum_{i=1}^{N} a_i d_i^{k2} \cos(d_i^k\omega_j^k) + \omega_j^k \sum_{i=1}^{N} a_i d_i^k \Delta d_i^k \cos(d_i^k\omega_j^k) \\ + \sum_{i=1}^{N} a_i \Delta d_i^k \sin(d_i^k\omega_j^k) = 0 \end{aligned} \tag{8}$$

Assuming that $\Delta\varepsilon_i$ is small, we perturb the element spacings iteratively until all the sidelobes are equalized. At each iteration, Δd_i and $\Delta\omega_j$ are found in (7) and (8) respectively and then the sidelobes are updated. It is to be noted that to insure unique solutions of Δd_i and $\Delta\omega_j$ at each iteration, the number of sidelobes to be controlled should be equal to the number of element spacings N.

In the perturbation method, only the initially chosen sidelobes, the number of which is equal to the number of microphone spacings to be determined, are controlled during the entire perturbation process and other sidelobes are uncontrollable. Also, the sidelobe performance may be degraded due to the error related to updated locations of the sidelobes. In the thinned arrays, the number of the sidelobes in the initial pattern is more than the number of element spacings, so that some of the sidelobes can not be controlled. So, the conventional perturbation method is not suitable for optimizing the sidelobes of the thinned microphone arrays.

The basic idea of the modified perturbation method is that the locations of the new set of maximum sidelobes are found from the array factor (1) numerically at each iteration instead of calculating $\Delta\omega_j$ using (8) as in the conventional approach. Then the perturbation of spacing is determined by (7) based on the numerically found ω_j. Thus if N maximum sidelobes are chosen and updated at each iteration instead of updating only the initially chosen L sidelobes, we can prevent any sidelobes from being higher than the specified threshold level.

3.2 Perturbation of Array Weight

The array weights are perturbed with uniform microphone spacing such that the $(k+1)$th weight with a perturbation Δa_i^k at the kth iteration is given by

$$a_i^{k+1} = a_i^k + \Delta a_i^k \tag{9}$$

Substituting (9) and the resulting ω_j^{k+1} and ε^{k+1} as in (4) and (5) into (1) and (2) with the similar approximations to the case of spacing perturbation, we have

$$\Delta\varepsilon_j^k = 2\sum_{i=1}^{N} \Delta a_i^k \cos(d_i\,\omega_j^k) \tag{10}$$

$$\sum_{i=1}^{N} \Delta a_i^k d_i \sin(d_i\,\omega_j^k) + \sum_{i=1}^{N} \Delta\omega_j^k a_i^k d_i \cos(d_i\,\omega_j^k) = 0 \tag{11}$$

To avoid the difficulties of the conventional method in updating the sidelobes for the thinned arrays, the sidelobes are located numerically as in the spacing perturbation and then the array weights are updated. This process will be continued to find an optimum beam pattern.

3.3 Considerations on Performance Improvement

To improve the sidelobe performance, we may perturb the array weight and element spacing alternatively. Also, we may use a linearly space-tapered thinned array. In this approach, the thinned array is firstly tapered in terms of element spacing, and then only the array weight is perturbed to achieve an equalized sidelobe level. It is shown that these approaches yield a lower sidelobe compared with the normal methods.

IV. Simulation Results

A filled linear array of half wavelength-spaced 41 microphones is employed in the simulation. The number of microphones is reduced by 25% (10 microphones reduced) to form a thinned array. An LU decomposition routine in IMSL is used to solve a set of linear equations for the differentials of spacing or weight. The beam pattern of the 41-element filled array optimized by microphone spacing with uniform array weight is shown in Fig. 2. Also, the beam pattern optimized by array weight with uniform microphone spacing is shown in Fig. 3. It is observed that the sidelobe level by array weight is about 5 dB lower than that by microphone spacing. Now, we reduce the number of microphones by 10 to form a 31-element thinned array with the length of which is the same as that of the filled array. The beam pattern of the thinned array optimized by microphone spacing is shown in Fig. 4. The corresponding microphone spacings are listed in Table 1. The center and two end microphones are fixed to maintain the array symmetry and length. It is observed that the sidelobes are equalized to about −20 dB which is about the same level as that of the 41-element filled array. Also, Fig. 5 shows the beam pattern optimized by array weight and the corresponding array weights are listed in Table 2. It is shown that the level of the sidelobe is −26 dB which is a little lower than that in the filled array while the width of the mainbeam is a little wider than that of the filled array. From the simulation results, it is demonstrated that the sidelobe perfor-

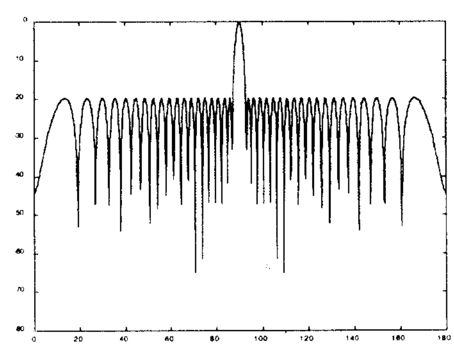

Figure 2. Beam pattern of the 41-element array optimized by microphone spacing.

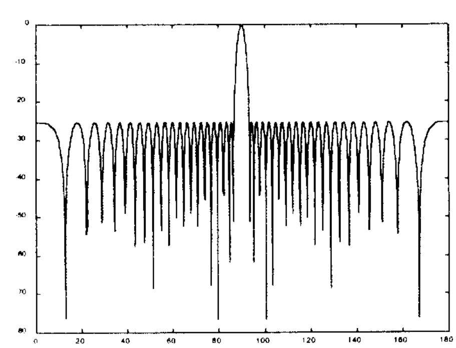

Figure 3. Beam pattern of the 41-element array optimized by array weight.

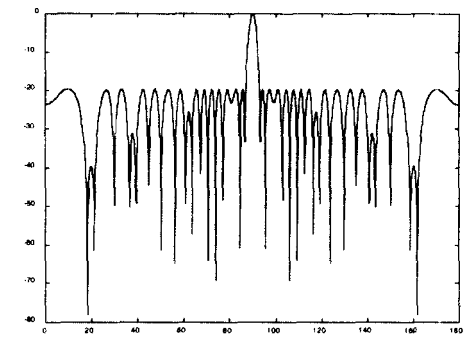

Figure 4. Beam pattern of the 31-element thinned array optimized by microphone spacing.

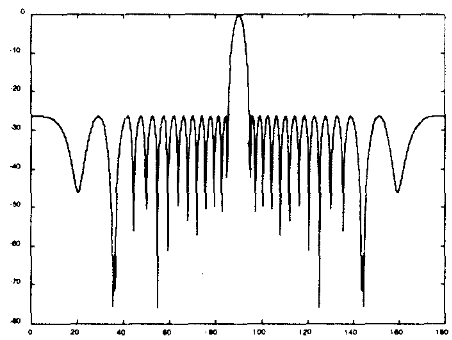

Figure 5. Beam pattern of the 31-element thinned array optimized by array weight.

mance of the thinned array is very close to that of the filled array. To improve the sidelobe performance, we first perturb the weights of the thinned array to find an intermediate optimum pattern and then perturb the spacing for a final optimum pattern. The resulting beam pattern is shown in Fig. 6. The beam pattern optimized only by the microphone spacing with uniform array weight is shown is Fig. 7. It is shown that the sidelobe optimized by both the weight and spacing in Fig. 6 is a little lower than that by spacing only in Fig. 7. The numerical data for Fig. 6 are shown in Table 3. The sidelobe performance may be improved by a space-tapering. We taper the spacings in the thinned array and then perturbation

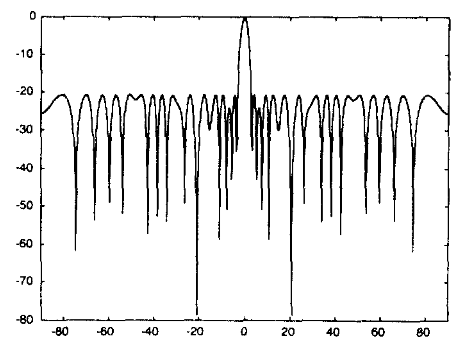

Figure 6. Beam pattern of the 31-element thinned array alternatively optimized by array weight and microphone spacing.

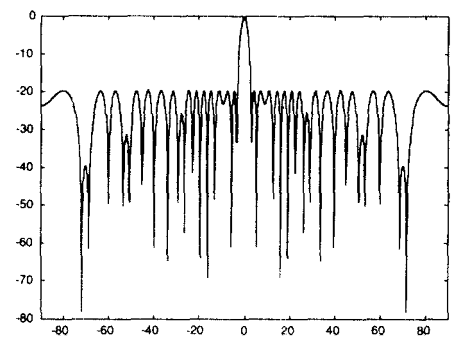

Figure 7. Beam pattern of the 31-element thinned array optimized by microphone spacing.

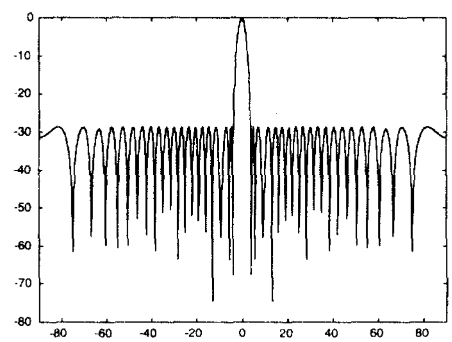

Figure 8. Beam pattern of the 31-element space-tapered thinned array optimized by array weight.

Table 1.

microphone index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
initial spacing(λ/2)	1.33	2.67	4.00	5.33	6.67	8.00	9.33	10.67	12.00	13.33	14.67	16.00	17.33	18.67	20.00
final spacing(λ/2)	0.86	1.69	2.75	3.85	4.93	6.44	8.08	9.17	10.65	11.73	12.65	14.06	15.19	18.35	20.00

Table 2.

microphone index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
initial weight($\times 10^{-2}$)	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22
final weight($\times 10^{-2}$)	4.20	3.32	3.75	3.42	3.18	3.21	2.79	2.63	2.49	2.00	1.96	1.69	1.09	1.68	2.09

Table 3.

microphone index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
initial spacing(λ/2)	1.33	2.67	4.00	5.33	6.67	8.00	9.33	10.67	12.00	13.33	14.67	16.00	17.33	18.67	20.00
initial weight($\times 10^{-2}$)	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22
optimum weight($\times 10^{-2}$)	4.20	3.32	3.75	3.42	3.18	3.21	2.79	2.63	2.49	2.00	1.96	1.69	1.09	1.68	2.09
optimum spacing(λ/2)	0.83	1.85	3.03	4.67	5.60	6.41	7.48	8.63	9.87	10.97	13.12	14.45	16.34	18.08	20.00

Table 4.

microphone index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
tapered spacing(λ/2)	1.00	1.86	2.88	3.97	5.12	6.33	7.60	8.94	10.33	11.79	13.30	14.89	16.53	18.23	20.00
initial weight($\times 10^{-2}$)	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22
optimum weight($\times 10^{-2}$)	3.40	3.53	3.63	3.64	3.65	3.61	3.59	3.58	3.56	3.41	3.08	2.55	1.91	1.31	2.30

of the array weight is applied to find an optimum beam pattern which is shown in Fig. 8. It is observed that the sidelobe is about 2 dB lowered compared with that by the uniform spacing case as in Fig. 5. The corresponding numerical data is shown in Table 4.

V. Conclusions

A modified version of the perturbation approach is proposed for a linear thinned microphone array. It is demonstrated that the sidelobes of the thinned arrays are controlled efficiently by the proposed method to become equalized successfully through the perturbation of array weight or microphone spacing.

An alternative perturbation approach is applied in a thinned microphone array with respect to microphone spacing and array weight. It is shown that the sidelobe performance is improved a little compared with that by spacing only perturbation. Also, a linearly space-tapered thinned array is perturbed by array weight to improve the sidelobe performance. It is demonstrated that the space-tapering yields a much lower sidelobe than the uniform configuration in a Dolph-Chebyshev sense.

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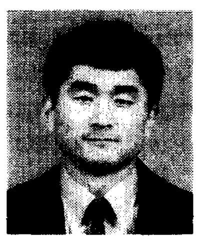

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