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Numerical Prediction of Turbulent Heat Transfer to Low Prandtl Number Fluid Flow through Rod Bundles

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Abstract — The turbulent heat transfer to low Prandtl number fluid flow through rod bundles is analyzed using k–ε two-equation model. For the prediction of the turbulent flow field, an anisotropic eddy viscosity model is used. In the analysis of the temperature field, the effects of various parameters such as geometry, Reynolds and Prandtl numbers are considered. The calculation is made for Prandtl numbers from 0.001 to 0.1 in order to analyze the heat transfer to low Prandtl number fluid such as liquid metals. The numerical results show that for small P/D (Pitch/Diameter) geometries low Prandtl number makes severe changes of the rod surface temperature.

1. Introduction

The convective heat transfer problems of the flow with non-circular cross section are commonly met in various engineering applications. In the turbulent flow through non-circular channel, there exists transverse transport on the cross sectional area such as secondary flow and strong anisotropic turbulent diffusion and laminar region near the corner[1,2]. The interaction of the above two phenomena brings difficulties in the prediction of heat transfer in non-circular flow field.

In this study, rod bundles are adopted as flow domain. Rod bundles are widely used in commercial nuclear fuel assemblies. The knowledge of flow and temperature fields in rod bundles is indispensible for the fuel performance analysis during normal operating conditions and for the insurance of the structural integrity during abnormal transients. Recently, for the disposal of long-lived high-level radioactive waste such as spent fuel, many researches on TR (Transmutation Reactor) are carried on. Lead, lead/bismuth, or lead/lithium are recommended as coolants of TR due to their nuclear and chemical characteristics. Hence, the information on the heat transfer behavior of such liquid metal coolant flow is very important in the selection of the coolant and the thermal hydraulic design of TRs. Although many studies on the heat transfer to liquid metal have been accomplished and many commercial CFD (Computational Fluid Dynamics) codes have been developed, the results seem to be unsatisfactory because of the lack of knowledge on turbulent flow through rod bundles and the limitation of turbulent models[3-6].

This work is concerned with numerical analysis of flow and temperature fields in rod bundles using the code developed by Kim and Park[7]. Especially, in order to predict the turbulent heat transfer to low Prandtl number flow, the calculation is performed with Prandtl number ranging from 0.001 to 0.1.

2. Mathematical Model

2-1. Governing Equations

The k–ε two-equation model which is commonly used for the analysis of turbulent flow field is adopted to predict the heat transfer to the flow through the infinitely-arrayed bare rod bundles as shown in Fig. 1. The transverse transport is simulated using the anisotropic turbulent diffusion model of Kim and Park[8] which is based on flow pulsation phenomenon observed in rod bundle flow fields. However, the secondary flow which may contribute to transverse transport is ignored because its contribution is much less than that of anisotropic eddy

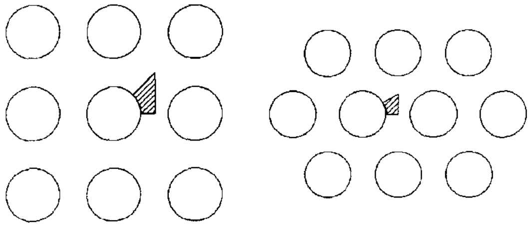

Fig. 1. Schematic of rod bundle geometries.

diffusion. It is assumed that the flow is steady and axially fully-developed. Also the working fluid is regarded as incompressible.

The aim of this study is to examine the heat transfer behavior according to the variation of parameters such as Reynolds number and P/D as well as Prandtl number. The governing equations are expressed in dimensionless form. Kinematic viscosity ν and friction velocity u* are used as basic parameters for the non-dimensionalization. So, the velocity and the length scale are u* and ν/u*, respectively. On the other hand, the energy equation is non-dimensionalized with temperature scale ν/u* ($\partial T/\partial x_3$).

In Cartesian tensor notation, the dimensionless governing equations are as follow:

• Dimensionless axial momentum equation

$$-\frac{\partial}{\partial x_i^+}\left[(\delta_{ij}+\nu_{ij}^+)\frac{\partial U_3^+}{\partial x_j^+}\right]=-\frac{4}{Re^*} \tag{1}$$

$$Re^*=\frac{u^*D_H}{\nu} \tag{2}$$

• Dimensionless turbulent kinetic energy equation (k^+ equation)

$$-\frac{\partial}{\partial x_i^+}\left[(\delta_{ij}+\nu_{ij}^+/\sigma_k)\frac{\partial k^+}{\partial x_j^+}\right]=P_k^+-\varepsilon^+ \tag{3}$$

$$P_k^+=-\overline{u_1^+u_3^+}\frac{\partial U_3^+}{\partial x_1^+}-\overline{u_2^+u_3^+}\frac{\partial U_3^+}{\partial x_2^+} \tag{4}$$

where σ_k=1.0

• Dimensionless turbulent kinetic energy dissipation rate equation (ε^+ equation)

$$-\frac{\partial}{\partial x_i^+}\left[(\delta_{ij}+\nu_{ij}^+/\sigma_\varepsilon)\frac{\delta\varepsilon^+}{\delta x_j^+}\right]$$

$$=C_{\varepsilon 1}f_{\varepsilon 1}\frac{\varepsilon^+}{k^+}P_k^+-C_{\varepsilon 2}f_{\varepsilon 2}\frac{\varepsilon^{+2}}{k^+} \tag{5}$$

where σ_ε=1.3, $C_{\varepsilon 1}$=1.44, and $C_{\varepsilon 2}$=1.92

• Dimensionless energy equation

$$-\frac{\partial}{\partial x_i^+}\left[(\delta_{ij}/Pr+\nu_{ij}^+/Pr_T)\frac{\partial T^+}{\partial x_j^+}\right]=-U_3^+ \tag{6}$$

where Pr_T=0.9

The subscripts i and j can be 1 and 2 which denote normal and parallel direction to the wall and the superscript + means non-dimensionalized variable. As for energy equation, there are many turbulent Prandtl number models but the suitable models for rod bundle geometry are seldom found. In this study, 0.9 is used as a turbulent Prandtl number through the work because it is popular in the turbulent heat transfer calculation. Further studies are required in this area.

2-2. Anisotropic eddy viscosity model

Kim and Park[8] derived anisotropic factor of turbulent diffusion based on flow pulsation phenomenon which is observed in rod bundle flow fields and successfully applied it to the flow field analysis[9]. In this work, Kim and Park's model[9] is used to predict the anisotropic feature of turbulence.

• Reynolds stress

$$-\overline{u_i^+u_3^+}=\nu_{ij}^+\frac{\partial U_3^+}{\partial x_j^+}\qquad(\text{for } i\neq j,\ \nu_{ij}^+=0) \tag{7}$$

$$\nu_{11}^+=C_\mu f_\mu\frac{k^{+2}}{\varepsilon^+},\qquad \nu_{22}^+=n\nu_{11}^+ \tag{8}$$

where C_μ=0.09

• Anisotropic factor model

$$n=\left[\frac{6\bar{n}}{\sqrt{\pi}}\exp\left\{-\left(\frac{6y^+}{Re^*}\right)^2\right\}+1\right]^{(1-\theta/\theta_{max})} \tag{9}$$

$$\bar{n}=\frac{1+2a_xb\dfrac{(z_{FP}/D)(\delta/D)}{g/D}Str}{1+2a_y(z_{FP}/D)Str} \tag{10}$$

In equations (9) and (10), y^+ is dimensionless distance from the wall. Also, a_x and a_y are velocity factors for parallel and normal to the wall respectively, where

$$a_x = 1.0 - 0.15\left(\frac{g}{D}\right), \quad a_y = 0.36\left(\frac{g}{D}\right) \tag{11}$$

are used. The path length of hypothetical flow representing flow pulsation occurred in rod bundle turbulent flow z_{FP} is approximated:

$$\frac{z_{FP}}{D} \approx \frac{\pi}{\sqrt{2}}\sqrt{b^2\left(\frac{\delta}{D}\right)^2 + \left(\frac{g}{D}\right)^2} \tag{12}$$

and δ is centroid-to-centroid distance of the channel. The shape factor b is introduced to reflect the effect of obstacles which may be located in the path of flow pulsation, and

$$b = \begin{cases} 1 & \text{for } \textit{square array} \\ \frac{2}{3} & \text{for } \textit{triangular array} \end{cases} \tag{13}$$

are used. Finally, Strouhal number Str, based on the principal frequency of flow pulsation f_p, rod diameter, and friction velocity, is expressed as

$$Str = \frac{f_p D}{u^*} \tag{14}$$

and the correlation of Wu and Trupp[9] is adopted:

$$Str^{-1} = 0.822\left(\frac{g}{D}\right) + 0.144 \tag{15}$$

The detail explanation of the model is found in References[8,9].

2-3. Low-Reynolds number model

The ultimate goal of thermal hydraulic analysis of nuclear rod bundles is to find the rod surface temperature, so the rod surface is included in the calculation domain and Lam and Bremhorst model[10] is used as a low-Reynolds number model. That is, as a wall boundary condition no-slip condition is used instead of wall function commonly used in the turbulent flow analysis.

- Lam and Bremhorst damping model

$$f_\mu = (1 - \exp(-B_\mu R))^2 (1 + \frac{D_\mu}{R_t}) \tag{16}$$

$$f_{\varepsilon 1} = 1 + (\frac{A_{\varepsilon 1}}{f_\mu})^3, \quad f_{\varepsilon 2} = 1 - \exp(-R_t^2) \tag{17}$$

$$R = k^{+1/2} y^+, \quad R_t = \frac{k^{+2}}{\varepsilon^+} \tag{18}$$

where B_μ=0.0165, D_μ=20.5, and $A_{\varepsilon 1}$=0.05

2-4. Boundary conditions

The problem domain is enclosed by rod surface and symmetry boundaries, so two types of boundary conditions are used: no-slip condition for rod surface and symmetry condition for symmetry boundary. On the rod surface, all components of the velocities and turbulent kinetic energy are zero by the no-slip condition. However, for the dissipation rate of turbulent kinetic energy, the boundary conditions are neither exact nor simple. Furthermore, some of them, even used commonly, are likely to cause the divergence of solutions especially at high Reynolds number. Thus, in this study, Kim and Park's boundary condition[7] is applied:

$$\varepsilon_w^+ = \frac{3k_{w+1}^+}{k_{w+2}^+ - k_{w+1}^+}\,\varepsilon_{w+1}^+ \tag{19}$$

The subscripts w, w+1, and w+2 denote the node of the wall, the first node next to the wall, and the second next to the wall, respectively.

On the symmetry boundaries, the normal gradients of axial velocity, turbulent kinetic energy and its dissipation rate are set to zero.

In the analysis of temperature field, uniform heat flux along the rod wall is assumed. The uniform heat flux assumption is more realistic than uniform temperature assumption.

3. Flow and Temperature Field Analysis

The computer code used in this work was developed in the studies of Kim and Park[7,9], which adopts finite element method. The detailed description of the code can be found in the above studies.

As mentioned above, the turbulent thermal flow field in rod bundles is analyzed varying three parameters: Reynolds number, Prandtl number, and P/D. Especially, in order to obtain information on the thermal hydraulics of liquid metal coolant nuclear reactors, the calculation is performed for

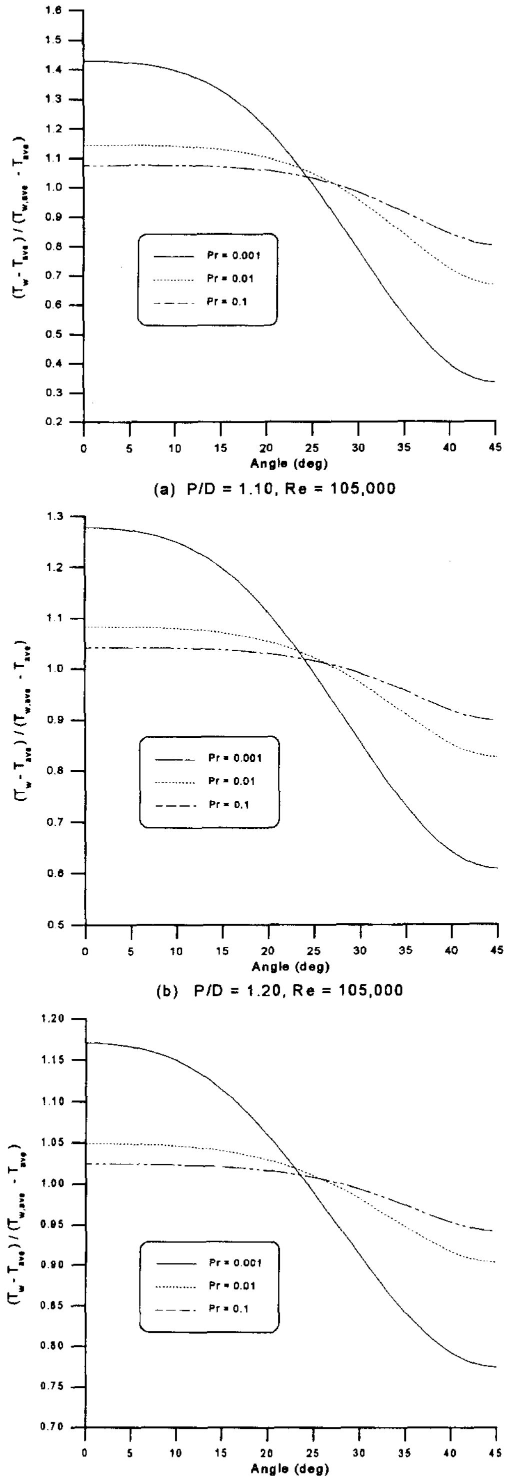

Fig. 2. Wall temperature vriation for a square array.

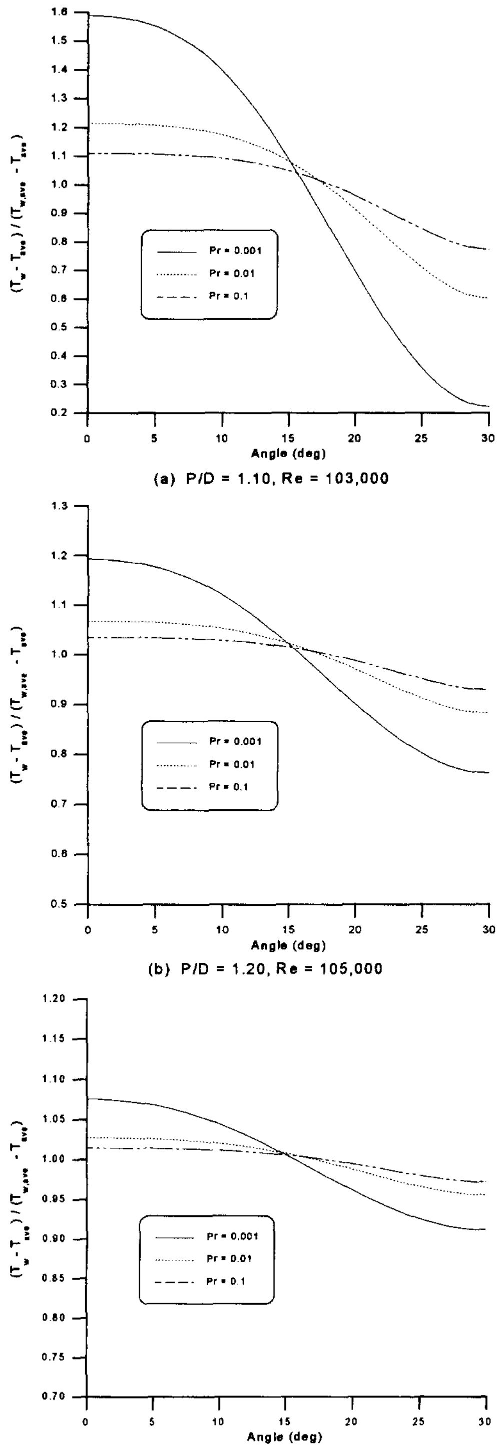

Fig. 3. Wall temperature variation for a triangular array.

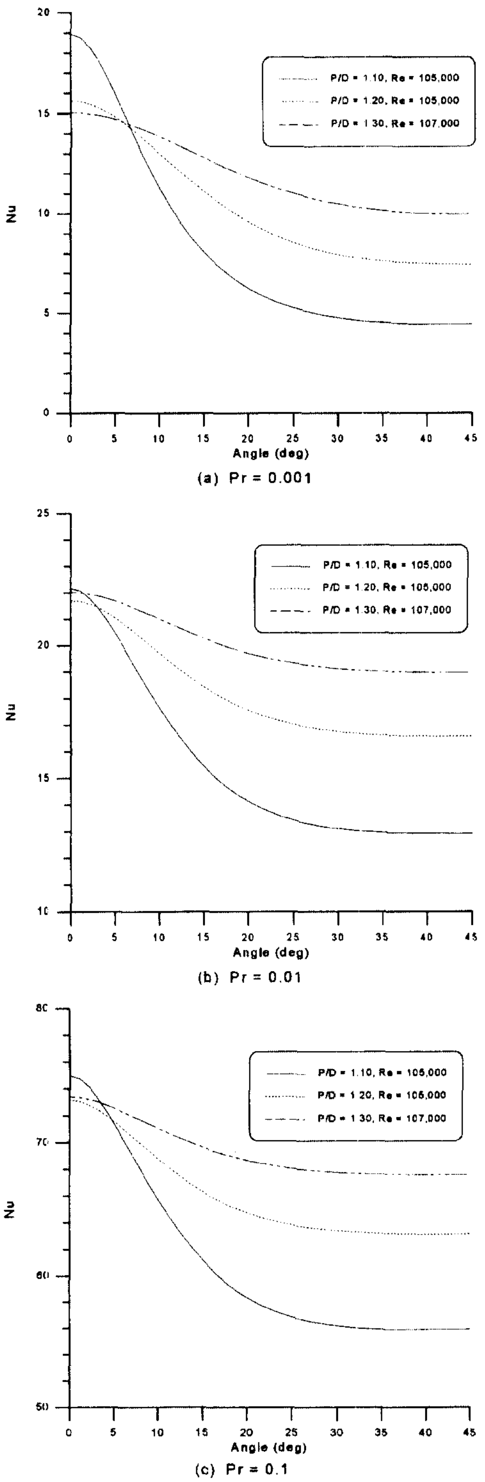

Fig. 4. Local Nusselt number variation for a square array.

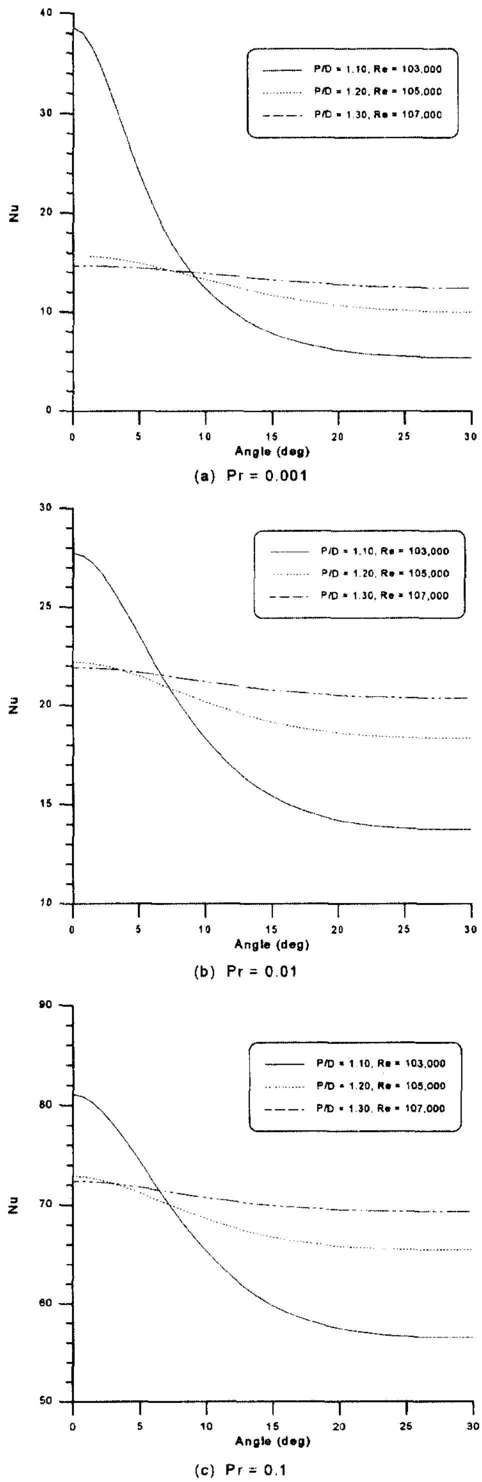

Fig. 5. Local nusselt number variation for a triangular array.

low Prandtl numbers varying 0.001 to 0.1.

The temperature distributions along the rod surface according to P/D and Prandtl number are shown in Figs. 2 and 3, which correspond to square and triangular arrays respectively. The overall behaviors are very similar to each other, but the magnitudes of the temperature variation are different. For both square and triangular arrays, bigger temperature variation are observed as P/D decreases. The temperature distribution of rod surface becomes almost uniform near P/D=1.30. As for the rate of change of the temperature difference, that of square array is smaller than that of triangular array. The temperature difference along rod surface becomes bigger as Prandtl number decreases. Hence, if low Prandtl number material such as liquid metal is selected as a working fluid of a nuclear reactor, structural problems may occur due to thermal stress by non-uniform temperature distribution along the fuel rod wall.

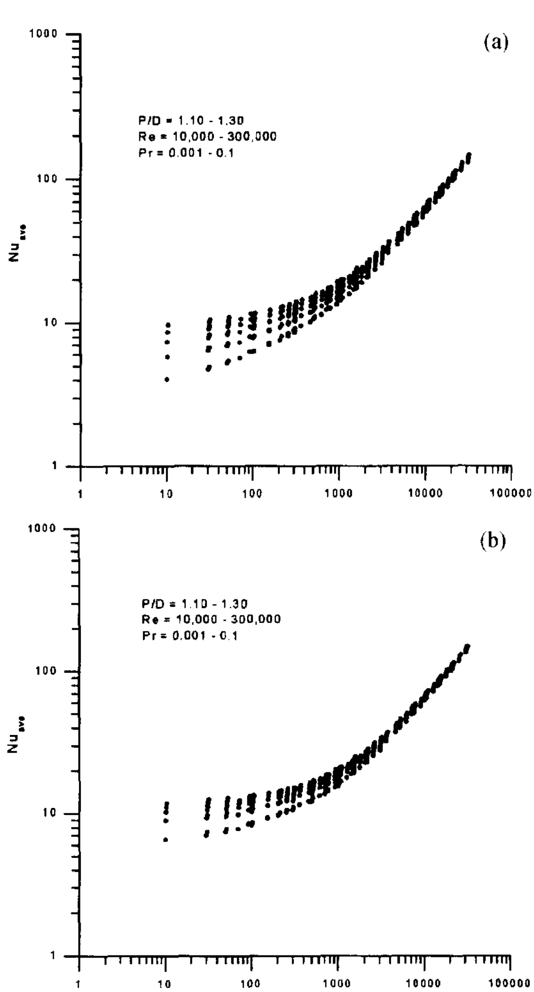

Fig. 6. Calculated average Nusselt numbers. a. square array, b. triangnlar array.

Figs. 4 and 5 show the local Nusselt number variation according to P/D for square and triangular array respectively when Reynolds is about 100,000. The local Nusselt number is more affected by P/D at low Prandtl number for triangular array than for square array.

In Fig. 6, average Nusselt numbers are plotted as a function of Péclet number. For Péclet number of above 10,000, P/D hardly affects Nusselt number. However, for Péclet number less than 10,000, P/D may have an considerable effect on Nusselt number.

4. Conclusion and Further Studies

The turbulent heat transfer in rod bundles is predicted numerically using a k-ε 2-equation model. For the calculation, the anisotropy of eddy diffusion manifested in rod bundle flow fields is considered. The heat transfer in rod bundles can be specified by Reynolds number, Prandtl number, and P/D, so that the calculation is made with varying these parameters. Especially, in order to examine the heat transfer for low Prandtl number fluid flow, the numerical results are obtained for Prandtl number from 0.001 to 0.1. For small P/D and small Prandtl number, the magnitude of rod surface temperature difference is considerably large. It is expected that these numerical results may be used in the selection of coolant material and in the thermal hydraulic design of TR (Transmutation Reactor) as whose coolant various liquid metals are proposed.

In this work, a constant value is used as turbulent Prandtl number because the suitable model for rod bundle geometry is not available. In fact, the influence of this parameter is very important to the turbulent heat transfer analysis. Therefore, for more accurate prediction, the proper turbulent Prandtl model for rod bundles should be developed and the analysis should be performed with the model. Also, this study fails to obtain a meaningful correlation of

Nusselt number with P/D, Reynolds and Prandtl number as a simple function such as power function like well-known Dittus-Boelter equation due to the complexity of effects of above major parameters on Nusselt number.

Acknowledgments

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Nomenclature

a_x, a_y : velocity coefficients
$A_{\varepsilon 1}$: turbulent model constant
b : shape factor
B_μ, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, C_μ: turbulent model constants
D : rod diameter
D_H : hydraulic diameter
D_μ : turbulent model constants
$f_{\varepsilon 1}$, $f_{\varepsilon 2}$, f_μ : damping factors used in Lam and Bremhorst low-Reynolds number k-ε model
g : gap size
k : turbulent kinetic energy
n : anisotropic factor
Nu : Nusselt number
p : pressure
P/D : pitch-to-diameter
Pe : Peclet number
P_k : turbulent kinetic energy production rate
Pr : Prandtl number
r : radial coordinate
Re : Reynolds number based on hydraulic diameter
Str : Strouhal number
T : Temperature
U_i : mean velocity of i direction
u_iu_j : Reynolds stress
x_i : coordinate of i direction
y : normal distance from the wall
z_{FP} : hypothetical path length of flow pulsation

Greek

δ : centroid-to-centroid distance
δ_{ij} : Kronecker delta
ε : dissipation rate of turbulent kinetic energy
θ : azimuthal coordinate
ν : molecular kinematic viscosity
ν_{ij} : anisotropic eddy viscosity
ρ : density
σ_k, σ_ε, σ_T : Prandtl number for turbulent kinetic energy and its dissipation rate, and turbulent Prandtl number
τ_w : wall shear stress

Subscript

ave : average
i,j : Cartesian index (1 for normal to the wall, 2 for parallel to the wall, 3 for axial direction)
w : wall

Superscript

\+ : nondimensionalized variable

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