

<Original Paper>

# Vibration Control of Flexible Structures Using ER Dampers

ER 댐퍼를 이용한 유연구조물의 진동제어

Seung-Bok Choi\* and Jae-Hong Lee\*\*

최 승 복 · 이 재 홍

(Received December, 16 1997 ; Accepted February, 4 1998)

**Key Words :** Vibration Control (진동제어), Electro-Rheological Damper(ER 댐퍼) Flexible Structure (유연구조물), Sliding Mode Control(슬라이딩모드제어)

## ABSTRACT

This paper addresses a sliding mode control of vibration in a flexible structure using ER(electro-rheological) dampers. A clamped-clamped flexible structure system supported by two short columns is considered. Three ER dampers to be operated in shear mode are designed on the basis of Bingham model of the arabic gum-based ER fluid, and attached to the flexible beam structure. After deriving the governing equation of motion and associated boundary conditions, a sliding mode controller is formulated to effectively suppress the vibration of the beam structure caused by sinusoidal and random excitations. In the formulation of the controller, parameter variations such as natural frequency deviation are treated to take into account the robustness of control system. The effectiveness of the proposed control system is confirmed by both simulation and experimental results.

## 요 약

본 논문에서는 전기장 강도에 따라 댐핑력 조율이 가능한 ER댐퍼를 이용하여 유연구조물의 진동제어를 수행하였다. 아라빅 검 ER유체를 자체 조성한 후 전단 모드하에서의 빙햄모델을 실험적으로 도출하고, 이를 근거로 알맞은 크기의 ER댐퍼를 설계제작하였다. ER 댐퍼를 장착한 양단고정형 유연구조물시스템을 구성하고, 제어기 설계를 위한 운동지배 방정식과 경계조건을 도출하였다. 시스템변수중 실제시스템에서 쉽게 발생할 수 있는 고유진동수와 감쇠비를 불확실 변수로 고려하여 슬라이딩모드 제어를 설계하였다. 정현파와 랜덤가진에 대한 제안된 제어시스템의 우수한 진동제어 효과를 컴퓨터 시뮬레이션과 실험을 통하여 입증하였다.

## 1. Introduction

Recently, with the aid of advanced technologies in computer and material sciences, high-performance structural systems have been designed for application in various research fields such as space structures, bridges, and robotics. In particular,

\* 정회원, 인하대학교 공과대학 기계공학과

\*\* 인하대학교 공과대학 기계공학과

the emergence of the smart materials has accelerated successful development of the advanced structural system. So far, the smart materials include electro-rheological (ER) fluids, piezoelectric materials, shape memory alloys, and optical fibers. These smart materials are employed primarily to vibration control of distributed parameter systems operating under variable service conditions.

ER fluids are materials which undergo significant instantaneous reversible changes in material characteristics when subjected to electric potentials. The most significant change is associated with complex shear moduli of the material, hence ER fluids can be usefully exploited in vibration-suppression situation where variable damping characteristics may be employed to effectively control the response by tailoring the properties of the ER fluid. The vibration control of flexible structures using the ER fluid can be achieved from two different methods. The first approach is to replace conventional viscoelastic materials by the ER fluid. So far, numerous researches have been undertaken in this way. Coulter and Duclos<sup>(1)</sup> formulated an analytical model for the ER fluid-embedded structures via the conventional sandwich beam theory, and presented a feasibility that the controllability of the complex shear moduli of the ER fluid itself can be utilized to obtain desired responses of the structures. Choi et al.<sup>(2)</sup> proposed a control logic to minimize the tip deflections of the ER fluid-based structures on the basis of field-dependent responses in the frequency domain. More recently, Choi et al.<sup>(3)</sup> developed a feedback control algorithm for vibration control of the ER fluid-based structure, and experimentally realized the control scheme to accomplish desired structural responses in the presence of variable excitations.

The second approach to achieve vibration control of flexible structures is to utilize an

ER damper. Originally, the idea of applying the ER damper to vibration control has been initiated in automotive engineering applications. For instance, Petek et al.<sup>(4)</sup> constructed a semi-active suspension system consisting of four ER dampers, and then evaluated its effectiveness for vibration isolation. This idea can be easily exploited in structural vibration control. Austin<sup>(5)</sup> applied the ER fluid to attenuate the longitudinal vibration in a flexible cylindrical structure. The attenuation characteristics have been evaluated through experimental and numerical results by imposing constant electric fields without consideration of feedback control algorithms. Wang et al.<sup>(6)</sup> proposed a sliding mode control scheme for vibration control of flexible structures with adaptable ER dampers. They assumed that as the applied voltage across the electrodes increases, both the viscous damping constant and the frictional damping force increase. This assumption is somewhat different from Bingham characteristic which is mostly well-known physical model of the ER fluid. The Bingham characteristic indicates that only the frictional damping force due to the yield stress of the ER fluid increases as the electric field increases, while the viscous damping force remains to be constant without regard to the electric field.<sup>(7-8)</sup>

The present study proposes an ER damper incorporating Bingham model of the ER fluid, and applies it to the vibration control of a flexible beam structure. Following the formulation of the governing equation of motion, a sliding mode controller is designed. In the synthesis of the controller, parameter variations such as natural frequency deviation are considered to demonstrate the robustness of control system. Forced vibrations caused by external excitations are effectively and robustly controlled by employing the proposed control methodology. Consequently, the main contribution of this study is to show how the sliding mode controller

associated the Bingham model-based-ER dampers can be satisfactorily employed for the forced-vibration control of a flexible beam structure subjected to parameter variations.

### 2. ER Damper Design

The flow motion of the ER fluid-based actuator can be classified by shear mode, flow mode and squeeze mode. The shear mode occurs when one of two electrodes moves linearly or rotationally. The ER devices featuring the shear mode include clutch systems, brake systems, and structural damper systems. The flow mode occurs when two electrodes are fixed like ER valve systems. The squeeze mode features a time-varying electrode gap processing the ER fluid vertically. An appropriate flow motion of the ER fluid should be identified

for right application devices. Fig. 1 presents a schematic diagram and its photograph of the ER damper proposed in this study. The ER damper mainly consists of a cup fixed on the ground and a moving cylinder attached to bottom-surface of the flexible structure(refer to Fig. 3). Thus, the moving cylinder is subjected to a vertical motion when the flexible structure vibrates. In other words, one electrode (fixed cup) is fixed, while the other electrode (moving cylinder) moves linearly resulting in the shear mode of the flow motion. Consequently, rheological property of the ER fluid should be evaluated in the shear motion.

Under the electric potential, a constitutive equation for the shear mode of the ER fluid has a form of Bingham plastic<sup>(8)</sup> :

$$\tau = \eta\dot{\gamma} + \tau_y(E), \quad \tau_y(E) = \alpha E^\beta \tag{1}$$

Here  $\tau$  is shear stress,  $\eta$  is viscosity,  $\dot{\gamma}$  is shear rate and  $\tau_y(E)$  is yield stress of the ER fluid. As evident from Eq. (1),  $\tau_y(E)$  is a function of the electric field  $E$  and exponentially increases with respect to the electric field. The proportional coefficient  $\alpha$  and exponent  $\beta$  are intrinsic values, which are functions of particle size, particle concentration and polarization factors such as particle conductivity. In this study, for the ER fluid, arabic gum and transformer oil are chosen as particles and carrier liquid, respectively. The size of the particles ranges from 26  $\mu\text{m}$  to 88  $\mu\text{m}$ . The weight

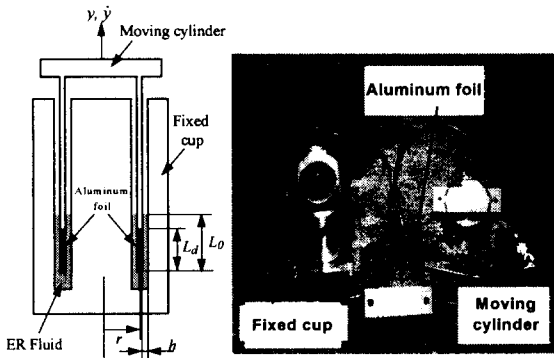


Fig. 1 A schematic diagram and photograph of the proposed ER damper

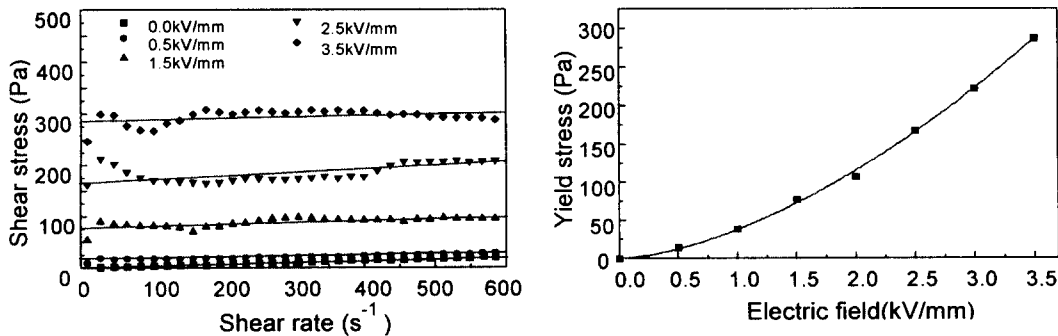


Fig. 2 Bingham property of the ER fluid

ratio of the particles to the ER fluid is 25 %. A couette type electroviscometer (Haake, VT-500) is employed to obtain the Bingham property of the ER fluid. Fig. 2 presents measured shear stress and yield stress of the ER fluid. The yield stress of the ER fluid is obtained using a linear regression of the shear stress. In other words, the intercept at zero shear rate is adopted as the (dynamic) yield stress. This is expressed by  $35.7E^{1.87}$  Pa as the form of Eq. (1). Here, the unit of  $E$  is kV/mm. This equation is used for the evaluation of field-dependent damping forces of the proposed ER damper.

The damping force in shear mode consists of two components : the force by the viscosity of the ER fluid, and the force by the yield stress of the ER fluid upon employing the electric field. Of course, the latter is controllable damping force. From the schematic diagram shown in Fig. 1, we may assume that  $(L_0 - y \approx L_d)$  since the vibration amplitude( $y$ ) considered in this study is relatively small (below 3 mm). In addition, we see that there are two electrode gaps. Thus, the damping force of the proposed ER damper can be obtained as follows.

$$F_{ER} = 4\pi r L_d \left[ \eta \frac{\dot{y}}{h} + \alpha E^\beta \cdot \text{sgn}(\dot{y}) \right] \quad (2)$$

Here  $h$  is electrode gap,  $L_d$  is electrode length of the moving cylinder,  $r$  is mean radius of the moving cylinder,  $\dot{y}$  is transverse velocity of the ER damper, and  $\text{sgn}(\cdot)$  represents signum function. After analyzing field-dependent damping force using Eq. (2), an appropriate size of the ER damper is designed and manufactured as shown in Fig. 1. The cup is made of aluminum, while the moving cylinder is made of acrylic tube. The inside and outside of the cylinder are coated with aluminum foils to provide the electrodes. The mean radius and length of the moving cylinder are 25 mm and 20 mm, respectively. The electrode gap is 2 mm.

### 3. Modeling of Structural System

Consider a flexible structure which has a continuous and uniform beam of length  $L$  as shown in Fig. 3. The structure is fixed at both ends and the surface-bonded piezoceramic is used to generate an external excitation, which results in the forced-vibration. When the Bernoulli-Euler beam theory is applied, the kinetic energy  $T$ , potential energy  $V$  of the structural system including the piezoceramic, and nonconservative work  $W_{nc}$  are obtained by

$$T = \frac{1}{2} \int_0^{l_1} m_a \left( \frac{\partial y}{\partial t} \right)^2 dx + \frac{1}{2} \int_{l_1}^{l_2} m_b \left( \frac{\partial y}{\partial t} \right)^2 dx + \frac{1}{2} \int_{l_2}^L m_a \left( \frac{\partial y}{\partial t} \right)^2 dx + \frac{1}{2} \int_{l_1}^{l_2} M_d \sum_{k=1}^m \delta(x - x_k) \left( \frac{\partial y}{\partial t} \right)^2 dx \quad (3)$$

$$V = \frac{1}{2} \int_0^{l_1} \frac{1}{E_a I_a} (E_a I_a \frac{\partial^2 y}{\partial x^2} + c V_1)^2 dx + \frac{1}{2} \int_{l_1}^{l_2} E_b I_b \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx + \frac{1}{2} \int_{l_2}^L \frac{1}{E_a I_a} (E_a I_a \frac{\partial^2 y}{\partial x^2} + c \cdot V_1)^2 dx \quad (4)$$

$$W_{nc} = \int_0^L (P(x, t) - U(x, t)) dx, \quad \text{where } U(x, t) = \sum_{k=1}^m \delta(x - x_k) F_{ER} \quad (5)$$

Here  $m_a$  is the mass density per unit length at the section A-A, and  $m_b$  is the one at the section B-B. The  $E_a I_a$  and  $E_b I_b$  are the effective bending stiffness at the

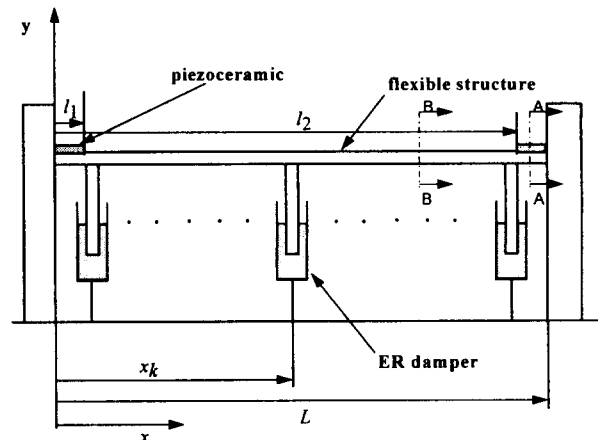


Fig. 3 A schematic diagram of the structural system with ER dampers



flexible structure has the section B-B. It is also noted that the output matrix  $C$  is related to  $m$  number of displacement sensors which are collocated with the ER dampers. On the other hand, the external force  $f_i(t)$  is given by

$$f_i(t) = -\frac{cV_1}{J} \left[ \int_0^h \phi_i^{(1)}(x) dx + \int_0^l \phi_i^{(3)}(x) dx \right] \quad (11)$$

The possible variations of the model parameters such as natural frequency and damping ratio can be occurred in practice by attaching a certain mass to the structure<sup>(9)</sup>. These variations can be expressed as follows:

$$\begin{aligned} \omega_i &= \omega_{0,i} + \delta\omega_i, \quad |\delta\omega_i| \leq \alpha_i \omega_{0,i} \\ \zeta_i &= \zeta_{0,i} + \delta\zeta_i, \quad |\delta\zeta_i| \leq \beta_i \zeta_{0,i} \end{aligned} \quad (12)$$

Here  $\omega_{0,i}$  and  $\zeta_{0,i}$  are the nominal natural frequency and the damping ratio of the  $i$ th mode, respectively. The  $\delta\omega_i$  and  $\delta\zeta_i$  are corresponding possible deviations. It is noted that the variations of the  $\delta\omega_i$  and  $\delta\zeta_i$  are bounded by the weighting factors  $\alpha_i$  and  $\beta_i$ , respectively. Substituting Eq.(12) into the system matrix  $A$  in Eq.(10) yields the following dynamic model, which divides the system matrix into the nominal part  $A_0$  and uncertain part  $\Delta A$  :

$$\dot{x}(t) = (A_0 + \Delta A)x(t) + Bu(t) + f(t), \quad y(t) = Cx(t) \quad (13)$$

Among numerous control strategies, we adopt a sliding mode control technique which has inherent robustness to the system uncertainties such as parameter variations. We first introduce the state errors in order to drive imposed vibration to zero for any arbitrary initial conditions as follows:

$$\begin{aligned} e_k &= y_k - y_{dk} = \sum_{i=1}^n \phi_i^{(2)}(l_k) x_i(t) - y_{dk} \\ \dot{e}_k &= \dot{y}_k - \dot{y}_{dk} = \sum_{i=1}^n \phi_i^{(2)}(l_k) \dot{x}_i(t) - \dot{y}_{dk} \end{aligned} \quad (14)$$

Here,  $y_k$  and  $y_{dk}$  represent actual and desired displacements at the sensor location of  $l_k$ , respectively. The problem now is to

design a sliding surface which guarantees stable sliding mode motion on the surface itself. Since we can treat the  $k$ th ER damper as a single input, we construct the  $k$ th sliding surface as follows:

$$s_k = g_k e_k + \dot{e}_k = g_k \left[ \sum_{i=1}^n \phi_i^{(2)}(l_k) x_i(t) - y_{dk} \right] + \left[ \sum_{i=1}^n \phi_i^{(2)}(l_k) \dot{x}_i(t) - \dot{y}_{dk} \right], \quad g_k > 0 \quad (15)$$

Then the following sliding condition is introduced to guarantee that the state variables of the system during the sliding mode motion are constrained to the sliding surface:

$$s_k \dot{s}_k < 0 \quad (16)$$

Now, we propose the following sliding mode controller which satisfies the sliding condition(16).

$$\begin{aligned} F_{ERk} = & -\frac{1}{P} \left\{ g_k \left[ \sum_{i=1}^n \phi_i^{(2)}(l_k) \dot{x}_i(t) - \dot{y}_{dk} \right] \right. \\ & + \left[ \sum_{i=1}^n \phi_i^{(2)}(l_k) (r_{2i-1} x_i(t) + r_{2i} \dot{x}_i(t)) - \ddot{y}_{dk} \right] \\ & \left. + [k_k + \sum_{i=1}^n |\phi_i^{(2)}(l_k)| (|-z_{2i-1} x_i(t)| + |-z_{2i} \dot{x}_i(t)|)] \text{sgn}(s_k) \right\} \quad (17) \end{aligned}$$

where,

$$\begin{aligned} P &= \sum_{i=1}^n \frac{\phi_i^{(2)}(l_k)}{I_i} \\ r_{2i-1} &= -\omega_{0,i}^2, \quad r_{2i} = -2\zeta_{0,i} \omega_{0,i} \\ z_{2i-1} &= -(2\omega_{0,i} \alpha_i \omega_{0,i} + \alpha_i^2 \omega_{0,i}^2) \\ z_{2i} &= -2(\beta_i \zeta_{0,i} \omega_{0,i} + \alpha_i \omega_{0,i} \zeta_{0,i} + \alpha_i \beta_i \omega_{0,i} \zeta_{0,i}) \\ k_k &> \sum_{i=1}^n \phi_i^{(2)}(l_k) |f_i(t)| \end{aligned} \quad (18)$$

We know that, in practice, it is not desirable to use the discontinuous control law (17), due to the chattering. Therefore we approximate the discontinuous control law by a continuous one inside the boundary layer width ( $\varepsilon_K$ ). To do this, we may replace ( $s_k$ ) in Eq.(17) by a saturation function,  $\text{sat}(s_k)$ , defined as

$$\text{sat}(s_k) = \begin{cases} \frac{s_k}{\varepsilon_k}, & |s_k| \leq \varepsilon_k \\ \text{sgn}(s_k), & |s_k| > \varepsilon_k \end{cases} \quad (19)$$

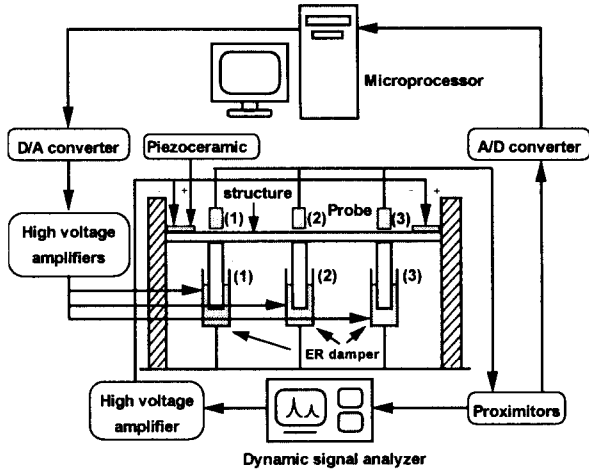


Fig. 4 A schematic diagram of experimental apparatus

In addition, we know that the proposed ER damper is semi-active. Therefore, the sliding mode controller (17), which is designed in an active manner, should be activated with the following condition.

$$F_{ERk} = \begin{cases} F_{ERk} \cdot F_{ERk} \cdot \left[ \sum_{i=1}^n \phi_i^{(2)}(l_k) \dot{x}_i(t) \right] > 0 \\ 0, & F_{ERk} \cdot \left[ \sum_{i=1}^n \phi_i^{(2)}(l_k) \dot{x}_i(t) \right] \leq 0 \end{cases} \quad (20)$$

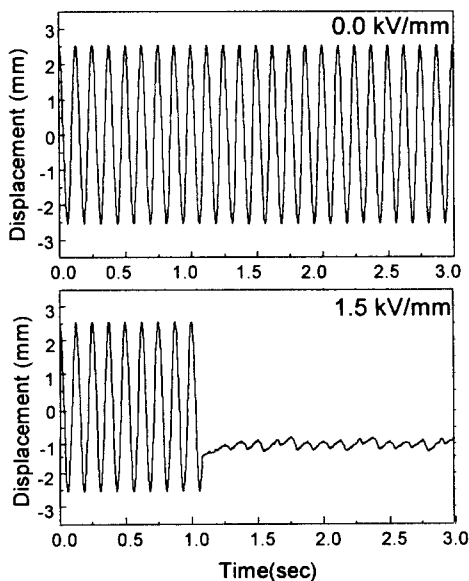
Above conditions indicate that if the sign of the control signal and velocity motion at

the damper position is same, the controller is to be activated. Otherwise, control input is set to zero. This physically implies that the control scheme in the semiactive mode only assures the increment of energy dissipation<sup>(10)</sup>. Now, the control electric field for the  $k$ th ER damper is determined from  $F_{ERk}$  in Eq. (8) :

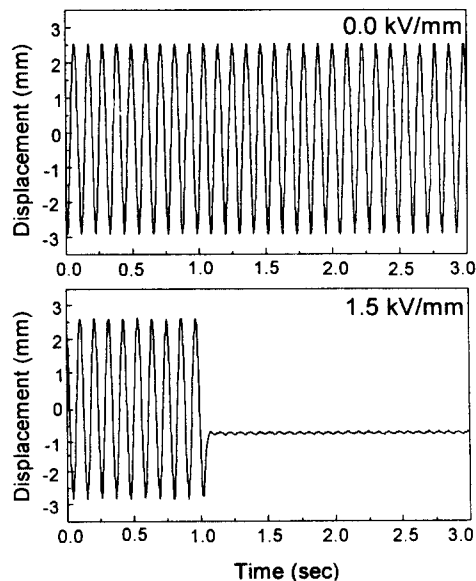
$$E = \left[ \frac{1}{\alpha} \left\{ \frac{F_{ERk} \cdot \text{sgn} \left( \sum_{i=1}^n \phi_i^{(2)}(l_k) \dot{x}_i(t) \right)}{4\pi r L_d} - \left( \sum_{i=1}^n \phi_i^{(2)}(l_k) \dot{x}_i(t) \eta / h \right) \right\} \right]^{\frac{1}{\beta}} \quad (21)$$

## 5. Results and Discussions

A schematic diagram of the experimental apparatus and associated instrumentation is presented in Fig. 4. Three ER dampers are attached to the structure, and three noncontacting displacement sensors (proximitors) are collocated to the ER dampers. Each ER damper is installed at the location of  $L/4$ ,  $L/2$  and  $3L/4$ , respectively. The beam structure is made of stainless steel, and its thickness, width and length are 0.5 mm, 40



(a) Simulated



(b) Measured

Fig. 5 Control responses with constant electric fields

mm and 600 mm, respectively. When the structure vibrates due to the excitation from the piezoceramic, the displacements are sensed and feed back to the microprocessor through the analog/digital converter. Depending upon the information of the displacement and its derivative, control input field is determined by means of the proposed sliding mode controller. The control field from the microprocessor is applied to the ER fluid domain of the damper after being amplified via a high-voltage amplifier, which has a gain of 1000. In the controller implementation, the maximum electric field is limited by 2.5 kV/mm due to the breakdown of electrodes. The sampling frequency to

implement the controller is chosen by 1000 Hz. The control parameters are employed as follows :  $g_k = 2.0$ ,  $k_k = 55.0$  and  $\epsilon_k = 0.4$  for all dampers. In this work, two dominant flexible modes(first and second) are considered as primary modes to be controlled.

As a first step for control observation, we apply constant electric fields to the ER dampers. Fig. 5. presents the first-mode excited forced-vibration responses with constant field of 0 kV/mm and 1.5 kV/mm, respectively. The response is measured from the sensor 2(middle). It is clearly observed that imposed vibration is quickly and effectively suppressed upon employing the electric field, but the suppressed displacement

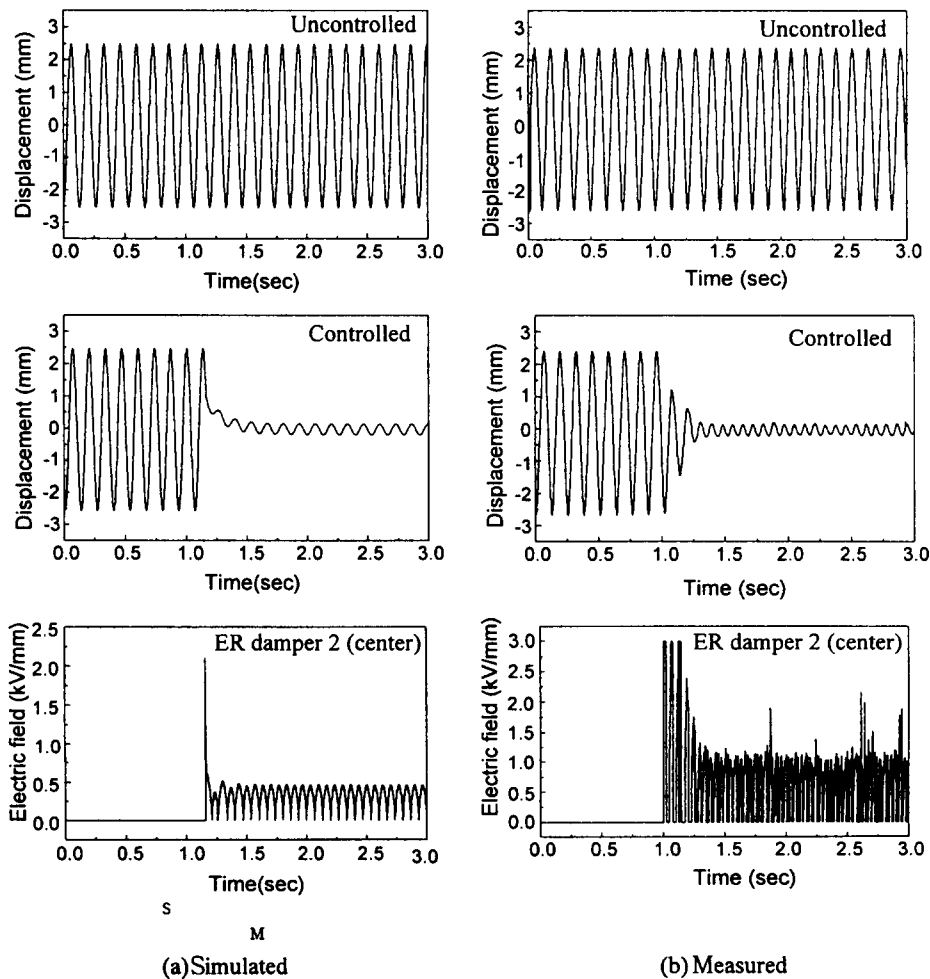


Fig. 6 Control responses for the first-mode excitation



does not go to zero. This phenomenon is arisen from Bingham behavior of the ER fluid in the presence of the electric field. In other words, the ER damper produces a Coulomb-like damping forces by applying constant electric field<sup>(6)</sup>. The proposed ER damper is a semi-active type, thus the control field should be employed by considering the direction of vibratory motion of the actuator.

Figure 6 presents control responses with the sliding mode controller for the first-mode excitation. The response is measured from the sensor 2(middle). To demonstrate the robustness of the control system, a certain mass(6.6 g) is added to the structure. This

causes the first mode natural frequency to be changed from 8 Hz to 7.5 Hz, and the damping ratio from 0.048 to 0.040. The imposed forced-vibration is quickly suppressed to almost zero upon activating the controller. As expected, the control electric field is employed according to the semi-active manner characterizing the condition given by Eq. (20). It is also noted that there exists a favorable agreement between the simulation and experimental results. This advocates the validity of the proposed model as well as control logic. The vibration controllability for the second-mode excitation(17 Hz) is measured from the sensor 1(side) and shown in Fig. 7. Since the response of the

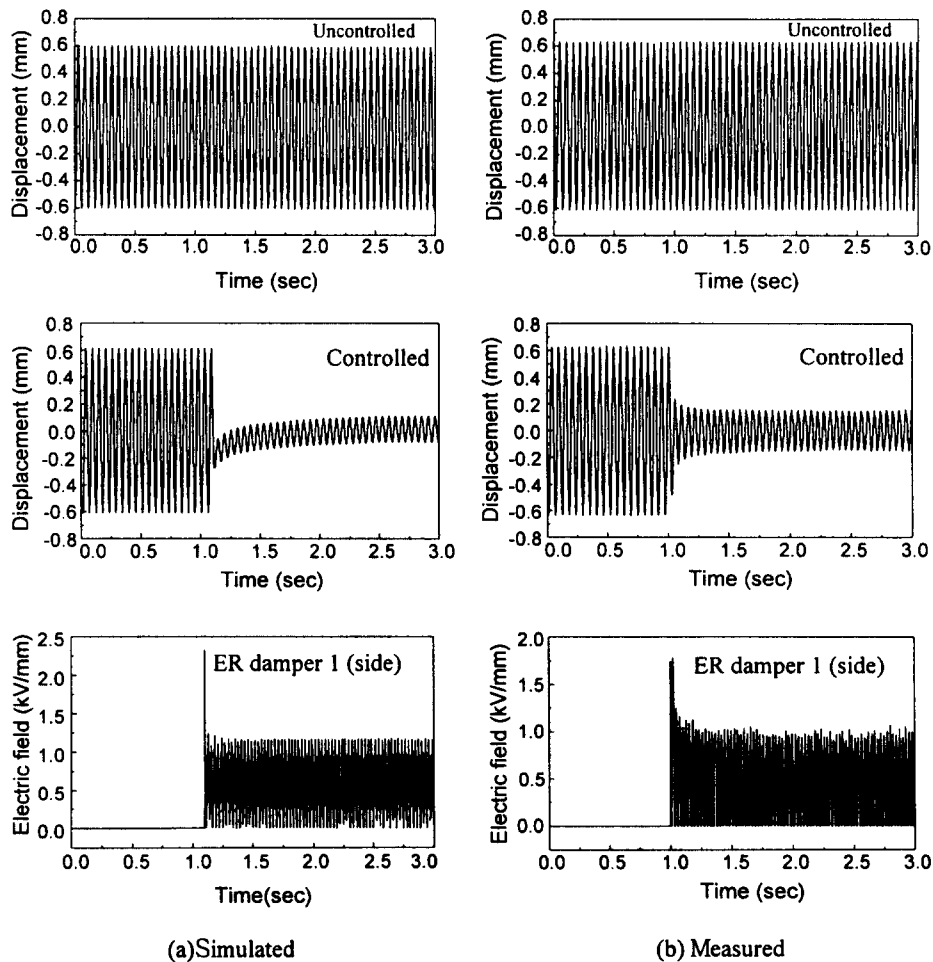


Fig. 7 Control responses for the second-mode excitation

ER damper is very fast, the excited vibration is quickly and favorably suppressed by applying the control electric field to the side ER dampers 1 and 3. The ER damper 2(center), of course, is not activated in this case since the damper is located at the nodal point of the second-mode shape. In addition, in order to demonstrate a practical feasibility of the proposed ER damper system a random vibration is considered. Fig. 8 presents the uncontrolled and controlled responses for random excitation. It is clearly observed that the vibration level is

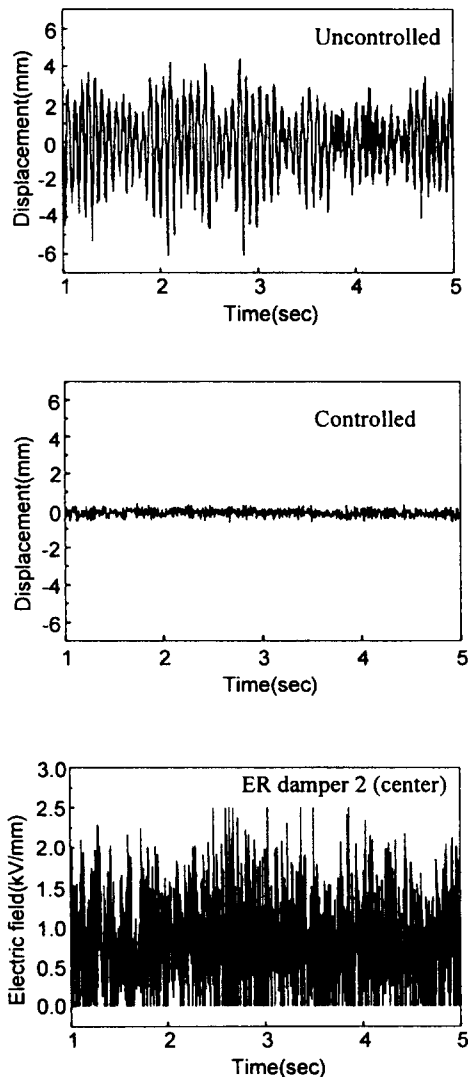


Fig. 8 Control response for random excitation

substantially reduced by applying the control electric field to the ER dampers. The results presented in this study are quite self-explanatory justifying that the proposed ER damper system is very effective for vibration control of flexible structures subjected to system uncertainties and random vibration.

## 6. Conclusions

Vibration control of a flexible beam structure was undertaken in a closed-loop control fashion with adaptable ER dampers. Following the derivation of governing equations and associated boundary conditions, a sliding mode controller was formulated on the basis of sliding mode condition. The controller was successfully implemented showing effective controllability of the forced-vibrations. In addition, the control robustness to the parameter variations such as natural frequency deviation has been demonstrated. It is finally remarked that comparative works with other types of ER dampers such as squeeze mode or/and with other types of control schemes need to be done in the near future.

## References

- (1) Coulter, J. P., and Duclos, T. G., 1989, Applications of Electrorheological Materials in Vibration Control, *Proceedings of the 2nd International Conference on ER Fluids*, pp. 300~325.
- (2) Choi, Y., Sprecher, A. F., and Conrad, H., 1992, Response of Electro-Rheological Fluid-Filled Laminate Composites to Forced-Vibration, *Journal of Intelligent Material Systems and Structures*, Vol. 3, No. 1, pp. 17~29.
- (3) Choi, S. B., Park, Y. K., and Cheong, C. C., 1996, Active Vibration Control of Intelligent Composite Laminated Structures Incorporating an Electro-Rheological Fluid.

- Journal of Intelligent Material Systems and Structures*, Vol. 7, No. 4, pp. 411~419.
- (4) Petek, N. K., Romstadt, D. J., Lizell, M. B., and Weyenberg, T. R., 1995, Demonstration of an Automotive Semi-Active Suspension Using Electro-Rheological Fluid, *SAE Paper* No. 950586.
- (5) Austin, S. A., 1993, The Vibration Damping Effect of an Electrorheological Fluid, *ASME J. of Vibration and Acoustics*, Vol. 115, No. 1, pp. 136~140.
- (6) Wang, K. W., Kim, Y. S., and Shea, D. B., 1994, Structural Vibration Control via Electrorheological-Fluid-Based Actuators with Adaptive Viscous and Frictional Damping, *Journal of Sound and Vibration*, Vol. 177, No. 2, pp. 227~237.
- (7) Bonnecaze, R. T., and Brady, J. F., 1992, Yield Stresses in Electrorheological Fluids, *Journal of Rheology*, Vol. 36, No. 1, pp. 73~113.
- (8) Ginder, J. M., and Ceccio, S. L., 1995, The Effect of Electrical Transients on the Shear Stresses in Electrorheological Fluids, *Journal of Rheology*, Vol. 39, No. 1, pp. 211~234.
- (9) Choi, S. B., and Lee, C. H., 1997, Force Tracking Control of a Flexible Gripper Driven by Piezoceramic Actuators, *ASME J. of Dynamic Systems, Measurement, and Control*, Vol. 119, No. 3, pp. 439~446.
- (10) Leitmann, G., 1994, Semiactive Control for Vibration Attenuation, *Journal of Intelligent Material Systems and Structures*, Vol. 5, No. 5, pp. 841~846.