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Application of Dynamic Programming to Optimization of a System Reliability

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Abstract

This paper is concerned with the optimization of a system reliability. Two kinds of the reliability model for optimal allocation of parallel redundancy are considered. The algorithm for solving the optimal redundancy problem is proposed by the use of dynamic programming(DP) method. The problem is approached with a standard DP formulation and the DP algorithm is applied to the model and then the optimal solution is found by the backtracking method. The method is applicable to the models having no constraints or having a cost constraint subject to a specified minimum requirement of the system reliability. A consequence of this study is that the developed computer program packages are implemental for the optimization of the system reliability.

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1. Introduction

In this paper, primarily we are concerned with an optimization of a system reliability. Specifically, we consider a system consisting of N-*stages*. Every stage i consists of X_i units of type i in parallel, so that stage i functions if and only if at least one of X_i units of type i functions, where $i = 1, \cdots, N$.

Everett[1] has considered the problem of determining X_i's that yield the maximum reliability subject to a cost constraint. Kettelle[2] presents an algorithm for allocating X_i's so as to maximize the system reliability without exceeding an allowable cost.

In most reliability problems two basic methods of improving the reliability of a system are 1)increase the reliability of each component and 2)add redundant components(in parallel, stand-by, etc.)[3]. This paper considers only the latter case. We will consider a problem in which there are no constraints such that the problem is to find the redundancy to maximize the whole system reliability at each stage i of a mixed system with N-stages in series where units are in parallel at each stage. And assume that a minimum requirement of a system reliability, Rs, min should be obtained. Then the optimal design(allocation) problem to maximize the system reliability will be considered at a minimum cost where the maximum system reliability, Rs is greater than or equal to Rs, min i. e., Rs $\geq$ Rs, min in this problem.

In this study, the dynamic programming method is used to solve those problems. The N-variable decision problems are transformed into N single-variable decision problems. Then the problem is restructured as N-stage sequential decision problems. At each stage, the single-variable decision problem will be solved sequentially.

In section 2, a system reliability problem is formulated as an N-stage mixed system without constraints and another problem is formulated so as to maximize the system reliability with a cost constraint subject to a given reliability requirement. The N-stage reliability model will be converted to the N-stage dynamic programming model and then dynamic programming formulation of the model will be derived in section 3. The computational algorithm by dynamic programming method will be developed and then applied to the typical numerical examples in section 4. The computational results will be discussed and the optimal solution will be obtained by the backtracking method in section 5. Finally the conclusions will be given in the final section.

2. Formulation of the System Reliability Models

In the N-stage mixed system, at least one unit of type i must be used at each stage. The whole system reliability of N-stage mixed system can be improved by adding the parallel redundancies at each stage.

Two kinds of the N-stage mixed system will be considered in this section. Consider the N-stage mixed system shown in Figure 1 having (X_i-1) parallel redundancies at each stage i[4].

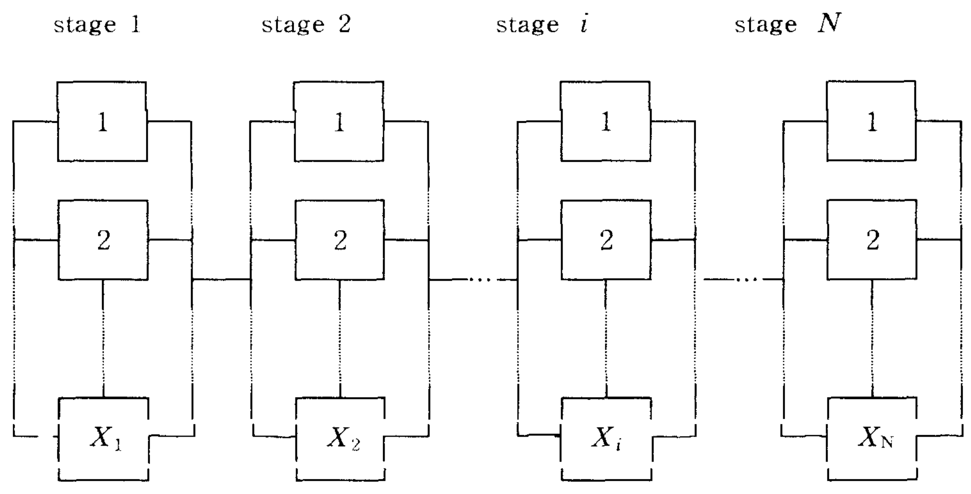

Figure 1. A mixed system with N-stages in series where units are in parallel at each stage.

If X_i units with reliability p_i are used at stage i, the reliability of stage i is represented as :

$$R_i = 1 - q_i^{X_i}$$

$$\text{where } q_i = 1 - p_i , \quad i = 1, \cdots , N.$$

The reliability of the N-stage mixed system is given as :

$$R_S = \prod_{i=1}^{N} R_i \quad .$$

We assume that P is the profit obtained when the mixed system operates successfully. The system reliability, R_S is the fraction of the trials that are successful and hence the expected profit for the system is $P\,R_S$. Suppose that the costs C_i of the redundant units of the i-th stage include the construction costs and the operating costs. Then the total cost, T_c of the system with redundancies is given as follows :

$$T_c = \sum_{i=1}^{N} C_i X_i \ .$$

The net profit N_p for the whole system is the profit less the total cost, i. e.,

$$N_p = P\,R_S - T_c = P\,R_S - \sum_{i=1}^{N} C_i X_i \ .$$

Hence the problem of the N-stage mixed system with no constraints can be formulated as follows :

$$\textit{Maximize} \ N_p = P \prod_{i=1}^{N} R_i - \sum_{i=1}^{N} C_i X_i \ .$$

In this N-stage mixed system, no constraints are imposed on the problem where let us call this kind of problem as type 1 problem.

Suppose that the minimum requirement of a system reliability, $R_{S,\min}$ is given, and it should be determined that the minimum cost allocation of an N-stage mixed system satisfies that the system reliability, R_S is greater than or equal to $R_{S,\min}$, i.e., $R_S \geq R_{S,\min}$. In general, the problem can be formulated as follows[5]:

$$Maximize \quad R_S = \prod_{i=1}^{N} [1 - q_i^{X_i}] ,$$

$$\text{subject to } g_1 = \sum_{i=1}^{N} C_i X_i \leq b$$

$$\text{and } R_{S,\min} \leq R_S \leq 1$$

where $q_i = 1 - p_i$ and let us call this kind of problem as type 2 problem.

3. Dynamic Programming Formulation of the Model

The dynamic programming(DP) technique is applicable to multi-stage(or sequential) decision problems. This technique converts such a problem to a series of single-stage optimization problems.

Let D_i be the set of all possible decision alternatives available at stage i, its elements are denoted by $d_i \in D_i$, and let S_i be the set of all possible states at stage i, its elements are denoted by $s_i \in S_i$, i = 1, $\cdots$, N. Let $T_i(s_i, d_i)$ be a ransformation function from s_i to s_{i-1} and $r_i(s_i, d_i)$ be a return function at stage i. Then the general multi-stage decision problems may be converted to a series of single-stage decision problems as being depicted in Figure 2[6].

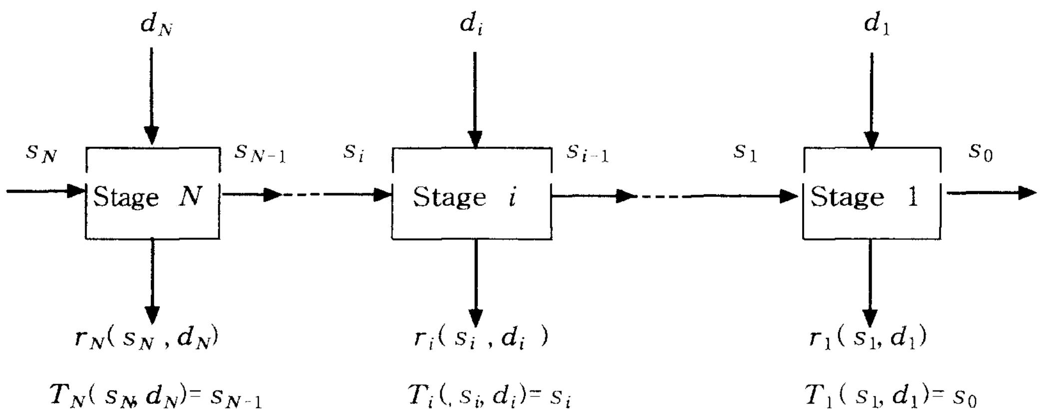

Figure 2. N-stage Dynamic Programming Representation.

Consider a system reliability model as type 1 problem in general. The dynamic programming formulation of the model is derived as a multi-stage decision problems mentioned previously in the followings. For the first stage, the optimal design is determined for the single decision variable, X_1 , by the recursive equation as :

$$f_1(V_2) = \max_{X_1}[\, r_1(s_1,\ d_1)] = \max_{X_1} (\, P\, V_1 - C_1 X_1) \quad \cdots\cdots\cdots\cdots (1)$$

where $V_2 = \prod_{i=N+1}^{2} Rs,i$ is the probability that all upstream stages work, r_1 is a return function at stage 1, $V_1 = V_2 R_{s,1} = V_2[1 - q_1^{X_1}]$ and Rs,i is the reliability of the i-th stage with X_i parallel units, i. e., $Rs,i = 1 - q_i^{X_i}$ and $R_{s,N+1} = V_{N+1} = 1$. We continue this process until (i-1)th stage. Then for the i-th stage, the recursive functional equation is represented as :

$$f_i(V_{i+1}) = \max_{X_i}[\, r_i(s_i\ , d_i)] = \max_{X_i} [\, f_{i-1}(V_i) - C_i X_i]. \quad \cdots\cdots\cdots (2)$$

If the optimal design for the subsystem including the stages, N-1, N-2, ⋯ , and 1 is known, the stage N can be designed by the recursive equation as :

$$f_N(V_{N+1}) = \max_{X_N}[\, r_N(s_N\ , d_N)] = \max_{X_N} [\, f_{N-1}(V_N) - C_N X_N\ ]. \quad \cdots\cdots (3)$$

Similarly, consider another system reliability model as type 2 problem in general. The dynamic programming formulation of the model also can be derived in the followings. The recursive equations of the model are formulated as[7] :

$$f_1(b) = \max_{X_1^l < X_1 < X_1^u} [1 - q_1^{X_1}], \quad \cdots\cdots\cdots\cdots\cdots (4)$$

$$f_2(b) = \max_{X_2^l < X_2 < X_2^u} [\ (1 - q_2^{X_2})\ f_1(b - g_{12}(X_2))], \quad \cdots\cdots (5)$$

$\cdots\cdots$

$$f_N(b) = \max_{X_N^l < X_N < X_N^u} [\ (1 - q_N^{X_N})\ f_{N-1}(b - g_{1N}(X_N))] \quad \cdots\cdots (6)$$

where X_j^l , j=1,2, ... ,N is the minimum integer number used at each stage. Usually X_j^l = 1, j = 1, ... ,N and X_j^u , j = 1, ... ,N is the maximum integer number used at each stage such that

$$\sum_{\substack{p=1 \\ p \neq j}}^{N} g_{1p}(X_P^l) + g_{1j}(X_j) < b.$$

4. Application of DP Algorithm to Numerical Examples

Consider two numerical examples with five and four stages where the whole system profit p associated with the final product, the costs C_i , the reliabilities R_i of each of the units, b the maximum allowable cost and Rs, min the minimum requirement of a system reliability are presented in Table 1.

Table 1. Two Numerical Examples

Type 1 Problem			Type 2 Problem		
Stage	Cost	Reliability	Stage	Cost	Reliability
1	1.0	0.75	1	5.4	0.85
2	0.55	0.425	2	4.3	0.75
3	0.80	0.65	3	3.2	0.70
4	0.75	0.50	4	2.1	0.80
5	0.45	0.333			
P = 20.0	*No*	*Constraints*	b = 73		Rs,min = 0.99

We illustrate the application of the computational algorithm based on the DP method to the two numerical examples in this section. Consider the type 1 problem in Table 1. Substituting the costs, C_i, and the reliabilities, R_i, of each of the units into the equations (1) through (3), then the recursive dynamic programming formulations of the problem can be represented as follows :

$$f_1(V_2) = \max_{X_1} [\ 20.0 \cdot V_1 - 1.0\,X_1]\quad \text{where}\quad V_1 = V_2\ [\ 1. - (1-0.75)^{X_1}\ ],\ \cdots (7)$$

$$f_2(V_3) = \max_{X_2} [\ f_1(V_2) - 0.55\,X_2]\quad \text{where}\quad V_2 = V_3\ [\ 1. - (1-0.425)^{X_2}\ ],\ \cdots (8)$$

$$f_3(V_4) = \max_{X_3} [\ f_2(V_3) - 0.80\,X_3]\quad \text{where}\quad V_3 = V_4\ [\ 1. - (1-0.65)^{X_3}\ ],\ \cdots (9)$$

$$f_4(V_5) = \max_{X_4} [\ f_3(V_4) - 0.75\,X_4]\quad \text{where}\quad V_4 = V_5\ [\ 1. - (1-0.50)^{X_4}\ ]\ \cdots (10)$$

and $$f_5(V_6) = \max_{X_5} [f_4(V_5) - 0.45\,X_5]\quad \cdots\cdots\cdots\cdots (11)$$

where $V_5 = V_6\ [\ 1. - (1-0.333)^{X_5}]$ and $V_6 = 1.0$.

The computational algorithm by DP method will be applied to this problem sequentially. At stage 1, the first maximum reliability solution will be found for the optimal X_1 for a spectrum of V_2 values. Since the reliability value, V_2 is a probability value between 0 and 1, V_2 may be allowed to take a value at each discrete point, say, 0.1, 0.2, ..., 1.0 in this study. For a particular V_2 value, V_1 in equation (7) will be calculated for each different X_1 parallel components.

After the right hand side of equation (7) for given different reliability value V_1 is computed, the optimal(maximum) return value, $f_1(V_2)$ will be searched by one-dimensional search technique. In the case of V_2 = 1 typically, the computed results are presented in Table 1a. The summary of the optimal $f_1(V_2)$ and the optimal parallel X_1 components for the specified different values, V_2 = 1.0, 0.9, ... , 0.1 are presented in Table 1b .

Similarly equation (8) is used to find the optimal value of $f_1(V_2) - C_2X_2$, for each value of V_3. In the process of calculations, the value of $f_1(V_2)$ can be obtained by interpolation method. For example, V_2 is given as 0.9371 from the equation (8) in the case of V_3 = 1.0 and X_2 = 5 . The value of $f_1(V_2)$ for given V_2 = 0.9371, is determined to be 15.5715 by interpolation of $f_1(0.9)$ = 14.8750 and $f_1(1.0)$ = 16.7500 that are obtained in the stage 1 optimization.

Table 1a. Results in the case of V_2=1 where MAX stands for the Maximum.

V_2	$PV_1 - C_1X_1$	X_1	V_1	$f_1(V_2)$
1.00	0.0000	0	0.0000	
1.00	14.0000	1	0.7500	
1.00	**16.7500**	**2**	**0.9375**	**MAX**
1.00	16.6875	3	0.9844	
1.00	15.9219	4	0.9961	
1.00	14.9805	5	0.9990	
1.00	13.9951	6	0.9998	
1.00	12.9988	7	0.9999	
1.00	11.9997	8	1.0000	
1.00	10.9999	9	1.0000	

Table 1b. Summary Table of the Optimal Results at Stage 1.

V_2	$f_1(V_2)$	X_1	V_1
1.00	16.7500	2	0.9375
0.90	14.8750	2	0.8438
0.80	13.0000	2	0.7500
0.70	11.1250	2	0.6563
0.60	9.2500	2	0.5625
0.50	7.3750	2	0.4688
0.40	5.5000	2	0.3750
0.30	3.6250	2	0.2812
0.20	2.0000	1	0.1500
0.10	0.5000	1	0.0750

Similarly, the summary table of the optimal results at stage 2 is presented in Table 2.

Table 2. Summary Table of the Optimal Results at Stage 2.

V_3	$f_2(V_3)$	X_2	V_2	$f_1(V_2)$
1.00	12.8215	5	0.9371	15.5715
0.90	11.0643	5	0.8434	13.8143
0.80	9.3072	5	0.7497	12.0572
0.70	7.5500	5	0.6560	10.3000
0.60	5.8202	4	0.5344	8.0202
0.50	4.1502	4	0.4453	6.3502
0.40	2.4802	4	0.3563	4.6802
0.30	1.0482	3	0.2430	2.6982
0.20	0.3625	1	0.0850	0.9125
0.10	0.3625	1	0.0425	0.9125

And also similarly equation (9) and (10) can be employed to find the optimal value $f_3(V_4)$ at stage 3 and $f_4(V_5)$ at stage 4 respectively. The summary tables of the optimal results at stage 3 and stage 4 are also presented in Table 3 and Table 4 respectively.

Table 3. Summary Table of the Optimal Results at Stage 3.

V_4	$f_3(V_4)$	X_3	V_3	$f_2(V_3)$
1.00	9.6681	3	0.9571	12.0681
0.90	7.9863	3	0.8614	10.3863
0.80	6.3045	3	0.7657	8.7045
0.70	4.6309	3	0.6700	7.0309
0.60	2.9928	2	0.5265	4.5928
0.50	1.5273	2	0.4387	3.1273
0.40	0.1785	2	0.3510	1.7785
0.30	0.0000	0	0.0000	0.0000
0.20	0.0000.	0	0.0000	0.0000
0.10	0.0000	0	0.0000	0.0000

Table 4. Summary Table of the Optimal Results at Stage 4.

V_5	$f_4(V_5)$	X_4	V_4	$f_3(V_4)$
1.00	5.6170	4	0.9375	8.6170
0.90	4.0403	4	0.8438	7.0403
0.80	2.4677	4	0.7500	5.4677
0.70	0.9475	3	0.6125	3.1975
0.60	0.0000	0	0.0000	0.0000
0.50	0.0000	0	0.0000	0.0000
0.40	0.0000	0	0.0000	0.0000
0.30	0.0000	0	0.0000	0.0000
0.20	0.0000	0	0.0000	0.0000
0.10	0.0000	0	0.0000	0.0000

Finally, equation (11) is used to search for X_5 which maximizes $f_4(V_5) - C_5X_5$ for only $V_6 = 1.0$, since V_{N+1} is always 1. The computational results at stage 5 are presented in Table 5.

Table 5. Summary of the Optimal Results at Stage 5 where MAX stands for the Maximum.

V_6	X_5	V_5	$f_4(V_5)$	C_5X_5	$f_4(V_5) - C_5X_5$	$f_5(V_6)$
1.00	0	0.0000	0.0000	0.0000	0.0000	
1.00	1	0.3330	0.0000	0.4500	-.4500	
1.00	2	0.5551	0.0000	0.9000	-.9000	
1.00	3	0.7033	0.9971	1.3500	-.3529	
1.00	4	0.8021	2.5003	1.8000	0.7003	
1.00	5	0.8680	3.5368	2.2500	1.2868	
1.00	6	0.9119	4.2286	2.7000	1.5286	
1.00	**7**	**0.9413**	**4.6909**	**3.1500**	**1.5409**	**MAX**
1.00	8	0.9608	4.9993	3.6000	1.3993	
1.00	9	0.9739	5.2050	4.0500	1.1550	

The computer program for type 1 problem has been developed in FORTRAN language. This computer program package can be used to find the optimal system reliability and the optimal system design(allocation) for this kind of the problem in general. The flow diagram of the program is shown in Figure 1 and the detailed computer program package can be accessed from the author.

Consider the type 2 problem in Table 1. To attain the minimum system reliability, Rs,min, let $R_i(X_i) = 1-(1-R_i)^{X_i}$, then at stage 1, $R_1(3)=1-(1-0.85)^3=0.9966$ which is greater than 0.99. Hence, the minimum number of components for stage 1 is $X_1^l=3$. Similarly, the minimum number of components for stage 2, 3 and 4, are determined in the same manner i. e., $(X_1^l,X_2^l,X_3^l,X_4^l)=(3,4,4,3)$ respectively.

For the first stage, the optimal design should be determined by the function as :

$f_1(b) = \max_{X_1^l \leq X_1 \leq X_1^u} [R_1(X_1)]$ where $X_1^l = 3$ and X_1^u is bounded by the cost constraint.

For a spectrum of b values, the optimization problem can be solved for the optimal X_1. Thus, for each value of b between 52.5, which is the minimum cost to attain Rs, min and 73.0, which is the maximum allowable cost, the optimal design will be found when all the upstream stage allocations are fixed by $(X_4^l,X_3^l,X_2^l)=(3,4,4)$. The optimal allocation for X_1 is shown in the following Table 6.

Table 6. The Optimal Allocation of for a spectrum of b values

b	X_4	X_3	X_2	X_1	R_s
52.50 - 57.90	3	4	4	3	0.97681
57,90 63.30	3	4	4	4	0.97963
63.30 - 68.70	3	4	4	5	0.98005
68.70 - 73.00	3	4	4	6	0.98011

Since the upstream fixed allocation, $(X_4^l,X_3^l,X_2^l)=(3,4,4)$ and it's cost is correspond to 36.3 in the case of a value b between 63.30 and 68.70, the rest of the cost can be used to allocate the optimal X_1 to be 5 and then the maximum system reliability, $f_1(b)=0.98005$ for 63.30 $\leq$ b $\leq$ 68.70. The optimal allocation and the

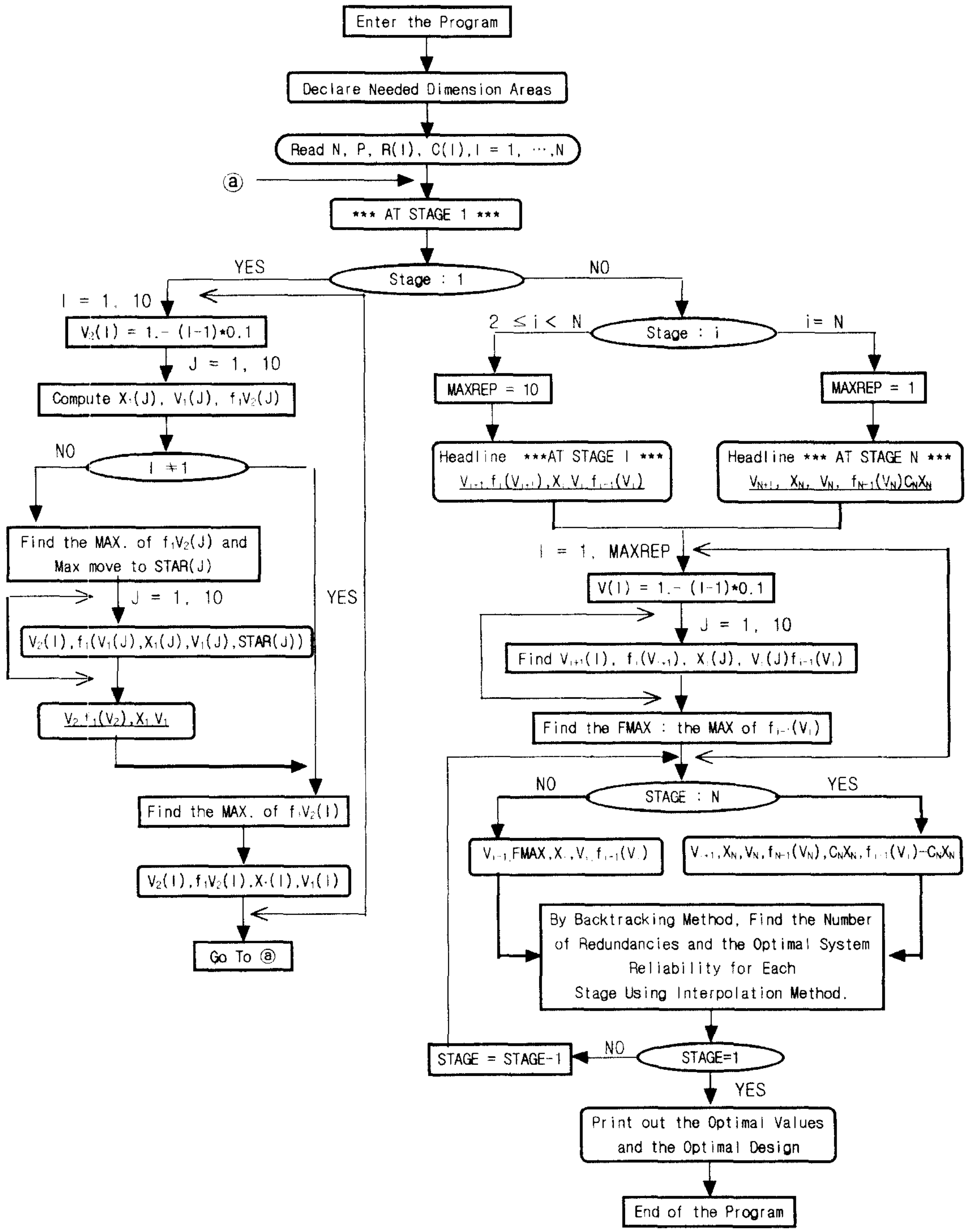

Figure 1. Flow Diagram for Computer Programming

maximum system reliability can be found for all possible b values under the minimum fixed allocations X_4^l, X_3^l and X_2^l.

The next step is to determine the optimal allocation of stage 2 and stage 1 where X_4^l and X_3^l are fixed to the upstream allocation of stage 4 and stage 3 respectively. The maximization will be carried out for b values from 53 to 73 in accordance with one unit discrete step. Similarly, the optimal allocation of stage 3, stage 2 and stage 1 are searched for the maximum system reliability where the only upstream allocation of stage 4, X_4^l is fixed. The summary tables of the optimal computational results for stage 2(and stage 1) and stage 3(and stage 2 and stage 1) are presented in Table 7 and Table 8 respectively.

The computer program for type 2 problem has been developed in FORTRAN language. The detailed program package can be accessed from the author.

Table 7. Computational Results for Stage 2

b	X_4^l	X_3^l	X_2	X_1	$f_2(b)$
53.0	3	4	4	3	0.97681
54.0	3	4	4	3	0.97681
55.0	3	4	4	3	0.97681
56.0	3	4	4	3	0.97681
57.0	3	4	5	3	0.97969
58.0	3	4	5	3	0.97969
59.0	3	4	5	3	0.97969
60.0	3	4	5	3	0.97969
61.0	3	4	5	3	0.97969
62.0	3	4	6	3	0.98040
63.0	3	4	5	4	0.98251
64.0	3	4	5	4	0.98251
65.0	3	4	5	4	0.98251
66.0	3	4	5	4	0.98251
67.0	3	4	6	4	0.98323
68.0	3	4	6	4	0.98323
69.0	3	4	6	4	0.98323
70.0	3	4	6	4	0.98323
71.0	3	4	7	4	0.98341
72.0	3	4	6	5	0.98365
73.0	3	4	6	5	0.98365

Table 8. Computational Results for Stage 3

b	X_4^l	X_3	X_2	X_1	$f_2(b)$
53.0	3	4	4	3	0.97681
54.0	3	4	4	3	0.97681
55.0	3	4	4	3	0.97681
56.0	3	5	4	3	0.98240
57.0	3	5	4	3	0.98240
58.0	3	5	4	3	0.98240
59.0	3	6	4	3	0.98407
60.0	3	5	5	3	0.98529
61.0	3	5	5	3	0.98529
62.0	3	5	5	3	0.98529
63.0	3	5	5	3	0.98629
64.0	3	6	5	3	0.98697
65.0	3	6	5	3	0.98697
66.0	3	5	5	4	0.98812
67.0	3	5	5	4	0.98812
68.0	3	5	5	4	0.98812
69.0	3	6	5	4	0.98981
70.0	3	6	5	4	0.98981
71.0	3	6	5	4	0.98981
72.0	3	7	5	4	0.99031
73.0	3	6	6	4	0.99053

5. Discussion of Computational Results

For the fifth stage process of the type 1 problem, the optimal system profit, $f_5(V_6)$, is 1.5409 units with the corresponding parallel components, $X_5=7$ and the optimal system reliability, $V_5=0.9413$ from Table 5.

By backtracking method, entering the fourth stage with $V_5=0.9413$, the optimal design of X_4 is 4 and V_4 is calculated from Table 4 by interpolation method and V_4 is equal to 0.8824. For the third stage process with V_4=0.8824, $X_3=3$ and then $V_3=0.8446$ from Table3. Entering the second stage process with V_3=0.8446, $X_2=5$ and $V_2=0.7915$ from Table 2. For the first stage process with V_2, the optimal design of X_1 is 2 and $V_1=0.7420$ at stage 1 from Table 1 by interpolation method.

Thus, the optimal allocation consists of 6 parallel redundancies for stage 5, 3 redundancies for stage 4, 2 redundancies for stage 3, 4 redundancies for stage 2 and 1 redundancy for stage 1. Therefore, this optimal design provide the optimal system reliability of 0.7420 at the optimal system profit of 1.5409 units. The optimal system values and the optimal design can be summarized in the following Table 9.

Table 9. The Optimal System Values and the Optimal Design

V_6	$f_5(V_6)$	X_5	V_5	$f_4(V_5)$
1.0	1.5409	7	0.9413	4.0909

Stage Number	5	4	3	2	1
Number of parallel Components(Redundancies)	7(6)	4(3)	3(2)	5(4)	2(1)
The Optimal System Reliability	0.9413	0.8824	0.8446	0.7915	0.7420

If the system has no parallel redundancies, then it has the system profit as

$$N_p = p\prod_{i=1}^{5} R_i - \sum_{i=1}^{5} C_i X_i$$

$= 20 \, [\, 0.75 * 0.425 * 0.65 * 0.50 * 0.333 \,]$

$- [\, 1.00 + 0.55 + 0.80 + 0.75 + 0.45 \,] = -0.86.$

The optimal system profit of the optimal system design is greater than the system profit of that with no parallel redundancies and their difference is 2.3009.

If the maximum allowable cost, b is 73 in the type 2 problem, then all possible allocations of stage 4 may be considered to cover the minimum system reliability, Rs, min. Finally, the optimal allocation to give the maximum system reliability will be seeked. The optimal allocation and the optimal system reliability of stage 4 is shown in the following Table 10. The computational results assert that the optimal allocation, (X_4, X_3, X_2, X_1) = (5,6,5,4) gives the maximum system reliability, R_S = 0.99747 that is greater than the minimum system reliability, R_S, min = 0.99 among all possible allocations.

Table 10. The Optimal Allocation and the Optimal System Reliability of Stage 4

b	X_4	X_3	X_2	X_1	$f_4(B)$
73	5	6	5	4	0.99747

6. Conclusions

We have shown in this paper that it is possible to find the optimal solution for N-stage mixed system reliability by the method of dynamic programming. Two kinds of the system reliability problems are considered such as one problem without constraints and another problem with a cost constraint subject to attaining the minimum requirement of a specified system reliability, Rs, min. After the problems are formulated, the models are approached with a standard dynamic programming formulation.

We have proposed two computational algorithm by the dynamic programming method. Also we have developed the computer program package using FORTRAN language according to the proposed algorithm and then the program is applied to two numerical examples to find the optimal solution.

The computational results show that the optimal system value and the optimal design(allocation) can be obtained by the backtracking method. The optimal design(allocation) also can be found to give the maximum system reliability that is greater than the minimum requirement level of the system reliability. The application of DP to two kinds of the models show that these versions are applicable to these types of model having 10 stages at most or fewer stages. But these also can be extended to the extent having more than 10 stages.

An important consequence of this study is that the developed algorithms are directly useful for finding the optimal allocation. The computational results are reported in which the performances of the algorithms work very well. Consequently, the computer program packages developed using dynamic programming are implemental for the optimization of the system reliability problems.

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