Robust Tracking Control for Uncertain Linear Systems using Linear Matrix Ineqaulities

선형행렬 부등식을 이용한 불확실한 선형시스템에 대한 강인 추적제어기

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요 약 : 본 논문에서는 상태행렬과 입력행렬에 시변 불확실성이 있는 선형시스템에 대한 강인 추적제어기를 제안한다. 본 논문에서 대상으로 하는 불확실성은 block-diagonally structured uncertainty와 norm bounded uncertainty인데 모두 정합조건을 만족시킬 필요는 없다. 폐루프 시스템이 불확실성하에서 안정할 수 있는 조건을 제시하고 이 조건이 선형행렬 부등식으로 나타낼 수 있음을 보인다. 추적 오차를 줄이고 오차 감소 비율을 증가시킬 수 있는 최적화 방법도 제안한다. 또한 불확실성의 크기가 0으로 줄어들면 추적 오차도 0으로 줄어듬을 보인다.

Keywords: time-varying uncertainties, tracking, robust control

I. Introduction

There are a lot of results on designing robust stabilizing controllers, which stabilize linear systems with parametric uncertainties [6][8][9][10]. But there are only a few results on designing robust tracking controllers, which make the parametric uncertain systems track the reference signal and simultaneously guarantee the closed loop stability in the presence of all allowable uncertainties. Parametric uncertainties may be classified into two cases. One is the case that the uncertain system should satisfy the matching condition, and the other is the case that the uncertain system need not satisfy the matching condition. In the robust stabilization problems, a lot of results on the uncertain systems without the matching condition have been developed [6][8][9][10]. In the robust tracking problems, however, the results only on the uncertain system with the matching condition have been developed [4][11][12]. Hence, we will consider a robust tracking control problem for the uncertain systems without the matching condition. In this paper, we consider the block-diagonally structured, norm-bounded, and time-varying uncertainties, which need not satisfy the matching condition. These types of uncertainties have been exploited in the recent researches on the robust stabilization problems. We propose conditions such that the tracking error is ultimately bounded and the closed loop stability is guaranteed for all allowable uncertainties. In the proposed conditions, there are some free parameters to allow flexibility in determining bound and decaying rate of the tracking error. We show that the proposed conditions can be expressed as Linear Matrix Inequalities (LMI) for which a lot of efficient algorithms have been developed. We also show that the bound of the tracking error can be decreased or the error decaying rate can be increased by solving a kind of LMI problem i.e., a Generalized Eigenvalue Problem (GEVP). This optimization problem can be solved via a lot of developed algorithms including the interior-point methods [1][7]. We also show that the tracking error exponentially decays to 0, when the norm bounds of uncertainties exponentially decay. In Section II, we define an error dynamic system for the uncertain system and formulate the robust tracking problem. In Section III, we present main results including robust tracking conditions for block-diagonally structured uncertain systems and some relevant LMI optimization problems. Finally, conclusions are given in Section IV.

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II. Problem statements

We consider the following linear time-invariant system with time-varying uncertainties :

$$\begin{aligned} \dot{x}(t) &= (A+\Delta A(t))x(t)+(B+\Delta B(t))u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ the control and $y \in R^l$ the output. $A \in R^{n\times n}, B \in R^{n\times m}$ and $C \in R^{l\times n}$ are constant matrices. $\Delta A(t) \in R^{n\times n}$ and $\Delta B(t) \in R^{n\times m}$ are time-varying uncertainties in the state matrix and the input matrix, respectively. The uncertainty pair $(\Delta A(t), \Delta B(t))$ belong to the prescribed sets that will be specified later. In this paper, we are interested in the state feedback and feedforward controller which has the following form :

$$u(t) = F_1 x(t) + F_2 y_*, \tag{2}$$

where F_1 is a constant feedback gain, F_2 is a constant feedforward gain and $y_* \in R^l$ is a given constant reference signal. The objective is to construct a robust tracking controller (2) which makes the tracking error $|y-y_*|$ bounded and simultaneously guarantees the quadratic stability of the closed loop system for all uncertainties which belong to the prescribed sets.

Now, we modify the uncertain system (1) in order to construct a tracking controller based on the idea in [3]. Assume that $l \le m$ and $rank\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n+l$.

Let an equilibrium state x_* and a control u_* satisfy

$$\begin{aligned} 0 &= Ax_* + Bu_* \\ y_* &= Cx_*, \end{aligned}$$

and the solution be of the form

$$\begin{bmatrix} x_* \\ u_* \end{bmatrix} = \Phi \begin{bmatrix} 0 \\ y_* \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} 0 \\ y_* \end{bmatrix}. \tag{3}$$

If $l < m$, then one solution may be obtained by

$$\Phi = \Sigma^T (\Sigma \Sigma^T)^{-1}, \tag{4}$$

where

$$\Sigma = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \tag{5}$$

and if $l=m$, then $\Phi = \Sigma^{-1}$. Now let us define new variables as

$$\begin{aligned} \tilde{x} &= x - x_*, \\ \tilde{u} &= u - u_*, \\ \tilde{y} &= y - y_*. \end{aligned}$$

Then the system (1) may be rewritten as

$$\begin{aligned} \dot{\tilde{x}}(t) &= [A + \Delta A(t)]\tilde{x}(t) + [B + \Delta B(t)]\tilde{u}(t) + w(t) \\ \tilde{y}(t) &= C\tilde{x}(t) \end{aligned} \tag{6}$$

where

$$w(t) = \Delta A(t) x_* + \Delta B(t) u_*. \tag{7}$$

The output of the system (6) is the tracking error and $w(t)$ can be regarded as time-varying disturbances generated by time-varying uncertainties and tracking commands. Now the tracking problem becomes the problem of finding a state feedback controller $\tilde{u} = F\tilde{x}$ to bound $\tilde{y}$ in the presence of allowable uncertainties and the disturbance corresponding to the uncertainties. If we find such a feedback gain F, then the robust tracking controller will be

$$\begin{aligned} u &= F(x - x_*) + u_* = Fx + [-F \;\; I_m]\begin{bmatrix} x_* \\ u_* \end{bmatrix} \\ &= Fx + [-F \;\; I_m]\,\Phi \begin{bmatrix} 0 \\ I_l \end{bmatrix} y_* \end{aligned} \tag{8}$$

Following the notation in (2), the feedback gain and the feedforward gain will be as follows :

$$\begin{aligned} &F_1\,(feedbackgain) = F \\ &F_2\,(feedforwardgain) = [-F \;\; I_m]\,\Phi \begin{bmatrix} 0 \\ I_l \end{bmatrix}. \end{aligned}$$

III. Main results

In this section, we propose conditions under which the tracking error is bounded and the quadratic stability of the closed loop system is guaranteed for all uncertainties which belong to the prescribed sets. We consider a block-diagonally structured, norm-bounded, and time-varying uncertainty set. We show that the proposed conditions can be expressed as LMIs. We also show that the bound of tracking error can be minimized or error decaying rate can be maximized via solving the GEVP subject to the LMIs, respectively. Before stating the main results, we introduce some notations and the definition of quadratic stability which is often used in the robust stabilization problems. $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ denote the minimum and maximum eigenvalue of the matrix X, respectively. $\|X\|$ denotes the 2-norm of the matrix X. $|x|$ denotes the Euclidian norm of the vector x. A matrix inequality $X > Y$ denotes that $X - Y$ is positive definite.

Definition 1 [6] : The uncertain system (1) is said to be quadratically stable with a linear state feedback control $u(t) = Fx(t)$, if there exist a positive definite symmetric matrix P and a constant $\varepsilon > 0$ such that, for any allowable uncertainty pair $(\Delta A(t), \Delta B(t))$, the derivative of the Lyapunov functional $V(x(t)) = x(t)^T P x(t)$ satisfies

$$\dot{V} \le -\varepsilon |x(t)|^2. \tag{9}$$

The following lemma will be used to prove the guaranteed bound of the tracking error under the robust tracking conditions that will be proposed in the following theorem.

Lemma 1 : Let a functional be $V(t) = z^T(t) P z(t)$. If there exist $a(>0), b \in R$ such that

$$\dot{V}(t) \le -aV(t) + b, \quad \forall \;\; t \ge t_0 \tag{10}$$

then

$$|z(t)| \le \begin{cases} \rho^{\frac{1}{2}} & \text{for } V(t_0) \le \frac{b}{a} \\ (\rho + (V(t_0) - \rho)e^{-a(t-t_0)})^{\frac{1}{2}} & \text{for } V(t_0) > \frac{b}{a} \end{cases} \tag{11}$$

where

$$\rho = (\lambda^{-1}_{\min}(P)\frac{b}{a}). \tag{12}$$

Proof : (10) implies

$$V(t) \le \frac{b}{a} + (V(t_0) - \frac{b}{a})e^{-a(t-t_0)}. \tag{13}$$

Since

$$\lambda_{\min}(P)|z(t)|^2 \le z^{T(t)} P z(t), \tag{14}$$

$$|z(t)|^2 \le \lambda^{-1}{}_{\min}(P)[\frac{b}{a}+(V(t_0)-\frac{b}{a})e^{-a(t-t_0)}] = (\rho+(V(t_0)-\rho)e^{-a(t-t_0)}).$$

This completes the proof. ■

Now, we consider the robust tracking problem for the block-diagonally structured uncertainties. Let us consider the following uncertainty set :

$$\Delta A(t) = D_A\Psi(t)E_A = \sum_{i=1}^{r} D_{A_i}\Psi_{i(t)}E_{A_i}$$
$$\Delta B(t) = D_B\Theta(t)E_B = \sum_{i=1}^{s} D_{B_i}\Theta_{i(t)}E_{B_i},$$

and the uncertainty set is given by

$$U_1 = \{(\Delta A(t),\Delta B(t)): \|\Psi_i(t)\|\le\gamma_{a_i}, \|\Theta_j(t)\|\le\gamma_{b_j}, \ \forall\ t\ge t_0,\ i=1,2,\cdots,r \text{ and } j=1,2,\cdots,s\}, \quad (15)$$

where Ψ and Θ are norm bounded uncertainties with the following structures :

$$\Psi=\begin{bmatrix}\Psi_1 & 0 & \cdots & 0\\ 0 & \Psi_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \Psi_r\end{bmatrix},\quad \Theta=\begin{bmatrix}\Theta_1 & 0 & \cdots & 0\\ 0 & \Theta_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \Theta_s\end{bmatrix}, \quad (16)$$

where $D_A, D_B, E_A,$ and E_B are known matrices with appropriate dimensions. As shown in (15), the uncertainty pair in U_1 need not satisfy the matching condition. Without loss of generality, we assume that $\gamma_{a_i}=\gamma_{b_j}=1$. Before we state the next theorem, let us introduce the following notations :

$$\widehat{D_A}=\sum_{i=1}^{r}\mu_i^{-1}D_{A_i}D_{A_i}^T,\ \widehat{E_A}=\sum_{i=1}^{r}\mu_i E_{A_i}^T E_{A_i}, \quad (17)$$

$$\widehat{D_B}=\sum_{i=1}^{s}\nu_i^{-1}D_{B_i}D_{B_i}^T,\ \widehat{E_B}=\sum_{i=1}^{s}\nu_i E_{B_i}^T E_{B_i}, \quad (18)$$

where μ_i's and ν_i's are some positive numbers.

Now we propose a robust tracking condition for the system (1) with the uncertainties (15) using the control with the type (2).

Theorem 1 : Assume that there exist a matrix F, a positive definite matrix P and a positive number $\varepsilon_1>0$ which satisfy the following inequality:

$$(A+BF)^TP+P(A+BF)+\varepsilon_1P+2P(\widehat{D_A}+\widehat{D_B})P+\widehat{E_A}+F^T\widehat{E_B}F<0, \quad (19)$$

for some positive μ_i's and ν_i's. Then with the state feedback controller $\tilde{u}=F\tilde{x}$, the tracking error is bounded as follows :

$$|\tilde{y}(t)|\le\begin{cases}\delta & \text{for } \delta_0\le\frac{M}{\varepsilon_1}\\ (\delta^2+(\delta_0-\delta^2)e^{-\varepsilon_1(t-t_0)})^{\frac{1}{2}} & \text{for } \delta_0>\frac{M}{\varepsilon_1}\end{cases} \quad (20)$$

where

$$\delta = (\lambda^{-1}{}_{\min}(P)\frac{M}{\varepsilon_1})^{\frac{1}{2}}$$
$$\delta_0 = \tilde{x}(t_0)^TP\tilde{x}(t_0)$$
$$M = (\|\widehat{E_A}\|\|\Phi_{12}\|^2+\|\widehat{E_B}\|\|\Phi_{22}\|^2)|y_*|^2.$$

Proof : Define a Lyapunov functional $V(\tilde{x}(t))$ as follows :

$$V(\tilde{x}(t)) := \tilde{x}^T(t)P\tilde{x}(t). \quad (21)$$

Using the notation $\widehat{A}=A+BF$ and, $\widehat{\Delta}=\Delta A+\Delta BF$ the corresponding Lyapunov derivative is given by

$$\dot{V}(\tilde{x}(t)) = \tilde{x}^T[(\widehat{A}+\widehat{\Delta})^TP+P(\widehat{A}+\widehat{\Delta})]\tilde{x} + w^TP\tilde{x}+\tilde{x}^TPw \quad (22)$$

Using the fact that

$$X^TY+Y^TX\le\beta X^TX+\frac{1}{\beta}Y^TY, \quad (23)$$

for any matrices X and Y and for any $\beta>0$, we obtain

$$(\widehat{A}+\widehat{\Delta})^TP+P(\widehat{A}+\widehat{\Delta})\le A^TP+PA+F^TB^TP+PBF + \sum_i^r\mu_iE_{A_i}^TE_{A_i}+P(\sum_i^r\mu_i^{-1}D_{A_i}D_{A_i}^T)P + F^T(\sum_i^s\nu_iE_{B_i}^TE_{B_i})F+P(\sum_i^s\nu_i^{-1}D_{B_i}D_{B_i}^T)P,$$

and

$$w^TP\tilde{x}+\tilde{x}^TPw=(\sum_i^rD_{A_i}\Psi_iE_{A_i}x_*+\sum_i^sD_{B_i}\Theta_iE_{B_i}u_*)^TP\tilde{x} + \tilde{x}^TP(\sum_i^rD_{A_i}\Psi_iE_{A_i}x_*+\sum_i^sD_{B_i}\Theta_iE_{B_i}u_*)$$
$$\le|x_*|^2\|\sum_i^r\mu_iE_{A_i}^TE_{A_i}\|+\tilde{x}^TP(\sum_i^r\mu_i^{-1}D_{A_i}D_{A_i}^T)P\tilde{x} + |u_*|^2\|\sum_i^s\nu_iE_{B_i}^TE_{B_i}\|+\tilde{x}^TP(\sum_i^s\nu_i^{-1}D_{B_i}D_{B_i}^T)P\tilde{x}.$$

Hence (22) becomes,

$$\dot{V}\le\tilde{x}^T[(A+BF)^TP+P(A+BF)+2P(\widehat{D_A}+\widehat{D_B})P + \widehat{E_A}+F^T\widehat{E_B}F]\tilde{x}+|x_*|^2\|\widehat{E_A}\|+|u_*|^2\|\widehat{E_B}\|.$$

Then, if (19) holds,

$$\dot{V}\le-\varepsilon_1\tilde{x}^TP\tilde{x}+|x_*|^2\|\widehat{E_A}\|+|u_*|^2\|\widehat{E_B}\|. \quad (24)$$

Lemma 1 and (24) imply (20). ■

From the solutions which satisfy the condition in Theorem 1, we can obtain the robust tracking controller with the type (2) using the relation (8).

In [11], a method to guarantee the asymptotic tracking property for uncertain systems was proposed, while a method to guarantee the tracking error bound for uncertain systmes is proposed in Theorem 1[11]. employed a dynamic feedback control in which on-line computation is needed, and assumed time-invariant uncertainties which should satisfy the matching condition. On the other hand, in this paper, we employ a constant feedback and constant feedforward control in which only off-line computation is needed, and assumed time-varying uncertainties which need not satisfy the matching condition.

If we use the control gain F satisfying the inequality condition (19), the quadratic stability of the closed loop system with the uncertainties (15) will be guaranteed. To investigate the closed loop stability of the robust tracking controller proposed in Theorem 1, we have only to consider the state feedback closed loop system :

$$\dot{x}(t)=(A+BF+\Delta A+\Delta BF)x(t). \tag{25}$$

Theorem 2 : Assume the hypotheses of Theorem 1. Then the closed loop system (25) is quadratically stable with the state feedback controller $u(t)=Fx(t)$ for all uncertainties $(\Delta A(t),\Delta B(t))\in U_1$.

Proof : Define a Lyapunov functional as :

$$L(x(t)):=x^T(t)Px(t). \tag{26}$$

Then

$$\begin{aligned}\dot{L} \le\ & x^T(t)[(A+BF)^TP+P(A+BF)+\widehat{E_A} \\ & +F^T\widehat{E_B}F+2P(\widehat{D_A}+\text{at}\,D_B)P]x(t) \\ \le\ & -\varepsilon_1 x^T(t)Px(t) \\ \le\ & -\varepsilon_1\lambda_{\min}(P)|x(t)|^2.\end{aligned} \tag{27}$$

Hence the closed loop system (25) is quadratically stable with the state feedback controller $u(t)=Fx(t)$. ■

The following corollary shows that the tracking error with the control in Theorem 1 decays to zero in the steady state if there are no uncertainties.

Corollary 1 : Assume that there exist a matrix F, a positive definite matrix P and a positive number $\varepsilon_1>0$ which satisfy the following inequality:

$$(A+BF)'P+P(A+BF)+\varepsilon_1 P<0.$$

Then it holds that

$$|\tilde{y}(t)|\le\tilde{x}(t_0)^TP\tilde{x}(t_0)e^{-\varepsilon_1(t-t_0)}\ \ \forall\ t\ge t_0 \tag{28}$$

when $\Delta A=0$ and $\Delta B=0$.

Proof : In this case, $M=0$. Hence the proof is trivial from Lemma 1. ■

Now, we show that the robust tracking condition (19) is equivalent to an LMI. In Theorem 1, the design parameter ε_1 determines the decaying rate of the tracking error, and δ means the guaranteed error bound. If we prescribe ε_1 to guarantee error decaying rate, we can convert the condition (19) to the following LMI problem :

$$\Omega_1(X,Y,\varepsilon_1)<0$$

where

$$\Omega_1(X,Y,\varepsilon_1)=\begin{bmatrix}X_1 & X & Y\\ X & Z_1 & 0\\ Y & 0 & Z_2\end{bmatrix}, \tag{29}$$

$$X_1=AX+XA^T+\varepsilon_1X+BY+Y^TB^T+2(\widehat{D_A}+\widehat{D_B}),$$

$X=P^{-1}$, $Y=FP^{-1}$, $Z_1=-\widehat{E_A}^{-1}$, and $Z_2=-\widehat{E_B}^{-1}$.

If there exist X and Y satisfying the LMI problem (29), then the robust tracking controller becomes

$$u(t)=YX^{-1}x(t)+[-YX^{-1}\ \ I_m]\,\Phi\begin{bmatrix}0\\ I_l\end{bmatrix}y_*.$$

Now we face two kinds of optimization problems. One is to minimize the error bound and the other is to maximize the error decaying rate ε_1. In Theorem 1, if we find a solution to minimize $\lambda_{\min}{}^{-1}(P)$, i.e. $\lambda_{\max}(P^{-1})$, we can minimize the error bound δ. Similarly, if we find a solution to maximize ε_1, we can maximize the error decaying rate. The solutions of both problems can be found by solving the following GEVPs subject to the corresponding LMIs. The tracking error bound can be decreased by the following problem :

$$\underset{X,\,Y}{\text{Minimize}}\ \lambda_{\max}(X) \tag{30}$$

subject to

$$\Omega_1(X,Y,\varepsilon_1)<0, \tag{31}$$

where ε_1 is a given positive number.

The error decaying rate can be increased by the following problem :

$$\underset{X,\,Y}{\text{Maximize}}\ \varepsilon_1 \tag{32}$$

subject to

$$\Omega_1(X,Y,\varepsilon_1)<0$$

These types of optimization problems belong to GEVPs. Recently, many algorithms including interior-point methods and ellipsoid algorithms for GEVPs have been developed. For details, see [1][2][7] and references therein. Now, we comment on the offset-free tracking property of the proposed controller for the uncertainties which decay exponentially. Let us consider the following unstructured uncertainty set :

$$\begin{aligned}U_2=\ &\{(\Delta A(t),\Delta B(t)):\|\Psi_i(t)\|\le e^{-\gamma_a t},\|\Theta_{j(t)}\|\le e^{-\gamma_b t},\\ &\forall\ t\ge t_0,\ i=1,2,\cdots,r\ \text{and}\ j=1,2,\cdots,s\},\end{aligned} \tag{33}$$

where γ_a and γ_b may be known or not. We observe that $U_2\subset U_1$ with $\gamma_{a_i}=\gamma_{b_i}=1$. The following lemma will be used to show the offset-free tracking property for the proposed controller under the above uncertainties.

Lemma 2 : Under the hypothesis of Lemma 1, suppose that b is time varying and exponentially decaying, i.e. $\dot{V}(t)\le -aV(t)+b(t)$ and $b(t)\le k_1e^{-k_2t}$, then

$$|z(t)|\le\begin{cases}\rho^{\frac{1}{2}}e^{-k_2t} \\ \quad\text{for } a>k_2 \text{ and } V(t_0)\le\dfrac{k_1}{a-k_2}e^{-k_2t_0} \\ (\rho e^{-k_2t}+(V(t_0)-\rho e^{-k_2t})e^{-a(t-t_0)})^{\frac{1}{2}} \\ \quad\text{for } a>k_2 \text{ and } V(t_0)>\dfrac{k_1}{a-k_2}e^{-k_2t_0} \\ (\lambda^{-1}{}_{\min}(P)V(t_0)e^{-a(t-t_0)}+k_1(t-t_0)e^{-at})^{\frac{1}{2}}\ \text{for } a=k_2 \\ (V(t_0)-\rho e^{-k_2t})e^{-\frac{a}{2}(t-t_0)}\ \text{for } a<k_2\end{cases} \tag{34}$$

where

$$\rho=(\lambda_{\min}{}^{-1}(P)\frac{b}{a-k_2}).$$

Proof : In this case, $V(t)$ will be

$$V(t)\le V(t_0)e^{-a(t-t_0)}+\int_{t_0}^{t}b(\tau)e^{-a(t-\tau)}d\tau \tag{35}$$

The rest of the proof is similar to that of Lemma 1.■

In the above lemma, we observe that the state $z(t)$ exponentially decays, which will be used to show that the tracking error also exponentially decays in the following corollary.

Corollary 2 : Assume the hypotheses of Theorem 1. Then with the controller $\tilde{u}=F\tilde{x}$, the tracking error $\tilde{y}(t)$ exponentially decays to 0 as t goes to ∞.

Proof : Using the inequalities in the proof of Theorem 1, we obtain :

$$\dot{V}\leq -\varepsilon_1 \tilde{x}^T P \tilde{x} + |x_*|^2 \| \widehat{E_A} \| e^{-\gamma_a t} + |u_*|^2 \| \widehat{E_B} \| e^{-\gamma_b t}$$

Hence, from Lemma 2, we conclude that the tracking error exponentially decays to 0. ■

IV. Conclusion

In this paper, we consider the robust tracking control problem for parametric uncertain systems which need not satisfy the matching condition. It is noted that that uncertain systems must satisfy the matching conditions in the existing robust tracking methods for parametric uncertain systems. We proposed the robust tracking controllers for linear systems with block-diagonally structured, norm-bounded, and time-varying uncertainties in both the state and the input matrix. These types of uncertainties have been widely investigated in the robust stabilization methods. Even though the proposed robust tracking controller adopts constant feedback gain and constant feedforward gain, it guarantees the tracking error bound and the quadratic stability of the closed loop for the above time-varying uncertainties. If the norm bounds of uncertainties exponentially decay, the proposed controller guarantees the offset-free tracking property. We showed that the proposed tracking controller can be obtained by solving LMIs. In the proposed controller, there are some free parameters which can be utilized in determining the bound and the decaying rate of the tracking error. By involving GEVPs which can be solved by recently developed algorithms, we showed that the error bound can be minimized or the error decaying rate can be maximized.

A subject of future researches would be to extend the proposed state feedback controllers to the output feedback controllers which would be easily extended using the method in [5]. Another subject would be to design a robust tracking controller for the cases of time-varying reference signals and model-following problems. It is expected that a slight modification of the methods proposed in this paper will make it possible to guarantee the tracking error bound and the closed loop stability for both cases.

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