〈論 文〉 3-4-9

전류모드 제어의 소신호 모델링

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Small Signal Modeling of Current Mode Control

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ABSTRACT

The mathematical interpretation of a practical sampler which is useful to obtain the small signal models for the peak and average current mode controls is proposed. Due to the difficulties in applying the Shannons sampling theorem to the analysis of sampling effects embedded in the current mode control, several different approaches have been reported. However, these approaches require the information of the inductor current in a discrete expression, which restricts the application of the reported method only to the peak current mode control. In this paper, the mathematical expressions of sampling effects on a current loop which can directly apply the Shannons sampling theorem are newly proposed, and applied to the modeling of the peak current mode control. By the newly derived models of a practical sampler, the models in a discrete time domain and a continuous time domain are obtained. It is expected that the derived models are useful for the control loop design of power supplies. The effectiveness of the derived models are verified through the simulation and experimental results.

Key Words: Peak current mode control, Buck converter, Sampling effect

1. Introduction

The current mode control has been quite popular in recent years, and has been the subject of extensive researches. Although the current mode control has several advantages such as built-in overload protection and easy load-sharing of multiple converters,[1][2] it also possesses the problem of a current loop instability. To characterize the current loop instability problem, several models have been reported such as discrete time and sampled data ones.[3][4] These models are useful for predicting the behaviors of the current mode controlled converter. It is, however, difficult to obtain the design insights of this converter from the reported models.

To overcome these problems, the continuous time models considering the sampling effects on a current loop have been presented to modify the low frequency models.[5][6][7] By using these approaches, the continuous time small signal models can be more accurate compared to the conventional low frequency model. To modify the high frequency behavior of the low frequency models, the practical sampler was introduced in[6]. This practical sampler was used to sample by using a series of pulses, not impulses. This property of a practical sampler makes it difficult to apply the Shannons sampling theorem to obtain the continuous time model for the current mode control. To solve this problem, the previous works are mainly dependent on the discrete expressions of a current loop. This dependence on the discrete expressions makes it difficult to apply these methods to the modeling of more complicated control such as an average current mode control.

Therefore, the mathematical interpretation of a practical sampler is proposed in this paper, which can be easily obtained and does not depend on the discrete expressions of a current loop. This makes it possible to

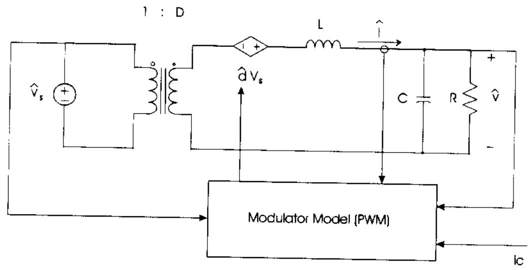


그림 1 벅 컨버터의 선형화 모델
Fig. 1 Linear circuit of a buck converter

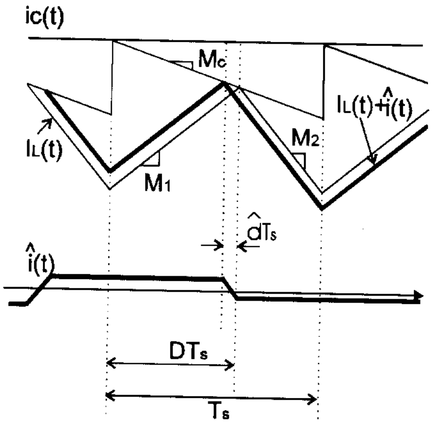


그림 2 전류모드제어의 기본파형
Fig. 2 Basic waveforms of a current-mode control

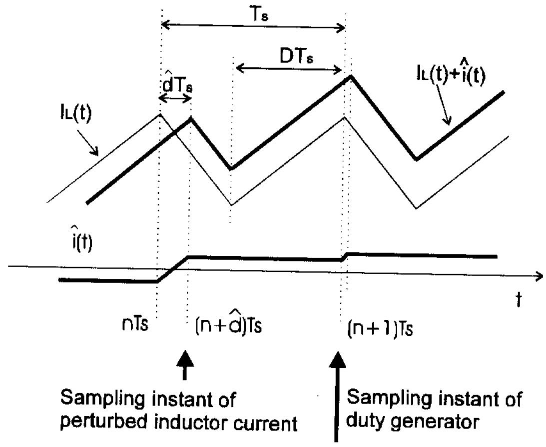


그림 3 인덕터 전류의 파형
Fig. 3 Waveforms of a inductor current

apply the proposed modeling method to a more complicated control. By the proposed modeling method, it is easy to obtain the models for a peak current mode control and an average current mode control. The model of a practical sampler is treated as two ideal samplers operated on the perturbed current and duty cycle generator with different sampling instant. And two different sampling instants are unified under the equivalent condition of the perturbed current. By the mathematical interpretation of a practical sampler, the small signal model of a current mode control can be easily obtained only with simple mathematical manipulation. In this paper, the derivation of the mathematical model of a practical sampler is focused on a peak current mode control. To show the validity of the proposed approach, the continuous time and discrete time models of a peak current mode controlled buck converter are derived and compared with the experiment results.

2. Power Stage Model

The model of a current mode control can be considered as the combination of a power stage model and a modulator model. Because the continuous time small signal model is especially very useful for the control loop design of power supplies, the averaging method is generally applied to the modeling of power converter. By applying the averaging method to the power converter, the conventional low frequency model can be obtained.[8] The low frequency model of a buck converter used for the power stage model of this paper is shown as

$$\begin{pmatrix}\frac{d}{dt}\hat{i}\\ \frac{d}{dt}\hat{v}\end{pmatrix} = \begin{pmatrix}0 & -\frac{1}{L}\\ \frac{1}{C} & -\frac{1}{RC}\end{pmatrix}\begin{pmatrix}\hat{i}\\ \hat{v}\end{pmatrix} + \hat{d}\begin{pmatrix}\frac{V_s}{L}\\ 0\end{pmatrix} + \hat{v}_s\begin{pmatrix}\frac{D}{L}\\ 0\end{pmatrix} \qquad (1)$$

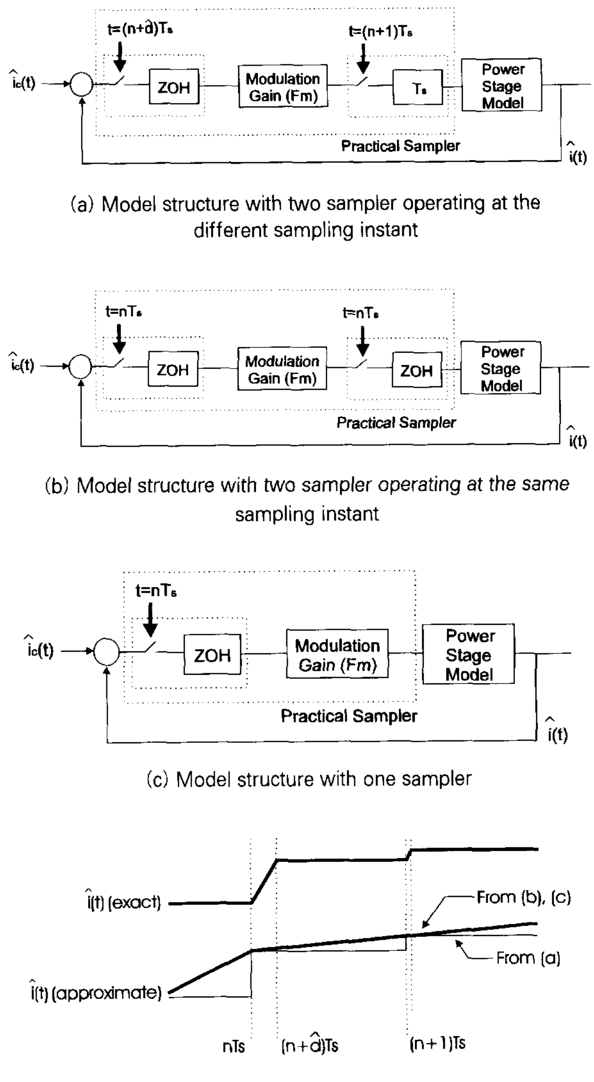
where $\hat{i}$ and $\hat{v}$ are a perturbed inductor current and a perturbed capacitor voltage, respectively. The linear equivalent circuit of a buck converter employing the modulator model is shown in Fig. 1.

3. Modulator Model

3.1 Basic Structure of Practical Sampler

Fig. 2 shows the waveforms of a peak current mode control modulator. As shown in this figure, the inductor current $i(t)$ consists of a steady-state current $I_L(t)$ and a small perturbed current $\hat{i}(t)$. It is noticed that the sampling effects is included in the waveform of a small perturbed current. As noted in[6], the response of a perturbed current $\hat{i}(t)$ can be considered as that of the low-frequency model connected in series with a practical sampler. The practical sampler is used to indicate that the sampling is done by a series of pulses, not impulses. This makes it difficult to use the Shannons sampling theorem. However, this difficulty can be overcome by developing the mathematical expression for a practical sampler as presented in this paper.

To obtain the mathematical expressions of the practical sampler in a peak current mode control, the expanded view of the inductor current is considered as shown in Fig. 3. As can be noticed in this figure, there are two slopes in the perturbed inductor current. One is kept zero over the interval between $(n+\hat{a})T_s$ Ts and $(n+1)T_s$, which shows the possibility of existence of an ideal sampler with zero order holder in a current loop. The other slope is a positive or negative one which determines the magnitude variation of a perturbed inductor current. The perturbed current caused by this slope can be rebuilt with the combination of the low frequency model of a power converter and the duty cycle modulator which contains an ideal sampler and a modulator gain. Therefore, the models of a practical sampler can be expressed with two ideal samplers operated at the instants of $(n+\hat{a})T_s$ Ts and $(n+1)T_s$, respectively. Considering the difference between two sampling instants, the model structure of a peak current mode control can be drawn as shown in Fig. 4(a). The response of a perturbed inductor current of this model structure is shown in Fig. 4(d). Fig. 4(a) shows that with different sampling instants, one of two samplers is used for the perturbed inductor current and the other is for the duty cycle generator. As can be well understood, since two samplers have the different sampling instants, the development of models for a peak current mode control is difficult. To unify the sampling instants of two samplers, the model structure is modified by adding another zero order holder as illustrated in Fig. 4(b). This modification is come from



(a) Model structure with two sampler operating at the different sampling instant

(b) Model structure with two sampler operating at the same sampling instant

(c) Model structure with one sampler

(d) Approximated perturbed current for each model structure

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Fig. 4 Model structure of a current-mode control

the equivalent condition of the perturbed current at the sampling instant, nTs. At the time of sampling instant, the perturbed inductor current response of the model shown in Fig. 4(a) is the same as that of Fig. 4(b). Since the modulator gain between two samplers is constant, two samplers can be reduced to one without affecting the results as shown in Fig. 4(c). Fig. 4(d) shows the relationship between the perturbed current and the sampling instant based on the derived models of Fig. 4.

3.2 Derivation of Discrete Time Model

The response of a peak current mode controlled converter is accurately predicted by the exact discrete-time and sampled-data models. Although the design insights can not be provided, the discrete time model is useful for understanding the behaviors of the converter. In this section, the derivation of a discrete time model of a peak current mode controlled converter from the proposed model structure is presented. The remaining works in deriving the discrete time model are only the mathematical manipulation of the proposed model structure of a peak current mode controlled converter. As an example, the buck converter is considered as a power stage model. The averaged model of a buck converter is expressed in (1). The gain of inductor current to duty cycle can be expressed as

$$F_i(s) = \frac{\hat{i}(s)}{\hat{d}(s)} = \frac{V_s}{Ls} = \frac{M_1 + M_2}{s} \tag{2}$$

where M_1 and M_2 are the on-time and off-time slopes of the inductor current, respectively. The expanded view of a peak current mode modulator is shown in Fig. 5. Assuming that the on-time slope, M_1, is constant over a switching period, the modulator gain of the circuit becomes

$$F_m = \frac{\hat{d}}{\hat{i}_c - \hat{i}} = \frac{1}{(M_1 + M_c)T_s} \tag{3}$$

where Mc is the slope of an external ramp. With the transfer function of a zero order holder as

$$H_{zoh}(s) = \frac{1 - e^{-sT_s}}{s} \tag{4}$$

the discrete-time domain expression of a combined gain of Fi(s) and $H_{zoh}(s)$ is derived as follows:

$$G(z) = Z(H_{zoh}(s)F_i(s)) = \frac{(M_1 + M_2)T_s}{z - 1} \tag{5}$$

From (5), the discrete time model of a current loop transfer function is obtained as follows:

$$T_i(z) = \frac{i(z)}{\hat{i}_c(z)} = \frac{a}{z - 1 + a} \tag{6}$$

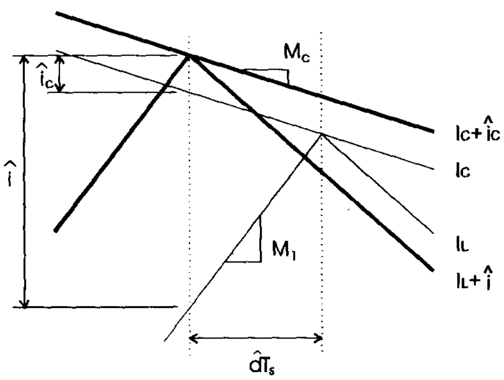


그림 5 변조 이득을 위한 전류 파형
Fig. 5 Modulator waveforms

where

$$a = \frac{M_1 + M_2}{M_1 + M_C}. \tag{7}$$

It can be seen that the derived discrete time model is the same as the model presented in previous works which explain the subharmonic oscillation phenomena in the current responses.[3][4][5] As shown in (6), the subharmonic oscillation condition of the inductor current is determined by the value of a. In case of $M_1 < M_2$ and of no applied external ramp, the value of a is greater than two, which results in the instability of an inductor, current. Therefore, the system instability at duty cycles greater than 0.5 without the external ramp can be explained using the derived discrete time model.

3.3 Derivation of Continuous Time Small Signal Model

As noted previously, the continuous time small signal model is especially useful for the control loop design of a power converter. Thus, this model including the sampling effects is derived from the proposed model structure. The gain, $T(s)$, including the zero order holder, modulation gain, and power stage model can be expressed as follows:

$$T(s) = F_m \frac{1 - e^{sT_s}}{s} \frac{V_s}{Ls}. \tag{8}$$

The perturbed inductor current, $\hat{i}(s)$, can be expressed from Fig. 4(c) as follows:

$$\hat{i}(s) = T(s)\left(\hat{i}_c^*(s) - \hat{i}^*(s)\right) \tag{9}$$

where * denotes the sampled quantity. By considering the sampling effects on the inductor current in (9), the following equation can be obtained as

$$\hat{i}(s) = T^*(s)\left(\hat{i}_c^*(s) - \hat{i}^*(s)\right) \tag{10}$$

where $T^*(s)$ is expressed as

$$T^*(s) = F_m G(z)\Big|_{z=e^{sT_s}} = \frac{a}{e^{sT_s} - 1}, \tag{11}$$

Combining (9) and (10), the equation of an inductor current can be expressed as

$$\hat{i}(s) = T(s)\frac{1}{1 + T^*(s)}\hat{i}_c^*(s), \tag{12}$$

With the assumption of slow variation of the reference signal $\hat{i}_c$, the following relation can be obtained as

$$\hat{i}_c(s) \approx \frac{1 - e^{-sT_s}}{s}\hat{i}_c^*(s). \tag{13}$$

The current loop transfer function from a control signal $\hat{i}_c$ to an inductor current $\hat{i}$ can be derived from (12) and (13) as follows:

$$\frac{\hat{i}(s)}{\hat{i}_c(s)} = \frac{s}{1 - e^{-sT_s}}\frac{T(s)}{1 + T^*(s)}. \tag{14}$$

Using the current loop gain including the sampling effects $T_s(s)$, the current loop transfer function can be rewritten as follows:

$$\frac{\hat{i}(s)}{\hat{i}_c(s)} = \frac{T(s)}{1 + T_S(s)}. \tag{15}$$

From the (14) and (15), the current loop gain can be derived as follows:

$$T_S(s) = \frac{\dfrac{s}{1 - e^{-sT_s}}T(s)}{1 + T^*(s) - \dfrac{s}{1 - e^{-sT_s}}T(s)}. \tag{16}$$

Therefore, the final form of an ideal sampler with a zero order holder approximating the sampling effects in a peak current-mode control can be expressed as follows:

$$T_{sh}(s) = \frac{1}{1 + T^*(s) - \dfrac{s}{1 - e^{-sT_s}}T(s)}. \tag{17}$$

Substituting (11) into (17) gives

$$T_{sh}(s) = \frac{1}{\dfrac{2a}{\pi\omega_s}s + \left(1 - \dfrac{a}{2}\right)} \tag{18}$$

with Pade approximation.[6] The Pade approximation is accurate up to half the switching frequency as follows:

$$e^{-sT_s} \cong \frac{1 - \dfrac{\pi}{\omega_s}s + \dfrac{4}{\omega_s^2}s^2}{1 + \dfrac{\pi}{\omega_s}s + \dfrac{4}{\omega_s^2}s^2}. \tag{19}$$

It is seen that the sampling effect can be considered as introducing an additional pole into the current loop in the low frequency model. By the presence of an additional pole, the crossover frequency of a current loop is changed, and the stability of the current response is affected. This result is coincided with that by Tan in.[6] And by rearranging the sampling gain $T_{sh}(s)$ for the feedback signal of a perturbed inductor current as in,[5] the sampling gain can be rewritten as

$$T_e(s) \cong 1 - \frac{T_s}{2}s + \frac{T_s^2}{\pi^2}s^2. \tag{20}$$

This result is also coincided with that by Ridley in[5]. Thus, the mathematical interpretation of a practical sampler is useful for modeling of the current mode control.

4. Simulation and Experimental Results

Fig. 6 shows the circuit diagram of a buck converter employing the peak current-mode control. To show the accuracy of the derived continuous time small signal model, the transient responses of the inductor current

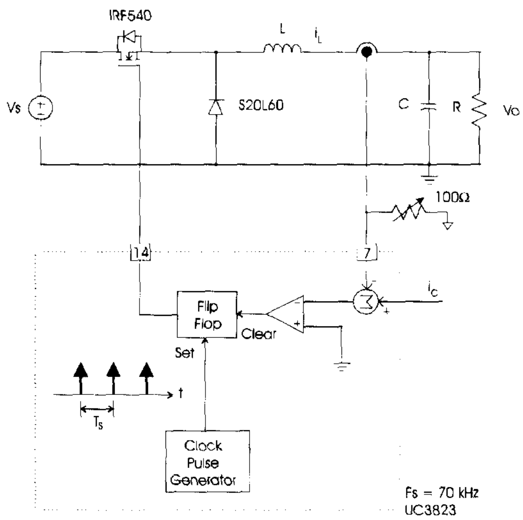
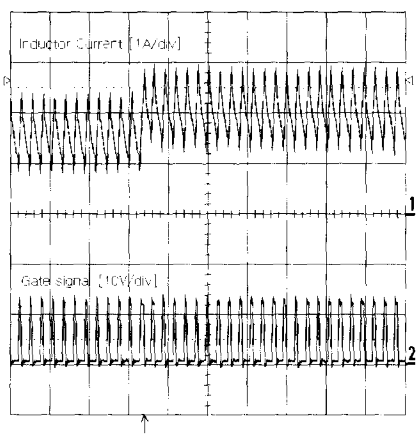


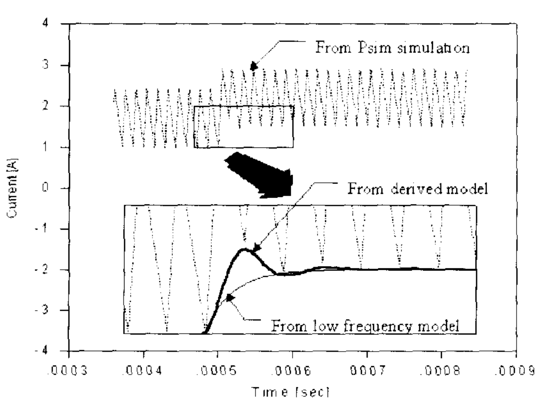
그림 6 전류모드제어의 회로도

Fig. 6 Circuit diagram of a current mode control

are examined under several different operating conditions. Fig. 7 shows the inductor current responses with an operating duty of $D=0.33$ for the step change of a reference signal. The experimental results for an inductor current and a gate signal are shown in Fig. 7(a). To verify the usefulness of the proposed model, simulation results are obtained and compared for the proposed model and the conventional low-frequency model in Fig. 7(b). For the convenient comparison, the results of a circuit level simulation using Psim is also presented in this figure, which has the same waveshape with the experimental results of Fig. 7(a). The low-frequency model reveals the incorrectness compared to the experimental result. The simulation and experimental results of an inductor current response, however, are well agreed with that of a derived continuous time small signal model.



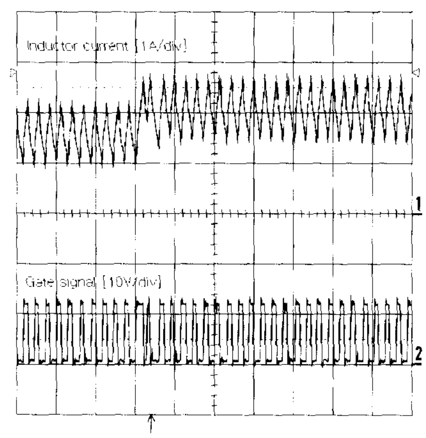
(a) Experimental results



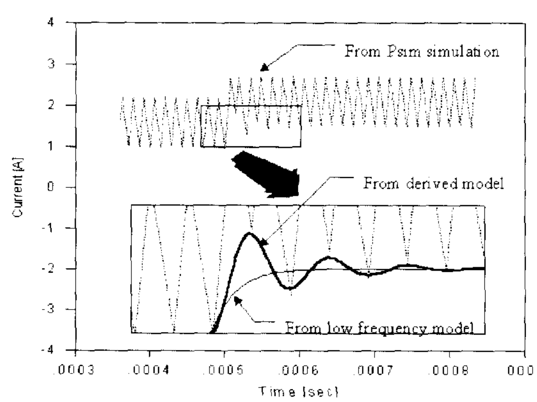
(b) Predicted results

그림 7 인덕터 전류파형(D=0.33)

Fig. 7 Inductor current responses with D=0.33



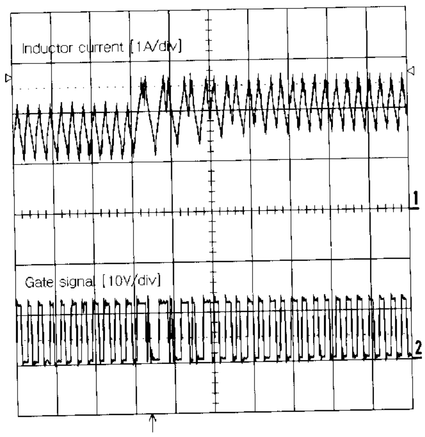
(a) Experimental results



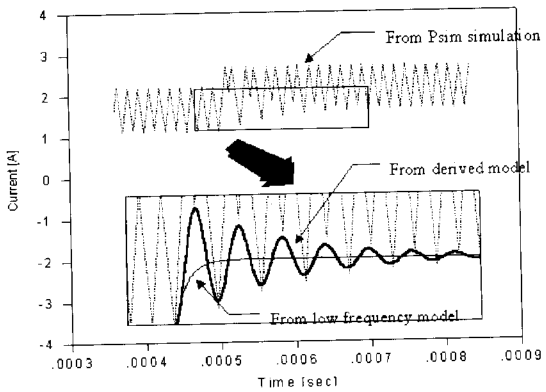
(b) Predicted results

그림 8 인덕터 전류파형(D=0.4)

Fig. 8 Inductor current responses with D=0.4



(a) Experimental results



(b) Predicted results

그림 9 인덕터 전류파형(D=0.46)
Fig. 9 Inductor current responses with D=0.46

In Figs. 8 and 9, the inductor current responses are shown with an operating duty of $D=0.4$ and $D=0.46$, respectively. As can be noticed in these figures, the response of the inductor current becomes oscillatory when the operating duty is increased. This phenomenon can be well predicted by the results of the root locus in Fig. 10, which is obtained from the derived continuous time small signal model. It can be also predicted in this figure that the system becomes unstable with duty cycles greater than 0.5 without the external ramp compensation, which is well illustrated in Fig. 11.

These results show that the derived model is useful for predicting the behaviors of the perturbed inductor current of power supplies employing the peak current mode control, and the model of a practical sampler in a peak current mode control is valid.

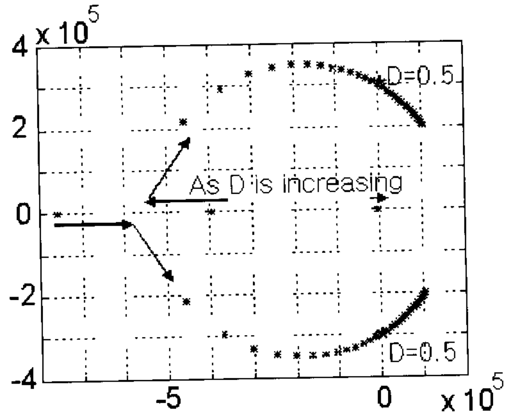


그림 10 전류 루프의 근궤적도
Fig. 10. Root loci of current loop transfer function

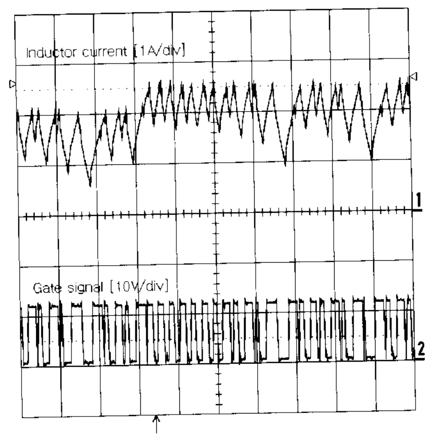


그림 11 인덕터 전류파형(D=0.6)
Fig. 11. Inductor current responses with D=0.6

5. Conclusion

In this paper, the mathematical interpretation of a practical sampler in a current control loop is proposed which is useful to obtain the small signal model for a current mode control. The practical sampler is treated as two ideal samplers which are operated on the perturbed current and the duty cycle generator with different sampling instant. Under the equivalent condition of the perturbed current at the sampling instant, two different sampling instants are unified. This makes it easier to obtain the discrete time and corresponding continuous time small signal models of the current mode control. Simulations and experiments are carried out for the peak current mode controlled buck converter to verify the usefulness of the derived model. Under the variations of the operating duty, the time response of an inductor current of the derived model is compared with that of the conventional low frequency model. From the derived small signal model, the root locus is obtained, which predicts the transient response of the converter system. Therefore, the usefulness of the practical sampler model and the derived model is verified.

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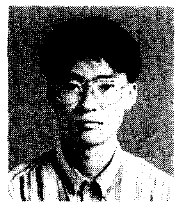

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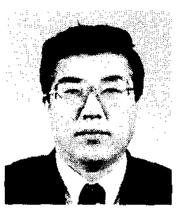

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